Title

Subtitle

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POLITECNICO
MILANO 1863

November 22, 2019

https://github.com/poliprojects/BNPlib

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Non-Parametric statistics

- Goal: density approximation
- **Infinite-dimensional** parameters, e.g. functions

Non-Parametric statistics (the Bayesian way)

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- Model:

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$$\mathcal{P} : \Omega \to M(S) \;\; \text{fixed}$$
$$\left[\; \omega \mapsto G(\cdot) \;\right]$$

- Model name: **BNP model**

Dirichlet Process prior

$$\begin{aligned} y_i | G &\overset{\text{iid}}{\sim} G \\ G &\sim DP(MG_0) \end{aligned}$$

- Parameters: $M > 0$, $G_0 \in M(S)$
- Defining property: $\forall \{B_{1:k}\}$ partition of S,

$$[G(B_1), \ldots, G(B_k)] \sim \mathrm{Dir}\left(MG_0(B_1), \ldots, MG_0(B_k)\right)$$

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- **Discreteness** (stick-breaking): $G(\cdot) = \sum_{k=1}^{+\infty} w_h \delta_{m_h}(\cdot)$
- **Conjugacy**: $G|\mathbf{y} \sim DP(MG_0 + \sum_i \delta_{y_i})$
- Polya urn representation:

$$\mathcal{L}(y_i | y_1, \ldots, y_{i-1}) \propto \sum_{h=1}^{i-1} \delta_{y_h}(y_i) + MG_0(y_i)$$

- $M \log n \ll n$ values

Continuous density estimation

- **Mixtures**:

$$y_i|G \sim f_G(y) = \int f_\vartheta(y)\,\mathrm{d}G(\vartheta)$$

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- Model name: **DPM model**

Continuous density estimation

- **Mixtures**:

$$y_i|G \sim f_G(y) = \int f_\vartheta(y)\,\mathrm{d}G(\vartheta)$$
$$G \sim DP(MG_0)$$

- Model name: **DPM model**
- Equivalent to:

$$\begin{aligned} y_i|\vartheta_i &\overset{\perp\!\!\!\perp}{\sim} f_{\vartheta_i} \\ \vartheta_i|G &\overset{\text{iid}}{\sim} G \\ G &\sim DP(MG_0) \end{aligned}$$

- ϑ_i “latent variables” $\forall i = 1, \ldots, n$

Clustering in the DPM

- Discreteness: the ϑ_i have one of the k **unique values** ϕ_j $(j = 1, \ldots, k)$
- $k \simeq M \log(n) \ll n$
- All i s.t. $\vartheta_i = \phi_j$ belong to cluster S_j $(j = 1, \ldots, k)$, and $n_j = |S_j|$

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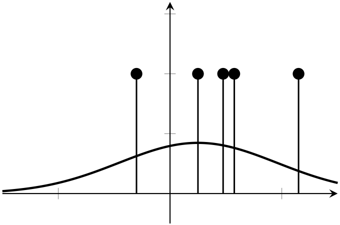
$$\mathcal{L}(\vartheta_i | \boldsymbol{\vartheta}_{-i}) \propto \sum_{j=1}^{k^-} n_j^- \delta_{\phi_j^-}(\vartheta_i) + M G_0(\vartheta_i)$$

$$\uparrow \qquad\qquad\qquad \uparrow$$



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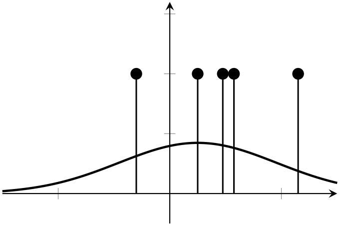

- Via **conjugancy** of the DP:

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Stuff

Bibliography

- Muller, Quintana, *Bayesian Nonparametric Data Analysis*
- Neal (2000), *Markov Chain Sampling Methods for Dirichlet Process Mixture Models*
- Ishwaran, James (2001), *Gibbs Sampling Methods for Stick-Breaking Priors*