```
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['C:\\Users\\polit\\.jupyter', 'd:\\html_doc\\math\\probability\\markov_chains\\env
\\etc\\jupyter', 'C:\\ProgramData\\jupyter']
D:\HTML_DOC\Math\Probability\Markov_Chains\env
C:\Program Files\Python38\python38.zip
C:\Program Files\Python38\DLLs
C:\Program Files\Python38\lib
C:\Program Files\Python38
d:\html_doc\math\probability\markov_chains\env

d:\html_doc\math\probability\markov_chains\env\lib\site-packages
d:\html_doc\math\probability\markov_chains\env\lib\site-packages\pip-20.2b1-py3.8.egg
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d:\html_doc\math\probability\markov_chains\env\lib\site-packages\Pythonwin
d:\html_doc\math\probability\markov_chains\env\lib\site-packages\IPython\extensions
C:\Users\polit\.ipython
current folder == D:\HTML_DOC\Math\Probability\Markov_Chains\env
```
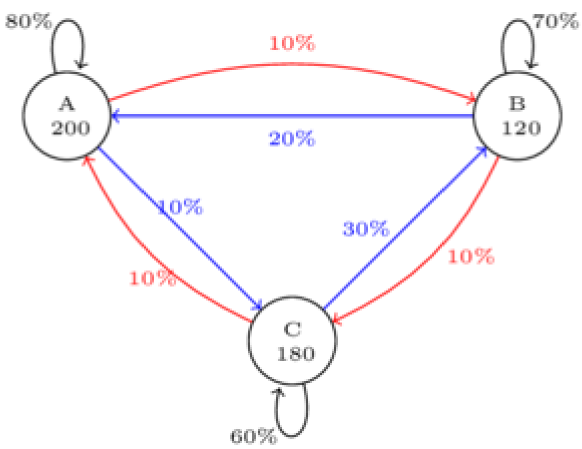



[Prob & Stats - Markov Chains (1 of 38) What are Markov Chains: An Introduction (https://youtu.be/Uz3JIp6EvIg)](https://youtu.be/Uz3JIp6EvIg)

$$[P_0] = \begin{array}{c} \overbrace{\begin{array}{ccc} A & B & C \end{array}}^{P\text{:from}} \\ \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ \underline{0.1} & \underline{0.1} & \underline{0.6} \end{bmatrix} \\ \begin{array}{ccc} 1.0 & 1.0 & 1.0 \end{array} \end{array} \begin{array}{c} to A \\ to B \\ to C \end{array} \quad [X_0] = \begin{array}{c} \overbrace{\begin{bmatrix} 0.40 & A = 200 \\ 0.24 & B = 120 \\ \underline{0.36} & \underline{C = 180} \end{bmatrix}}^{X_0} \\ \begin{array}{cc} 1.0 & 500 \end{array} \end{array}$$

$$\boxed{\begin{bmatrix} \text{Next State} \\ \text{Future State} \end{bmatrix} = \begin{bmatrix} \text{Matrix of} \\ \text{Transition} \\ \text{Probabilities} \end{bmatrix} * [\text{Current State}]}$$

$$[X_1] = [P] * [X_0]$$

$$
[X_1] = \overbrace{\begin{array}{ccc} A & B & C \\ \end{array}}^{P\text{:from}}
\begin{array}{c} \left[\begin{array}{ccc} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ \underline{0.1} & \underline{0.1} & \underline{0.6} \end{array}\right] \\ \begin{array}{ccc} 1.0 & 1.0 & 1.0 \end{array} \end{array}
\begin{array}{c} toA \\ toB \\ toC \end{array}
*
\overbrace{\begin{array}{c} \left[\begin{array}{cc} 0.40 & A = 200 \\ 0.24 & B = 120 \\ \underline{0.36} & \underline{C = 180} \end{array}\right] \\ \begin{array}{cc} 1.0 & 500 \end{array} \end{array}}^{X_0}
=
\overbrace{\left[\begin{array}{ccc} 0.8 * 0.4 & +0.2 * 0.24 & +0.1 * 0.36 \\ 0.1 * 0.4 & +0.7 * 0.24 & +0.3 * 0.36 \\ 0.1 * 0.4 & +0.1 * 0.24 & +0.6 * 0.36 \end{array}\right]}^{X_1}
=
$$

$$
[X_1] = \overbrace{\begin{array}{c} \left[\begin{array}{cc} 0.404 & A = 202 \\ 0.316 & B = 158 \\ \underline{0.280} & \underline{C = 140} \end{array}\right] \\ \begin{array}{cc} 1.0 & 500 \end{array} \end{array}}^{X_1}
$$

[Prob & Stats - Markov Chains (2 of 38) Markov Chains: An Introduction (Another Method) (https://youtu.be/3P8ZlIYgpvc)](https://youtu.be/3P8ZlIYgpvc)

[Prob & Stats - Markov Chains (3 of 38) Why Are Markov Chains Called "Markov Chains"? (https://youtu.be/ECrsoUtsKq0)](https://youtu.be/ECrsoUtsKq0)

$[X_1] = [P] * [X_0]$ $[X_2] = [P] * [X_1]$ $[X_3] = [P] * [X_2]$ $[X_4] = [P] * [X_3]$

$$
[X_1] = \overbrace{\begin{array}{c} \left[\begin{array}{c} 0.404 \\ 0.316 \\ \underline{0.280} \end{array}\right] \\ 1.0 \end{array}}^{X_1}
=
\overbrace{\begin{array}{ccc} A & B & C \end{array}}^{P}
\begin{array}{c} \left[\begin{array}{ccc} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ \underline{0.1} & \underline{0.1} & \underline{0.6} \end{array}\right] \\ \begin{array}{ccc} 1.0 & 1.0 & 1.0 \end{array} \end{array}
\overbrace{\begin{array}{c} \left[\begin{array}{c} 0.40 \\ 0.24 \\ \underline{0.36} \end{array}\right] \\ 1.0 \end{array}}^{X_0}
$$

$$
[X_2] = \overbrace{\begin{array}{c} \left[\begin{array}{c} \\ \\ \underline{} \end{array}\right] \\ 1.0 \end{array}}^{X_2}
=
\overbrace{\begin{array}{c} \left[\begin{array}{ccc} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ \underline{0.1} & \underline{0.1} & \underline{0.6} \end{array}\right] \\ \begin{array}{ccc} 1.0 & 1.0 & 1.0 \end{array} \end{array}}^{P}
\overbrace{\begin{array}{c} \left[\begin{array}{c} 0.404 \\ 0.316 \\ \underline{0.280} \end{array}\right] \\ 1.0 \end{array}}^{X_1}
=
\overbrace{\left[\begin{array}{ccc} 0.8 * 0.404 & +0.2 * 0.316 & +0.1 * 0.280 \\ 0.1 * 0.404 & +0.7 * 0.316 & +0.3 * 0.280 \\ 0.1 * 0.404 & +0.1 * 0.316 & +0.6 * 0.280 \end{array}\right]}^{X_2}
$$

$$
[X_2] = \overbrace{\begin{array}{c} \left[\begin{array}{c} 0.4144 \\ 0.3456 \\ \underline{0.2400} \end{array}\right] \\ 1.0 \end{array}}^{X_2}
$$

$X_0 = [0.4, 0.24, 0.36]$

```
[0.4  0.24 0.36]
[0.404 0.316 0.28 ]
[0.4144 0.3456 0.24  ]
[0.42464 0.35536 0.22   ]
```

[Prob & Stats - Markov Chains (4 of 38) Another Way to Calculate the Markov Chains (https://youtu.be/bBZrKmP020c)](https://youtu.be/bBZrKmP020c)

$$X_1 \quad = P \cdot X_0 \quad = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \begin{bmatrix} 0.40 \\ 0.24 \\ 0.36 \end{bmatrix} \quad = \begin{bmatrix} 0.404 \\ 0.316 \\ 0.280 \end{bmatrix}$$

$$X_2 \quad = P \cdot X_1 \quad = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \begin{bmatrix} 0.404 \\ 0.316 \\ 0.280 \end{bmatrix} \quad = \begin{bmatrix} 0.4144 \\ 0.3456 \\ 0.2400 \end{bmatrix}$$

$$X_3 \quad = P \cdot X_2 \quad = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \begin{bmatrix} 0.4144 \\ 0.3456 \\ 0.2400 \end{bmatrix} \quad = \begin{bmatrix} 0.42464 \\ 0.35536 \\ 0.22000 \end{bmatrix}$$

$$X_1 \quad = P^1 X_0 \quad = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \begin{bmatrix} 0.40 \\ 0.24 \\ 0.36 \end{bmatrix} \quad = \begin{bmatrix} 0.404 \\ 0.316 \\ 0.280 \end{bmatrix}$$

$$X_2 \quad = P^2 X_0 \quad = \begin{bmatrix} \\ \quad P^2 \quad \\ \\ \end{bmatrix} \begin{bmatrix} 0.40 \\ 0.24 \\ 0.36 \end{bmatrix} \quad = \begin{bmatrix} 0.4144 \\ 0.3456 \\ 0.2400 \end{bmatrix}$$

$$X_3 \quad = P^3 X_0 \quad = \begin{bmatrix} \\ \quad P^3 \quad \\ \\ \end{bmatrix} \begin{bmatrix} 0.40 \\ 0.24 \\ 0.36 \end{bmatrix} \quad = \begin{bmatrix} 0.42464 \\ 0.35536 \\ 0.22000 \end{bmatrix}$$

$$P^2 \quad = P \cdot P \quad = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} = \begin{bmatrix} \\ \qquad\qquad \\ \\ \end{bmatrix}$$

$$P^3 \quad = P^2 \cdot P \quad = \begin{bmatrix} \\ \quad P^2 \quad \\ \\ \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} = \begin{bmatrix} \\ \qquad\qquad \\ \\ \end{bmatrix}$$

```
[0.8 0.2 0.1][0.67 0.31 0.2][0.587 0.371 0.28][0.5347 0.4051 0.338]
[0.1 0.7 0.3][0.18 0.54 0.4][0.238 0.454 0.42][0.2778 0.4074 0.412]
[0.1 0.1 0.6][0.15 0.15 0.4][0.175 0.175 0.3 ][0.1875 0.1875 0.25 ]
[0.4  0.24 0.36]
```

$$P^2 = P \cdot P = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.67 & 0.31 & 0.2 \\ 0.18 & 0.54 & 0.4 \\ 0.15 & 0.15 & 0.4 \end{bmatrix}$$

$$P^3 = P^2 \cdot P = \begin{bmatrix} 0.67 & 0.31 & 0.2 \\ 0.18 & 0.54 & 0.4 \\ 0.15 & 0.15 & 0.4 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.587 & 0.371 & 0.28 \\ 0.238 & 0.454 & 0.42 \\ 0.175 & 0.175 & 0.30 \end{bmatrix}$$

```
[0.4  0.24 0.36]
[0.404 0.316 0.28 ]
[0.4144 0.3456 0.24  ]
[0.42464 0.35536 0.22   ]
```

$$X_1 = P^1 \cdot X_0 = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.40 \\ 0.24 \\ 0.36 \end{bmatrix} = \begin{bmatrix} 0.404 \\ 0.316 \\ 0.28 \end{bmatrix}$$

$$X_2 = P^2 \cdot X_0 = \begin{bmatrix} 0.67 & 0.31 & 0.2 \\ 0.18 & 0.54 & 0.4 \\ 0.15 & 0.15 & 0.4 \end{bmatrix} \cdot \begin{bmatrix} 0.40 \\ 0.24 \\ 0.36 \end{bmatrix} = \begin{bmatrix} 0.4144 \\ 0.3456 \\ 0.24 \end{bmatrix}$$

$$X_3 = P^3 \cdot X_0 = \begin{bmatrix} 0.587 & 0.371 & 0.28 \\ 0.238 & 0.454 & 0.42 \\ 0.175 & 0.175 & 0.30 \end{bmatrix} \cdot \begin{bmatrix} 0.40 \\ 0.24 \\ 0.36 \end{bmatrix} = \begin{bmatrix} 0.42464 \\ 0.35536 \\ 0.22 \end{bmatrix}$$

[Prob & Stats - Markov Chains (5 of 38) What Happens if the Markov Chain Continues? (https://youtu.be/9wBPa2eu_lc)](https://youtu.be/9wBPa2eu_lc)

$$X_1 = P^1 \cdot X_0 = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.40 \\ 0.24 \\ 0.36 \end{bmatrix} = \begin{bmatrix} 0.404 \\ 0.316 \\ 0.28 \end{bmatrix}$$

$$X_2 = P^1 \cdot X_1 = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.404 \\ 0.316 \\ 0.28 \end{bmatrix} = \begin{bmatrix} 0.4144 \\ 0.3456 \\ 0.24 \end{bmatrix}$$

$$X_3 = P^1 \cdot X_2 = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.4144 \\ 0.3456 \\ 0.24 \end{bmatrix} = \begin{bmatrix} 0.42464 \\ 0.35536 \\ 0.22 \end{bmatrix}$$

```
0= [0.4  0.24 0.36]
1= [0.404 0.316 0.28 ]
2= [0.4144 0.3456 0.24  ]
3= [0.42464 0.35536 0.22   ]
4= [0.432784 0.357216 0.21    ]
5= [0.4386704 0.3563296 0.205    ]
6= [0.44270224 0.35479776 0.2025    ]
7= [0.44537134 0.35337866 0.20125   ]
8= [0.44709781 0.35227719 0.200625  ]
```

$$X_4 = P^1 \cdot X_3 = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.42464 \\ 0.35536 \\ 0.22 \end{bmatrix} = \begin{bmatrix} 0.432784 \\ 0.357216 \\ 0.21 \end{bmatrix}$$

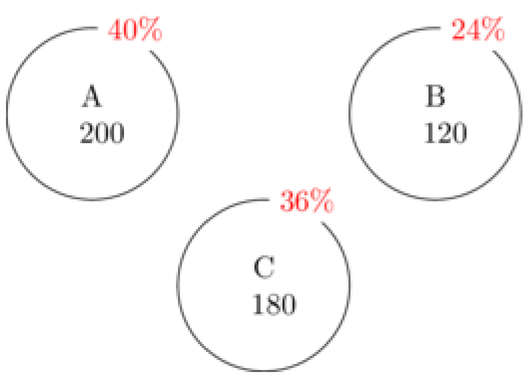
$$X_5 = P^1 \cdot X_4 = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.432784 \\ 0.357216 \\ 0.21 \end{bmatrix} = \begin{bmatrix} 0.4386704 \\ 0.3563296 \\ 0.205 \end{bmatrix}$$

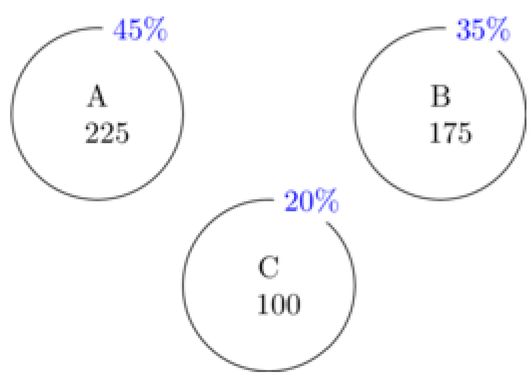
$$X_6 = P^1 \cdot X_5 = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.4386704 \\ 0.3563296 \\ 0.205 \end{bmatrix} = \begin{bmatrix} 0.44270224 \\ 0.35479776 \\ 0.2025 \end{bmatrix}$$

$$X_7 = P^1 \cdot X_6 = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.44270224 \\ 0.35479776 \\ 0.2025 \end{bmatrix} = \begin{bmatrix} 0.44537134 \\ 0.35337866 \\ 0.20125 \end{bmatrix}$$

$$X_8 = P^1 \cdot X_7 = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.44537134 \\ 0.35337866 \\ 0.20125 \end{bmatrix} = \begin{bmatrix} 0.44709781 \\ 0.35227719 \\ 0.200625 \end{bmatrix}$$

```
 8=0.4471 0.3523 0.2006
 9=0.4482 0.3515 0.2003
10=0.4489 0.3510 0.2002
11=0.4493 0.3506 0.2001
12=0.4496 0.3504 0.2000
13=0.4497 0.3502 0.2000
14=0.4498 0.3501 0.2000
15=0.4499 0.3501 0.2000
16=0.4499 0.3501 0.2000
17=0.4500 0.3500 0.2000
18=0.4500 0.3500 0.2000
19=0.4500 0.3500 0.2000
```

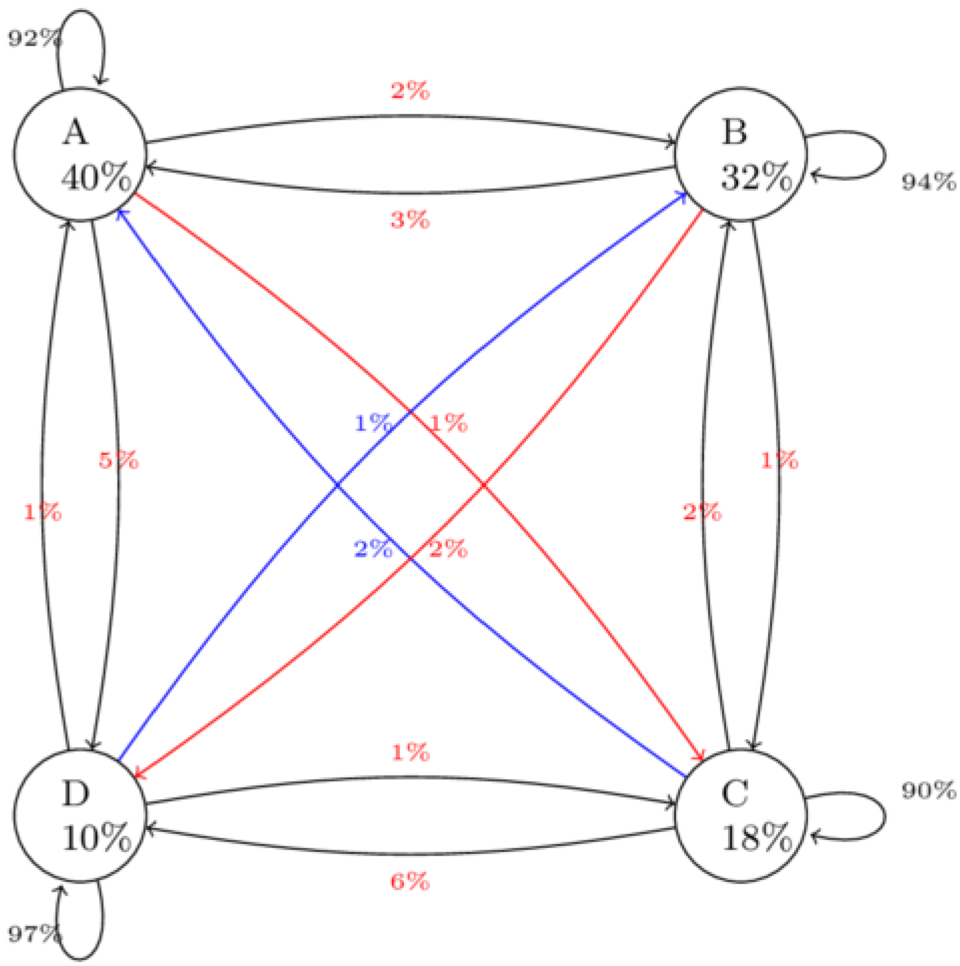




[Prob & Stats - Markov Chains (6 of 38) Markov Chain Applied to Market Penetration (https://youtu.be/xgvgN4fUqcs)](https://youtu.be/xgvgN4fUqcs)

$$
[P_0] = \overbrace{\begin{array}{cccc} A & B & C & D \end{array}}^{P\text{:from}}
$$

$$
[P_0] = \begin{array}{c} \begin{bmatrix} 0.92 & 0.03 & 0.02 & 0.01 \\ 0.02 & 0.94 & 0.02 & 0.01 \\ 0.01 & 0.01 & 0.90 & 0.01 \\ \underline{0.05} & \underline{0.02} & \underline{0.06} & \underline{0.97} \end{bmatrix} \\ \begin{array}{cccc} 1.0 & 1.0 & 1.0 & 1.0 \end{array} \end{array} \begin{array}{l} to A \\ to B \\ to C \\ to D \\ \end{array}
\qquad
[X_0] = \overbrace{\begin{bmatrix} 0.40 \\ 0.32 \\ 0.18 \\ \underline{0.10} \end{bmatrix}}^{X_0} \atop 1.0
\qquad
[\bar{X}] = \overbrace{\begin{bmatrix} 0.161 \\ 0.179 \\ 0.091 \\ \underline{0.569} \end{bmatrix}}^{X_0} \atop 1.0
$$




```
[0.92 0.03 0.02 0.01]
[0.02 0.94 0.02 0.01]
[0.01 0.01 0.9  0.01]
[0.05 0.02 0.06 0.97]
X[0] = [0.4  0.32 0.18 0.1 ]
X[ 50]: = 0.167367 0.186412 0.091172 0.555050
X[100]: = 0.161185 0.179146 0.090910 0.568760
X[150]: = 0.160963 0.178850 0.090909 0.569278
X[200]: = 0.160954 0.178838 0.090909 0.569299
X[250]: = 0.160954 0.178838 0.090909 0.569300
```

[**Prob & Stats - Markov Chains (7 of 38) Power of the Probability Matrix (https://youtu.be/KBCZ7o8XLKU)**](https://youtu.be/KBCZ7o8XLKU)

$$P^2 = P \cdot P = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix} = \begin{bmatrix} 0.66 & 0.17 \\ 0.34 & 0.83 \end{bmatrix}$$

$$P^3 = P^2 \cdot P = \begin{bmatrix} 0.66 & 0.17 \\ 0.34 & 0.83 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix} = \begin{bmatrix} 0.562 & 0.219 \\ 0.438 & 0.781 \end{bmatrix}$$

$$P^4 = P^3 \cdot P = \begin{bmatrix} 0.562 & 0.219 \\ 0.438 & 0.781 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix} = \begin{bmatrix} 0.4934 & 0.2533 \\ 0.5066 & 0.7467 \end{bmatrix}$$

$$P^5 = P^4 \cdot P = \begin{bmatrix} 0.4934 & 0.2533 \\ 0.5066 & 0.7467 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix} = \begin{bmatrix} 0.445380.27731 \\ 0.554620.72269 \end{bmatrix}$$

$$P^6 = P^5 \cdot P = \begin{bmatrix} 0.4934 & 0.2533 \\ 0.5066 & 0.7467 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix} = \begin{bmatrix} 0.411766 & 0.294117 \\ 0.588234 & 0.705883 \end{bmatrix}$$

$$\bar{P} = \bar{P} \cdot P = \left[\, \bar{P} \,\right] \cdot \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$$

$$\bar{P} = \bar{P} \cdot P = \begin{bmatrix} 0.3333 & 0.3333 \\ 0.6667 & 0.6667 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix} = \begin{bmatrix} 0.3333 & 0.3333 \\ 0.6667 & 0.6667 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

```
[0.8 0.1][0.66 0.17][0.562 0.219][0.4934 0.2533][0.44538 0.27731][0.411766 0.294117]
[0.2 0.9][0.34 0.83][0.438 0.781][0.5066 0.7467][0.55462 0.72269][0.588234 0.705883]
10 --------
[0.346516 0.326742]
[0.653484 0.673258]
20 --------
[0.333706 0.333147]
[0.666294 0.666853]
30 --------
[0.333344 0.333328]
[0.666656 0.666672]
40 --------
[0.333334 0.333333]
[0.666666 0.666667]
50 --------
[0.333333 0.333333]
[0.666667 0.666667]
60 --------
[0.333333 0.333333]
[0.666667 0.666667]
1/3 1/3
2/3 2/3
```

$\bar{P} \cdot X_0 =? = \bar{X}$ Stable distribution matrix

$$X_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \bar{P}X_0 = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 0 \\ \frac{2}{3} \cdot 1 + \frac{2}{3} \cdot 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} + 0 \\ \frac{2}{3} + 0 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix} = \bar{X}$$

$$X_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \quad \bar{P}X_0 = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} \frac{1}{3}\frac{1}{2} + \frac{1}{3}\frac{1}{2} \\ \frac{2}{3}\frac{1}{2} + \frac{2}{3}\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{6} + \frac{1}{6} \\ \frac{1}{3} + \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix} = \bar{X}$$

$$X_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \bar{P}X_0 = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot 1 \\ \frac{2}{3} \cdot 0 + \frac{2}{3} \cdot 1 \end{bmatrix} = \begin{bmatrix} 0 + \frac{1}{3} \\ 0 + \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix} = \bar{X}$$

[Prob & Stats - Markov Chains (8 of 38) What is a Stochastic Matrix? (https://youtu.be/VXntl6MqRlo)](https://youtu.be/VXntl6MqRlo)

[Prob & Stats - Markov Chains (9 of 38) What is a Regular Matrix? (https://youtu.be/YMUwWV1IGdk)](https://youtu.be/YMUwWV1IGdk)

[Prob & Stats - Markov Chains (10 of 38) Regular Markov Chain (https://youtu.be/loBUEME5chQ)](https://youtu.be/loBUEME5chQ)

$$P^n \cdot X_0 = \bar{X} - \text{Stable Distribution Matrix}$$

$$P_0 = \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix}, \qquad X_0 = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$$

$$P, \quad P^2, \quad P^3, \quad P^4, \quad P^5, \quad P^6, \quad \cdots, \quad \text{as Regular Markov chain}$$

$$P^2 = P \cdot P = \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix} \cdot \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix} = \begin{bmatrix} 0.34 & 0.33 \\ 0.66 & 0.67 \end{bmatrix}$$

$$P^3 = P^2 \cdot P = \begin{bmatrix} 0.34 & 0.33 \\ 0.66 & 0.67 \end{bmatrix} \cdot \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix} = \begin{bmatrix} 0.334 & 0.333 \\ 0.666 & 0.667 \end{bmatrix}$$

$$P^4 = P^3 \cdot P = \begin{bmatrix} 0.334 & 0.333 \\ 0.666 & 0.667 \end{bmatrix} \cdot \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix} = \begin{bmatrix} 0.3334 & 0.3333 \\ 0.6666 & 0.6667 \end{bmatrix}$$

$$\bar{P} = \bar{P} \cdot P = \left[\, \bar{P} \,\right] \cdot \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix} = \text{Stable Matrix}$$

$$\bar{P} = \bar{P} \cdot P = \begin{bmatrix} 0.3333 & 0.3333 \\ 0.6667 & 0.6667 \end{bmatrix} \cdot \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix} = \begin{bmatrix} 0.3333 & 0.3333 \\ 0.6667 & 0.6667 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

$$P^4 \cdot X_0 = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$$

```
[0.4 0.3][0.34 0.33][0.334 0.333][0.3334 0.3333][0.33334 0.33333][0.333334 0.333333]
[0.6 0.7][0.66 0.67][0.666 0.667][0.6666 0.6667][0.66666 0.66667][0.666666 0.666667]
6 --------
[0.333334 0.333333]
[0.666666 0.666667]
7 --------
[0.333333 0.333333]
[0.666667 0.666667]
8 --------
[0.333333 0.333333]
[0.666667 0.666667]
1/3 1/3
2/3 2/3
```

[Prob & Stats - Markov Chains (11 of 38) How to Check for a Stable Distribution Matrix (https://youtu.be/DeG8MIORxRA)](https://youtu.be/DeG8MIORxRA)

$$[P_0] = \begin{array}{c} \overbrace{\begin{array}{cc} A & B \end{array}}^{\text{FROM}} \\ \begin{bmatrix} 0.5 & 0.25 \\ \underline{0.5} & \underline{0.75} \end{bmatrix} \\ \begin{array}{cc} 1.0 & 1.0 \end{array} \end{array} \quad \begin{array}{c} to A \\ to B \end{array}$$

$$X_0 = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$$

Is the $\bar{P} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix}$ is **Stable Matrix** ?

Is the $\bar{X} = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$ is **Stable Distribution Matrix** ?

Checking:

$$P \cdot \bar{X} = \bar{X}$$

$$\begin{bmatrix} 0.5 & 0.25 \\ 0.5 & 0.75 \end{bmatrix} \cdot \begin{bmatrix} 0.333 \\ 0.667 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$$

Checking:

$$\bar{P} \cdot X_0 = \bar{X}$$

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$$

```
[0.333333 0.333333][0.333333 0.333333][-0.000000 0.000000]
[0.666667 0.666667][0.666667 0.666667][-0.000000 -0.000000]
[0.33333333 0.66666667] [0.33333333 0.66666667] [0. 0.]
1/3 1/3
2/3 2/3
```

[Prob & Stats - Markov Chains (12 of 38) How to Find a Stable 2x2 Matrix - Ex. 1 (https://youtu.be/cSKXAalhW6w)](https://youtu.be/cSKXAalhW6w)

$$P_0 = \begin{bmatrix} 0.5 & 0.25 \\ 0.5 & 0.75 \end{bmatrix}, X_0 = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$$

$$\bar{P} = ?, \bar{X} = ?$$

$P \cdot \bar{X} = \left[\, \bar{X} \,\right]$, Let $\bar{X} = \begin{bmatrix} A \\ B \end{bmatrix}$:

$$\begin{bmatrix} 0.5 & 0.25 \\ 0.5 & 0.75 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{cases} 0.5 \cdot A + 0.25 \cdot B & = A \\ 0.5 \cdot A + 0.75 \cdot B & = B \\ A + B & = 1 \end{cases} \Rightarrow \begin{cases} B & = 4 \times (A - 0.5A) \\ 0.5A & = B - 0.75B \\ A + B & = 1 \end{cases} \Rightarrow \begin{cases} B & = 2A \\ A & = 0.5B \\ A + 2A & = 1 \end{cases} \Rightarrow$$

$$\begin{cases} B & = 2A \\ A & = \frac{1}{3} \end{cases} \Rightarrow \begin{cases} B & = \frac{2}{3} \\ A & = \frac{1}{3} \end{cases}$$

$$\bar{X} = \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 0.33333333 \\ 0.66666667 \end{bmatrix}$$

$$\bar{P} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

```
[0.5 0.25][0.333333 0.333333]
[0.5 0.75][0.666667 0.666667]
[0.33333333 0.66666667]
1/3 1/3
2/3 2/3
```

[Prob & Stats - Markov Chains (13 of 38) How to Find a Stable 2x2 Matrix - Ex. 2 (https://youtu.be/C1hs3EQS_jo)](https://youtu.be/C1hs3EQS_jo)

$$P_0 = \begin{bmatrix} 0.9 & 0.5 \\ 0.1 & 0.5 \end{bmatrix}$$

$$\bar{P} = ?, \bar{X} = ?$$

$P \cdot \bar{X} = \bar{X}$, Let $\bar{X} = \begin{bmatrix} A \\ B \end{bmatrix}$:

$$\begin{bmatrix} 0.9 & 0.5 \\ 0.1 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{cases} 0.9 \cdot A + 0.5 \cdot B & = A \\ 0.1 \cdot A + 0.5 \cdot B & = B \\ A + B & = 1 \end{cases} \Rightarrow \begin{cases} B & = 2 \times (A - 0.9A) \\ A & = 10 \times (B - 0.5B) \\ A + B & = 1 \end{cases} \Rightarrow \begin{cases} B & = 0.2A \\ A & = 5B \\ 5B + B & = 1 \end{cases} \Rightarrow$$

$$\begin{cases} A & = 5B \\ B & = \frac{1}{6} \end{cases} \Rightarrow \begin{cases} A & = \frac{5}{6} \\ B & = \frac{1}{6} \end{cases}$$

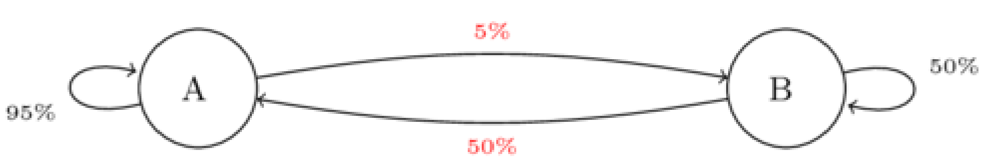
$$\bar{X} = \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 5/6 \\ 1/6 \end{bmatrix} = \begin{bmatrix} 0.83333333 \\ 0.16666667 \end{bmatrix}$$

$$\bar{P} = \begin{bmatrix} \frac{5}{6} & \frac{5}{6} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$$

```
[0.9 0.5][0.833333 0.833333]
[0.1 0.5][0.166667 0.166667]
[0.83333333 0.16666667]
5/6 5/6
1/6 1/6
```

[Prob & Stats - Markov Chains (14 of 38) How to Find a Stable 2x2 Matrix - Ex. 3 (https://youtu.be/vxdUtjoxWvE)](https://youtu.be/vxdUtjoxWvE)

$$\begin{array}{c|cc|c} & \overbrace{A \quad\quad B}^{P\text{:from}} & & \\ [P_0] = & \begin{bmatrix} 0.95 & 0.5 \\ \underline{0.05} & \underline{0.5} \end{bmatrix} & & \begin{matrix} to A \\ to B \end{matrix} \\ & 1.0 \quad 1.0 & & \end{array}$$



$$\bar{P} = ?, \bar{X} = ?$$

$P \cdot \bar{X} = \bar{X}$, Let $\bar{X} = \begin{bmatrix} A \\ B \end{bmatrix}$:

$$\begin{bmatrix} 0.95 & 0.5 \\ 0.05 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{cases} 0.95 \cdot A + 0.5 \cdot B & = A \\ 0.05 \cdot A + 0.5 \cdot B & = B \\ A + B & = 1 \end{cases} \Rightarrow \begin{cases} B & = 2 \times (A - 0.95A) \\ A & = 20 \times (B - 0.5B) \\ A + B & = 1 \end{cases} \Rightarrow \begin{cases} B & = 0.1A \\ A & = 10B \\ 10B + B & = 1 \end{cases} \Rightarrow$$

$$\begin{cases} A & = 10B \\ B & = \frac{1}{11} \end{cases} \Rightarrow \begin{cases} A & = \frac{10}{11} \\ B & = \frac{1}{11} \end{cases}$$

$$\bar{X} = \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 10/11 \\ 1/11 \end{bmatrix} = \begin{bmatrix} 0.909 \\ 0.091 \end{bmatrix}$$

$$\bar{P} = \begin{bmatrix} \frac{10}{11} & \frac{10}{11} \\ \frac{1}{11} & \frac{1}{11} \end{bmatrix} = \begin{bmatrix} 0.909 & 0.909 \\ 0.091 & 0.091 \end{bmatrix}$$

```
[0.95 0.5][0.909091 0.909091]
[0.05 0.5][0.090909 0.090909]
[0.90909091 0.09090909]
10/11 10/11
1/11  1/11
```

[Prob & Stats - Markov Chains (15 of 38) How to Find a Stable 3x3 Matrix (https://youtu.be/ZENBQj2qQ2k)](https://youtu.be/ZENBQj2qQ2k)

$$\begin{array}{cccc} & \overbrace{\begin{matrix} A & \quad B & \quad C \end{matrix}}^{P\text{:from}} & \\ [P_0] = & \begin{bmatrix} 0.8 & 0.2 & 0.3 \\ 0.1 & 0.7 & 0.1 \\ \underline{0.1} & \underline{0.1} & \underline{0.6} \end{bmatrix} & \begin{matrix} to A \\ to B \\ to C \end{matrix} \\ & \begin{matrix} 1.0 & 1.0 & 1.0 \end{matrix} & \end{array}$$

$$\bar{P} = ?, \bar{X} = ?$$

$P \cdot \bar{X} = \bar{X}$, Let $\bar{X} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$:

$$\begin{bmatrix} 0.8 & 0.2 & 0.3 \\ 0.1 & 0.7 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$\begin{cases} 0.8 \cdot A + 0.2 \cdot B + 0.3 \cdot C & = A \\ 0.1 \cdot A + 0.7 \cdot B + 0.1 \cdot C & = B \\ 0.1 \cdot A + 0.1 \cdot B + 0.6 \cdot C & = C \\ A + B + C & = 1 \end{cases} \Rightarrow \begin{cases} 0.2B + 0.3C & = A - 0.8A \\ 0.1A + 0.7B + 0.1C & = B \\ 0.1A + 0.1B + 0.6C & = C \\ A + B + C & = 1 \end{cases} \Rightarrow$$

$$\begin{cases} 0.2B + 0.3C &= 0.2A \\ A + 7B + C &= 10 \cdot B \\ A + B + C &= 1 \end{cases} \Rightarrow \begin{cases} 2B + 3C &= 2A \\ A + C &= 10B - 7B \\ A + B + C &= 1 \end{cases} \Rightarrow \begin{cases} B + \frac{3}{2}C &= A \\ A + C &= 3B \\ A + B + C &= 1 \end{cases} \Rightarrow$$

$$\begin{cases} B + \frac{3}{2}C &= A \\ (B + \frac{3}{2}C) + C &= 3B \\ A + B + C &= 1 \end{cases} \Rightarrow \begin{cases} B + \frac{3}{2}C &= A \\ \frac{3}{2}C + C &= 3B - B \\ A + B + C &= 1 \end{cases} \Rightarrow \begin{cases} B + \frac{3}{2}C &= A \\ \frac{5}{2}C &= 2B \\ A + B + C &= 1 \end{cases} \Rightarrow$$

$$\begin{cases} B + \frac{3}{2}C &= A \\ C &= \frac{4}{5}B \\ A + B + C &= 1 \end{cases} \Rightarrow \begin{cases} B + \frac{3}{2}C &= A \\ C &= \frac{4}{5}B \\ (B + \frac{3}{2}C) + B + (\frac{4}{5}B) &= 1 \end{cases} \Rightarrow$$

$$\begin{cases} B + \frac{3}{2}C &= A \\ C &= \frac{4}{5}B \\ \frac{5}{5}B + \frac{3}{2}(\frac{4}{5}B) + \frac{5}{5}B + \frac{4}{5}B &= 1 \end{cases} \Rightarrow \begin{cases} A &= B + \frac{3}{2}C \\ C &= \frac{4}{5}B \\ \frac{20}{5}B &= 1 \end{cases} \Rightarrow \begin{cases} A &= B + \frac{3}{2}C \\ C &= \frac{4}{5}B \\ B &= \frac{1}{4} \end{cases} \Rightarrow$$

$$\begin{cases} A &= \left(\frac{1}{4}\right) + \frac{3}{2}C \\ C &= \frac{4}{5}\left(\frac{1}{4}\right) \\ B &= \frac{1}{4} \end{cases} \Rightarrow \begin{cases} A &= \left(\frac{1}{4}\right) + \frac{3}{2}C \\ C &= \frac{1}{5} \\ B &= \frac{1}{4} \end{cases} \Rightarrow \begin{cases} A &= \left(\frac{1}{4}\right) \cdot \frac{5}{5} + \frac{3}{2}\left(\frac{1}{5}\right) \cdot \frac{2}{2} \\ C &= \left(\frac{1}{5}\right) \cdot \frac{4}{4} \\ B &= \left(\frac{1}{4}\right) \cdot \frac{5}{5} \end{cases} \Rightarrow \begin{cases} A &= \frac{5}{20} + \frac{6}{20} \\ B &= \frac{5}{20} \\ C &= \frac{4}{20} \end{cases} \Rightarrow$$

$$\bar{X} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \frac{11}{20} \\ \frac{5}{20} \\ \frac{4}{20} \end{bmatrix} = \begin{bmatrix} 0.55 \\ 0.25 \\ 0.20 \end{bmatrix}$$

$$\bar{P} = \begin{bmatrix} \frac{11}{20} & \frac{11}{20} & \frac{11}{20} \\ \frac{5}{20} & \frac{5}{20} & \frac{5}{20} \\ \frac{4}{20} & \frac{4}{20} & \frac{4}{20} \end{bmatrix}$$

```
[0.8 0.2 0.3][0.55 0.55 0.55]
[0.1 0.7 0.1][0.25 0.25 0.25]
[0.1 0.1 0.6][0.2  0.2  0.2 ]
[0.55 0.25 0.2 ]
11/20 11/20 11/20
1/4   1/4   1/4
1/5   1/5   1/5
```

[Prob & Stats - Markov Chains (16 of 38) Application Problem #1, Charity Contributions (https://youtu.be/87u7a2XGq1s)](https://youtu.be/87u7a2XGq1s)

Charity contributions

- of those who contribute 40% will not contribute next time
- of those who don't contribute 10% will not contribute next time

What is the Final Distribution matrix $\bar{X}$ = ?

Благотворительный взнос

- те кто вносит свой взнос 40% не будет участвовать в следующий раз
- те, кто не вносит взнос 10% не будет участвовать в следующий раз

Какая будет предельная/финальная Матрица распределения $\bar{X}$ =?

$$[P_0] = \begin{array}{c} \overbrace{\begin{array}{cc} Not & Contr \end{array}}^{P\text{:from}} \\ \begin{bmatrix} & 0.4 \\ \underline{0.1} & \underline{\quad} \end{bmatrix} \\ \begin{array}{cc} 1.0 & 1.0 \end{array} \end{array} \begin{array}{l} \text{to Not contribute} \\ \text{to Contribute} \end{array}$$

$$[P_0] = \begin{array}{c} \overbrace{\begin{array}{cc} Not & Contr \end{array}}^{P\text{:from}} \\ \begin{bmatrix} 0.9 & 0.4 \\ \underline{0.1} & \underline{0.6} \end{bmatrix} \\ \begin{array}{cc} 1.0 & 1.0 \end{array} \end{array} \begin{array}{l} \text{to Not contribute} \\ \text{to Contribute} \end{array}$$

$$\bar{P} = ?, \bar{X} = ?$$

$P \cdot \bar{X} = \bar{X}$, Let $\bar{X} = \begin{bmatrix} N \\ C \end{bmatrix}$:

$$\begin{bmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} N \\ C \end{bmatrix} = \begin{bmatrix} N \\ C \end{bmatrix}$$

$$\begin{cases} 0.9 \cdot N + 0.4 \cdot C & = N \\ 0.1 \cdot N + 0.6 \cdot C & = C \\ N + C & = 1 \end{cases} \Rightarrow \begin{cases} N + 6C & = 10C \\ N + C & = 1 \end{cases} \Rightarrow \begin{cases} N & = 4C \\ N + C & = 1 \end{cases} \Rightarrow \begin{cases} N & = 4C \\ (4C) + C & = 1 \end{cases} \Rightarrow$$

$$\begin{cases} N & = 4C \\ 5C & = 1 \end{cases} \Rightarrow \begin{cases} N & = 4C \\ C & = \frac{1}{5} \end{cases} \Rightarrow \begin{cases} N & = 4 \cdot \left(\frac{1}{5}\right) \\ C & = \frac{1}{5} \end{cases} \Rightarrow \begin{cases} N & = \frac{4}{5} \\ C & = \frac{1}{5} \end{cases}$$

$$\bar{X} = \begin{bmatrix} N \\ C \end{bmatrix} = \begin{bmatrix} \frac{4}{5} \\ \frac{1}{5} \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} \begin{array}{l} 80\% \\ 20\% \end{array}$$

$$\bar{P} = \begin{bmatrix} \frac{4}{5} & \frac{4}{5} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

```
[0.9 0.4][0.8 0.8]
[0.1 0.6][0.2 0.2]
[0.8 0.2]
4/5 4/5
1/5 1/5
```

[Prob & Stats - Markov Chains (17 of 38) Application Problem #2, Grocery Stores (https://youtu.be/Na4m8HSpZSo)](https://youtu.be/Na4m8HSpZSo)

2 Stores **A** and **B** :

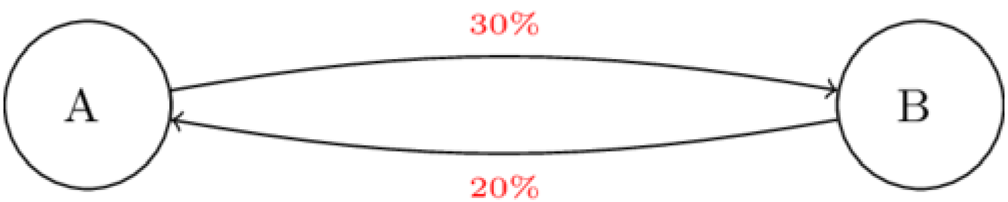
- 30% of customers shopping at **A** will switch to **B** every nex month
- 20% of customers shopping at **B** will switch to **A** every nex month

2 Магазина **A** и **B** :

- 30% покупателей из магазина **A** переходят в магазин **B** каждый следущий месяц
- 30% покупателей из магазина **B** переходят в магазин **A** каждый следущий месяц

Какая будет предельная/финальная Матрица распределения $\bar{X}$ =?

$$[P_0] = \begin{array}{c} \overbrace{\begin{array}{cc} A & B \end{array}}^{P\text{:from}} \\ \begin{bmatrix} & 0.2 \\ \underline{0.3} & \underline{\quad} \end{bmatrix} \\ \begin{array}{cc} 1.0 & 1.0 \end{array} \end{array} \begin{array}{l} \text{to A} \\ \text{to B} \end{array}$$



$$[P_0] = \begin{array}{c} \overbrace{\begin{array}{cc} A & B \end{array}}^{P\text{:from}} \\ \begin{bmatrix} 0.7 & 0.2 \\ \underline{0.3} & \underline{0.8} \end{bmatrix} \\ \begin{array}{cc} 1.0 & 1.0 \end{array} \end{array} \begin{array}{l} \text{to A} \\ \text{to B} \end{array}$$

$$\bar{P} = ?, \bar{X} = ?$$

$P \cdot \bar{X} = \bar{X}$, Let $\bar{X} = \begin{bmatrix} A \\ B \end{bmatrix}$:

$$\begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{cases} 0.7 \cdot A + 0.2 \cdot B & = A \\ 0.3 \cdot A + 0.8 \cdot B & = B \\ A + B & = 1 \end{cases} \Rightarrow \begin{cases} 7A + 2B & = 10A \\ \\ A + B & = 1 \end{cases} \Rightarrow \begin{cases} 2B & = 3A \\ \\ A + B & = 1 \end{cases} \Rightarrow \begin{cases} B & = \frac{3}{2}A \\ \\ A + B & = 1 \end{cases} \Rightarrow$$

$$\begin{cases} B & = \frac{3}{2}A \\ A + \left(\frac{3}{2}A\right) & = 1 \end{cases} \Rightarrow \begin{cases} B & = \frac{3}{2}A \\ \frac{5}{2}A & = 1 \end{cases} \Rightarrow \begin{cases} B & = \frac{3}{2}\left(\frac{2}{5}\right) \\ A & = \frac{2}{5} \end{cases} \Rightarrow \begin{cases} A & = \frac{2}{5} \\ B & = \frac{3}{5} \end{cases}$$

$$\bar{X} = \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \frac{2}{5} \\ \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} \begin{array}{l} 40\% \\ 60\% \end{array}$$

$$\bar{P} = \begin{bmatrix} \frac{2}{5} & \frac{2}{5} \\ \frac{3}{5} & \frac{3}{5} \end{bmatrix}$$

```
[0.7 0.2][0.4 0.4]
[0.3 0.8][0.6 0.6]
[0.4 0.6]
2/5 2/5
3/5 3/5
```

[Prob & Stats - Markov Chains (18 of 38) Application Problem #3, Brand Loyalty (https://youtu.be/7_NI_9L35t0)](https://youtu.be/7_NI_9L35t0)

3 Brands **A**, **B** and **C**:

- 10% of **A** shoppers will switch to **B** and 10% to **C**
- 20% of **B** shoppers will switch to **A** and 10% to **C**
- 10% of **C** shoppers will switch to **A** and 10% to **B**

3 Бренда **A**, **B** и **C**:

- 10% покупателей **A** будут покупать **B** и 10% на **C**
- 20% покупателей **B** будут покупать **A** и 10% на **C**
- 10% покупателей **C** будут покупать **A** и 10% на **B**

Какая будет предельная/финальная Матрица распределения $\bar{X}$ =?

$$\begin{array}{cc} & \overbrace{\begin{array}{ccc} A & B & C \end{array}}^{P\text{:from}} \\ [P_0] = & \begin{bmatrix} & 0.2 & 0.1 \\ 0.1 & & 0.2 \\ \underline{0.1} & \underline{0.2} & \underline{\quad} \end{bmatrix} \begin{array}{c} toA \\ toB \\ toC \end{array} \\ & \begin{array}{ccc} 1.0 & 1.0 & 1.0 \end{array} \end{array}$$

$$\begin{array}{cc} & \overbrace{\begin{array}{ccc} A & B & C \end{array}}^{P\text{:from}} \\ [P_0] = & \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.6 & 0.2 \\ \underline{0.1} & \underline{0.2} & \underline{0.7} \end{bmatrix} \begin{array}{c} toA \\ toB \\ toC \end{array} \\ & \begin{array}{ccc} 1.0 & 1.0 & 1.0 \end{array} \end{array}$$

$$\bar{P} = ?, \bar{X} = ?$$

$P \cdot \bar{X} = \bar{X}$, Let $\bar{X} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$:

$$\begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.6 & 0.2 \\ 0.1 & 0.2 & 0.7 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$\begin{cases} 0.8 \cdot A + 0.2 \cdot B + 0.1 \cdot C & = A \\ 0.1 \cdot A + 0.6 \cdot B + 0.2 \cdot C & = B \\ 0.1 \cdot A + 0.2 \cdot B + 0.7 \cdot C & = C \\ A + B + C & = 1 \end{cases} \Rightarrow \begin{cases} 8A + 2B + C & = 10A \\ A + 6B + 2C & = 10B \\ & \\ A + B + C & = 1 \end{cases} \Rightarrow$$

$$\begin{cases} C & = 10A - 8A - 2B \\ A + 2C & = 10B - 6B \\ A + B + C & = 1 \end{cases} \Rightarrow \begin{cases} 2A - 2B = C & \\ A + 2C & = 4B \\ A + B + C & = 1 \end{cases} \Rightarrow$$

$$\begin{cases} 2A - 2B & = C \\ A + 2(2A - 2B) & = 4B \\ A + B + C & = 1 \end{cases} \Rightarrow \begin{cases} 2A - 2B & = C \\ A + 4A - 4B & = 4B \\ A + B + C & = 1 \end{cases} \Rightarrow \begin{cases} 2A - 2B & = C \\ 5A & = 8B \\ A + B + C & = 1 \end{cases} \Rightarrow$$

$$\begin{cases} 2A - 2B & = C \\ \frac{5}{8}A & = B \\ A + B + C & = 1 \end{cases} \Rightarrow \begin{cases} 2A - 2\left(\frac{5}{8}A\right) & = C \\ B & = \frac{5}{8}A \\ A + B + C & = 1 \end{cases} \Rightarrow \begin{cases} \frac{16}{8}A - \frac{10}{8}A & = C \\ B & = \frac{5}{8}A \\ A + B + C & = 1 \end{cases} \Rightarrow$$

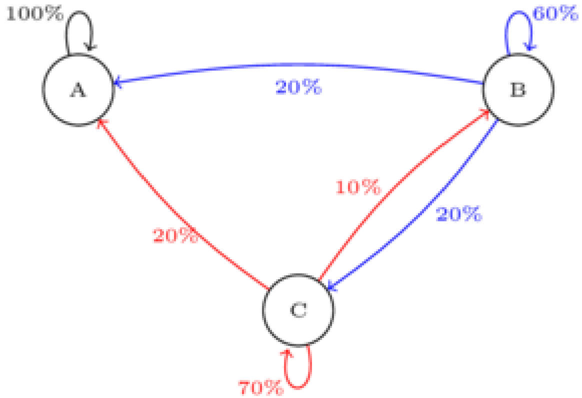
$$\begin{cases} \frac{6}{8}A & = C \\ B & = \frac{5}{8}A \\ \frac{8}{8}A + \left(\frac{5}{8}A\right) + \left(\frac{6}{8}A\right) & = 1 \end{cases} \Rightarrow \begin{cases} C & = \frac{6}{8}A \\ B & = \frac{5}{8}A \\ A & = \frac{8}{19} \end{cases} \Rightarrow \begin{cases} C & = \frac{6}{8}\left(\frac{8}{19}\right) \\ B & = \frac{5}{8}\left(\frac{8}{19}\right) \\ A & = \frac{8}{19} \end{cases} \Rightarrow \begin{cases} A & = \frac{8}{19} \\ B & = \frac{5}{19} \\ C & = \frac{6}{19} \end{cases}$$

$$P_0 \cdot \bar{X} = \bar{X} = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.6 & 0.2 \\ 0.1 & 0.2 & 0.7 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \frac{8}{19} \\ \frac{5}{19} \\ \frac{6}{19} \end{bmatrix} = \begin{bmatrix} 0.42105263 \\ 0.26315789 \\ 0.31578947 \end{bmatrix}$$

$$\bar{P} = \begin{bmatrix} \frac{8}{19}, \frac{8}{19}, \frac{8}{19} \\ \frac{5}{19}, \frac{8}{19}, \frac{8}{19} \\ \frac{6}{19}, \frac{8}{19}, \frac{8}{19} \end{bmatrix}$$

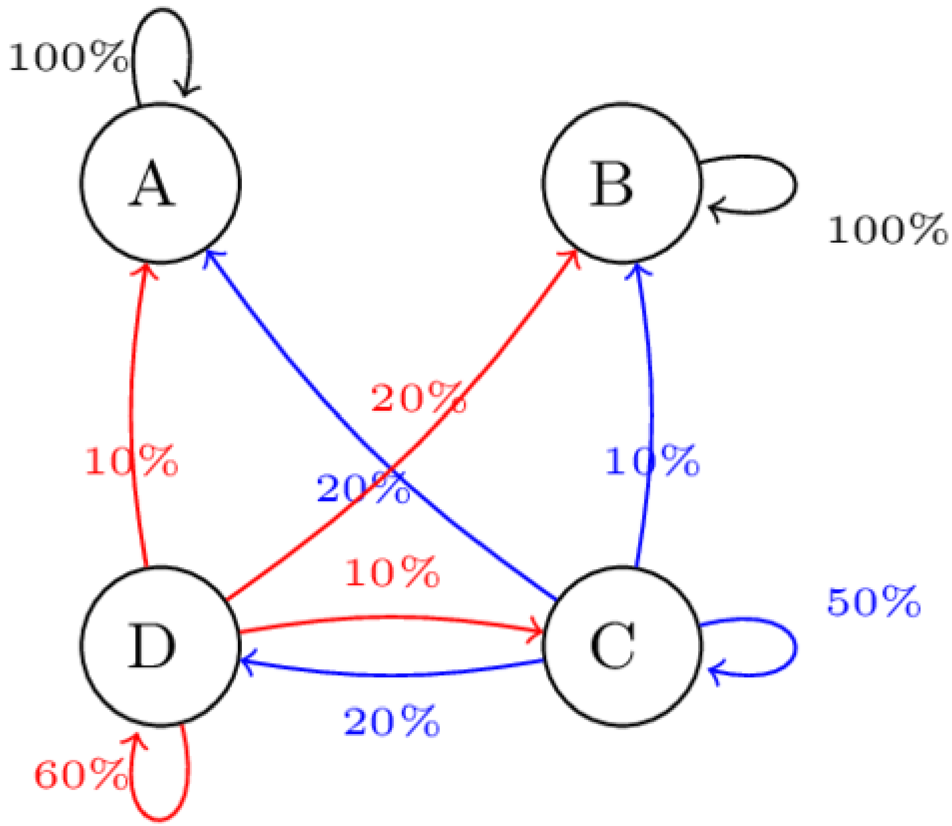
```
[0.8 0.2 0.1][0.421053 0.421053 0.421053]
[0.1 0.6 0.2][0.263158 0.263158 0.263158]
[0.1 0.2 0.7][0.315789 0.315789 0.315789]
[0.42105263 0.26315789 0.31578947]
8/19 8/19 8/19
5/19 5/19 5/19
6/19 6/19 6/19
```

[Prob & Stats - Markov Chains (19 of 38) Absorbing Markov Chains - Definition 1 (https://youtu.be/bpWV66hnRvQ)](https://youtu.be/bpWV66hnRvQ)



$$\begin{array}{ccc} & \overbrace{\begin{array}{ccc} A & B & C \end{array}}^{P\text{:from}} & \\ [P_0] = & \begin{bmatrix} 1 & 0.2 & 0.2 \\ 0 & 0.6 & 0.1 \\ \underline{0} & \underline{0.2} & \underline{0.7} \end{bmatrix} & \begin{array}{c} toA \\ toB \\ toC \end{array} \\ & \begin{array}{ccc} 1.0 & 1.0 & 1.0 \end{array} & \end{array}$$

```
[1 0.2 0.1][1 0.34 0.21][1 0.446 0.315][1 0.5306 0.4097][1 0.6003  0.49291]
[0 0.6 0.2][0 0.4  0.26][0 0.292 0.262][0 0.2276 0.2418][0 0.18492 0.21478]
[0 0.2 0.7][0 0.26 0.53][0 0.262 0.423][0 0.2418 0.3485][0 0.21478 0.29231]
[1 1 1]
[0 0 0]
[0 0 0]
1 1 1
0 0 0
0 0 0
```



$$[P_0] = \begin{array}{c} \overbrace{\begin{array}{cccc} \;A\; & \;B\; & \;C\; & \;D\; \end{array}}^{P\text{:from}} \\ \left[\begin{array}{cccc} 1 & 0 & 0.2 & 0.1 \\ 0 & 1 & 0.1 & 0.2 \\ 0 & 0 & 0.5 & 0.1 \\ \underline{0} & \underline{0} & \underline{0.2} & \underline{0.6} \end{array}\right] \\ \begin{array}{cccc} 1 & 1 & 1.0 & 1.0 \end{array} \end{array} \begin{array}{l} \\ to A \\ to B \\ to C \\ to D \\ \\ \end{array}$$

```
[1 0 0.2 0.1][1 0 0.32 0.18][1 0 0.396 0.24 ][1 0 0.446  0.2836][1 0 0.47972 0.31476]
[0 1 0.1 0.2][0 1 0.19 0.33][0 1 0.261 0.417][0 1 0.3139 0.4763][0 1 0.35221 0.51717]
[0 0 0.5 0.1][0 0 0.27 0.11][0 0 0.157 0.093][0 0 0.0971 0.0715][0 0 0.06285 0.05261]
[0 0 0.2 0.6][0 0 0.22 0.38][0 0 0.186 0.25 ][0 0 0.143  0.1686][0 0 0.10522 0.11546]
[1 0 1 0]
[0 1 0 1]
[0 0 0 0]
[0 0 0 0]
1 0 5/9 7/18
0 1 4/9 11/18
0 0 0   0
0 0 0   0
```

[Prob & Stats - Markov Chains (20 of 38) Absorbing Markov Chains - Definition 2 (https://youtu.be/S_QPpEELwZk)](https://youtu.be/S_QPpEELwZk)

Matrix of Transition Probabilities P:

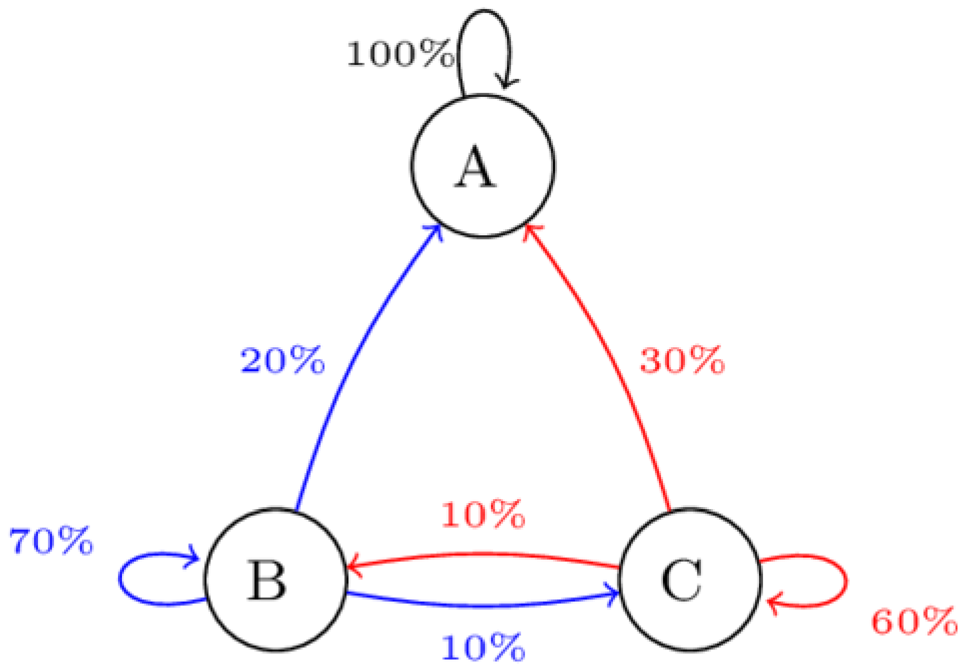
$$P = \begin{array}{c} \overbrace{\begin{array}{ccccc} S_1 & S_2 & S_3 & \dots & S_n \end{array}}^{\text{From}} \\ \begin{bmatrix} P_{11} & P_{12} & P_{13} & \dots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \dots & P_{2n} \\ P_{31} & P_{32} & P_{33} & \dots & P_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{P_{n1}} & \underline{P_{n2}} & \underline{P_{n3}} & \dots & \underline{P_{nn}} \end{bmatrix} \\ \begin{array}{ccccc} 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{array} \end{array} \begin{array}{c} \text{to } S_1 \\ \text{to } S_2 \\ \text{to } S_3 \\ \vdots \\ \text{to } S_n \end{array}$$

P_{ij} - Вероятность перехода из $S_j \longrightarrow S_i$.

Если $P_{ij} == 1$ значит состояние уже не меняется.

Если $P_{ij} == 1$ при $i == j$ и другие значения тогда $P_{ij} == 0$ получаем марковскую цепь с поглощением.

Марковская цепь с поглощением: $$P = \begin{array}{c} \overbrace{\begin{array}{ccc} S_1 & S_2 & S_3 \end{array}}^{P\text{:from}} \\ \begin{bmatrix} 0.6 & 0 & 0.1 \\ 0.3 & 1 & 0.2 \\ \underline{0.1} & \underline{0} & \underline{0.7} \end{bmatrix} \\ \begin{array}{ccc} 1.0 & 1.0 & 1.0 \end{array} \end{array} \begin{array}{c} \text{to } S_1 \\ \text{to } S_2 \\ \text{to } S_3 \end{array}$$



$$\begin{array}{cc} & \overbrace{\begin{array}{ccc} A & B & C \end{array}}^{P\text{:from}} \\ [P_0] = & \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0 & 0.7 & 0.1 \\ \underline{0} & \underline{0.1} & \underline{0.6} \end{bmatrix} \begin{array}{l} \text{to } A \\ \text{to } B \\ \text{to } C \end{array} \\ & \begin{array}{ccc} 1 & 1.0 & 1.0 \end{array} \end{array}$$

$$\bar{X} = ?$$

$P \cdot \bar{X} = \bar{X}$, Let $\bar{X} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$:

$$\begin{bmatrix} 1 & 0.2 & 0.3 \\ 0 & 0.7 & 0.1 \\ 0 & 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$\begin{cases} 1 \cdot A + 0.2 \cdot B + 0.3 \cdot C & = A \\ 0 \cdot A + 0.7 \cdot B + 0.1 \cdot C & = B \\ 0 \cdot A + 0.1 \cdot B + 0.6 \cdot C & = C \\ A + B + C & = 1 \end{cases} \Rightarrow \begin{cases} A + 0.2B + 0.3C & = A \\ \quad 0.7B + 0.1C & = B \\ \quad 0.1B + 0.6C & = C \\ A + B + C & = 1 \end{cases} \Rightarrow \begin{cases} A + 0.2B + 0.3C & = A \\ \quad 7B + C & = 10B \\ \quad B + 6C & = 10C \\ A + B + C & = 1 \end{cases} \Rightarrow$$

$$\begin{cases} A + 0.2B + 0.3C & = A \\ \quad C & = 3B \\ \quad B & = 4C \\ A + B + C & = 1 \end{cases} \Rightarrow \quad \begin{cases} C & = 3B \\ C & = \frac{B}{4} \end{cases} \quad \Rightarrow \begin{cases} ????? \\ ????? \end{cases}$$

$C = 3B$ и $C = \frac{B}{4}$ возможно только в одном случае, когда $B == 0$ и $C == 0$

$$\begin{bmatrix} 1 & 0.2 & 0.3 \\ 0 & 0.7 & 0.1 \\ 0 & 0.1 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

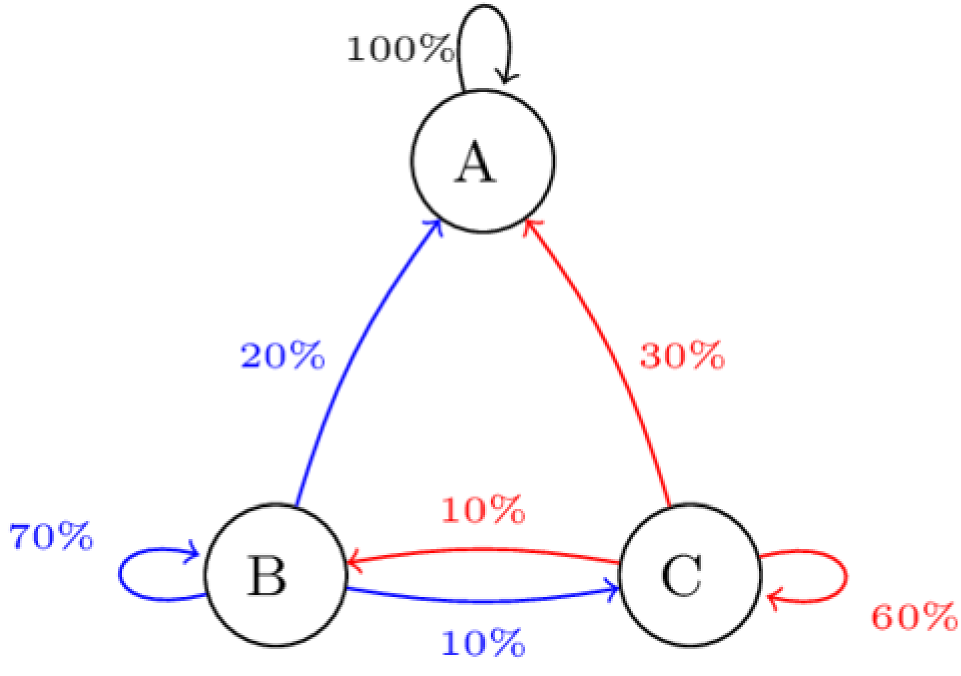
[Prob & Stats - Markov Chains (22 of 38) Absorbing Markov Chains - Example 2 (https://youtu.be/1bErNmzD8Sw)](https://youtu.be/1bErNmzD8Sw)

$$\begin{array}{cccc} & \overbrace{\begin{array}{ccc} A & B & C \end{array}}^{P\text{:from}} & \\ [P_0] = & \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0 & 0.7 & 0.1 \\ \underline{0} & \underline{0.1} & \underline{0.6} \end{bmatrix} & \begin{array}{l} \text{to } A \\ \text{to } B \\ \text{to } C \end{array} \\ & \begin{array}{ccc} 1 & 1.0 & 1.0 \end{array} & \end{array}$$

$$\bar{X} = ?$$

$$\bar{P} = ?$$

:




```
[1 0.2 0.3][1 0.37 0.5 ][1 0.509 0.637][1 0.62   0.7331][1 0.70731 0.80186]
[0 0.7 0.1][0 0.5  0.13][0 0.363 0.128][0 0.2669 0.1131][0 0.19814 0.09455]
[0 0.1 0.6][0 0.13 0.37][0 0.128 0.235][0 0.1131 0.1538][0 0.09455 0.10359]
[[1.00000000e+00 9.96135010e-01 9.97610450e-01]
 [0.00000000e+00 2.38954974e-03 1.47544012e-03]
 [0.00000000e+00 1.47544012e-03 9.14109625e-04]]
[1 1 1]
[0 0 0]
[0 0 0]
1 621394/623805 994876/997259
0 2383/997259   631/427669
0 631/427669    865/946276
[1. 0. 0.]
```

$$P = \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0 & 0.7 & 0.1 \\ 0 & 0.1 & 0.6 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0 & 0.7 & 0.1 \\ 0 & 0.1 & 0.6 \end{bmatrix}\begin{bmatrix} 1 & 0.2 & 0.3 \\ 0 & 0.7 & 0.1 \\ 0 & 0.1 & 0.6 \end{bmatrix} = \begin{bmatrix} 1 & 0.37 & 0.5 \\ 0 & 0.5 & 0.13 \\ 0 & 0.13 & 0.37 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0.37 & 0.5 \\ 0 & 0.5 & 0.13 \\ 0 & 0.13 & 0.37 \end{bmatrix}\begin{bmatrix} 1 & 0.2 & 0.3 \\ 0 & 0.7 & 0.1 \\ 0 & 0.1 & 0.6 \end{bmatrix} = \begin{bmatrix} 1 & 0.509 & 0.637 \\ 0 & 0.363 & 0.128 \\ 0 & 0.128 & 0.235 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 1 & 0.509 & 0.637 \\ 0 & 0.363 & 0.128 \\ 0 & 0.128 & 0.235 \end{bmatrix}\begin{bmatrix} 1 & 0.2 & 0.3 \\ 0 & 0.7 & 0.1 \\ 0 & 0.1 & 0.6 \end{bmatrix} = \begin{bmatrix} 1 & 0.62 & 0.7331 \\ 0 & 0.2669 & 0.1131 \\ 0 & 0.1131 & 0.1538 \end{bmatrix}$$

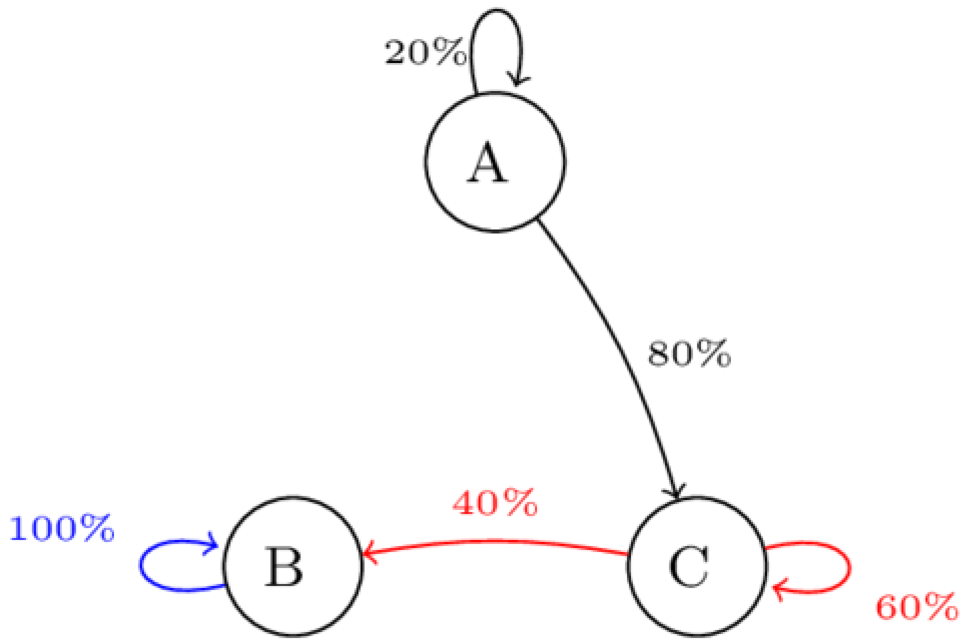
$$P^5 = \begin{bmatrix} 1 & 0.62 & 0.7331 \\ 0 & 0.2669 & 0.1131 \\ 0 & 0.1131 & 0.1538 \end{bmatrix}\begin{bmatrix} 1 & 0.2 & 0.3 \\ 0 & 0.7 & 0.1 \\ 0 & 0.1 & 0.6 \end{bmatrix} = \begin{bmatrix} 1 & 0.70731 & 0.80186 \\ 0 & 0.19814 & 0.09455 \\ 0 & 0.09455 & 0.10359 \end{bmatrix}$$

$$\vdots \qquad \vdots \qquad\qquad\qquad \vdots$$

$$P^{20} = \begin{bmatrix} 1 & 0.99334084 & 0.99588146 \\ 0 & 0.00411854 & 0.00254062 \\ 0 & 0.00254062 & 0.00157792 \end{bmatrix}\begin{bmatrix} 1 & 0.2 & 0.3 \\ 0 & 0.7 & 0.1 \\ 0 & 0.1 & 0.6 \end{bmatrix} = \begin{bmatrix} 1 & 0.99492673 & 0.99686296 \\ 0 & 0.00313704 & 0.00193623 \\ 0 & 0.00193623 & 0.00120081 \end{bmatrix}$$

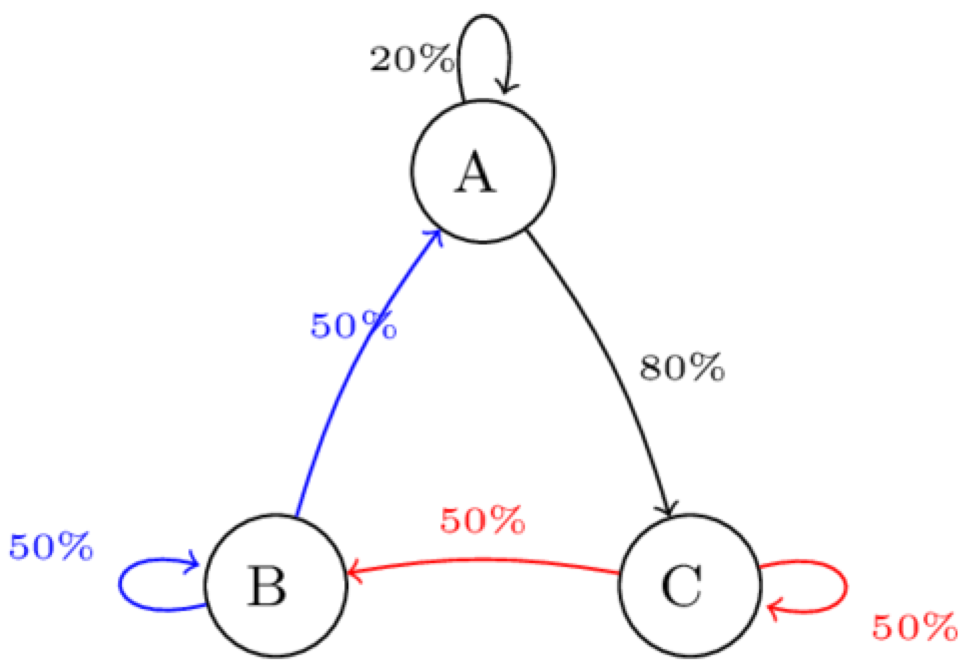
$$\bar{P} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \bar{X} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

[Prob & Stats - Markov Chains (23 of 38) Absorbing and Non-Absorbing Markov Chain (https://youtu.be/hMceS_HIcKY)](https://youtu.be/hMceS_HIcKY)

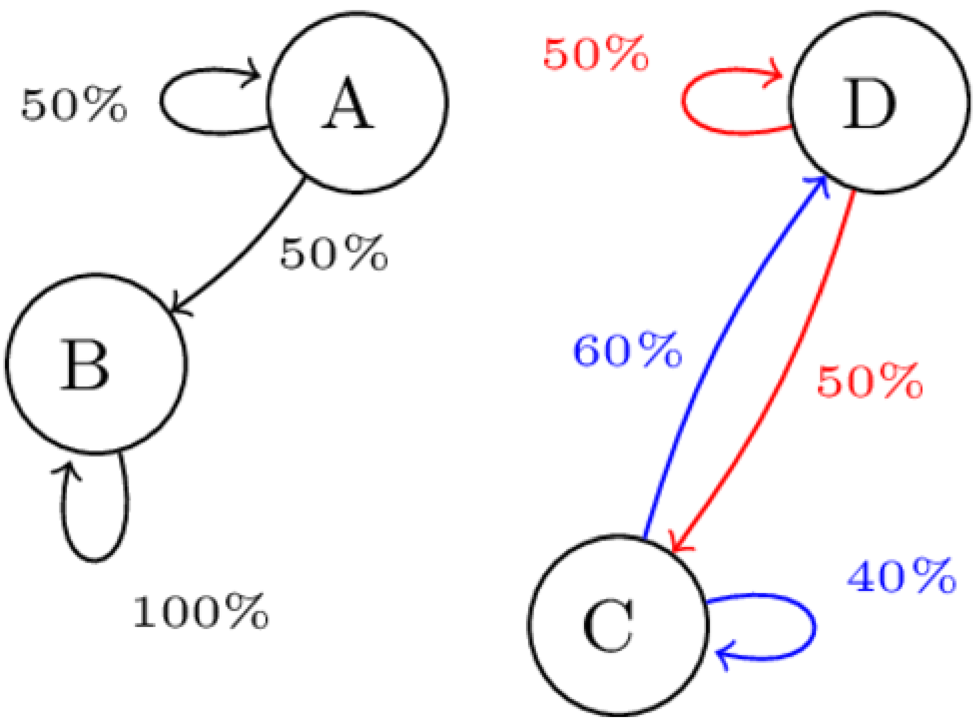


$$
\begin{array}{cc}
 & \overbrace{\begin{array}{ccc} A & B & C \end{array}}^{P\text{:from}} \\
P_0 = & \left[\begin{array}{c|cc} 1 & 0.2 & 0.3 \\ 0 & 0.7 & 0.1 \\ \underline{0} & \underline{0.1} & \underline{0.6} \end{array}\right] \begin{array}{l} \text{to } A \\ \text{to } B \\ \text{to } C \end{array} \\
 & \begin{array}{ccc} 1 & 1.0 & 1.0 \end{array}
\end{array}
$$

ABSORBING



$$
\begin{array}{c}
\overbrace{\begin{array}{ccc} \quad A \quad & \quad B \quad & \quad C \quad \end{array}}^{P\text{:from}} \\
P_0 = \begin{bmatrix} 0.2 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ \underline{0.8} & \underline{0} & \underline{0.5} \end{bmatrix}
\begin{array}{l} \text{to } A \\ \text{to } B \\ \text{to } C \end{array} \\
\begin{array}{ccc} 1.0 & 1.0 & 1.0 \end{array}
\end{array}
\qquad NOT \qquad ABSORBING
$$



$$
\begin{array}{c}
\overbrace{\begin{array}{cccc} A & B & C & D \end{array}}^{P\text{:from}} \\
P_0 = \left[\begin{array}{c:c:cc} 0.5 & 1 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.5 \\ \underline{0} & \underline{0} & \underline{0.6} & \underline{0.5} \end{array}\right]
\begin{array}{l} \text{to } A \\ \text{to } B \\ \text{to } C \\ \text{to } D \end{array} \\
\begin{array}{cccc} 1.0 & 1 & 1.0 & 1.0 \end{array}
\end{array}
\qquad NOT \qquad ABSORBING
$$

```
[0.5 1 0   0  ][0.75 0.5 0    0   ][0.625 0.75 0     0     ][0.688 0.625 0.000 0.000]
[0.5 0 0   0  ][0.25 0.5 0    0   ][0.375 0.25 0     0     ][0.312 0.375 0.000 0.000]
[0   0 0.4 0.5][0    0   0.46 0.45][0     0    0.454 0.455][0.000 0     0.455 0.455]
[0   0 0.6 0.5][0    0   0.54 0.55][0     0    0.546 0.545][0.000 0     0.545 0.546]
[[0.66666667 0.66666667 0.         0.        ]
 [0.33333333 0.33333333 0.         0.        ]
 [0.         0.         0.45454545 0.45454545]
 [0.         0.         0.54545455 0.54545455]]
[1 1 0 0]
[0 0 0 0]
[0 0 0 0]
[0 0 1 1]
[0.66666667 0.33333333 0.         0.        ]
```

[Prob & Stats - Markov Chains (24 of 38) Absorbing Markov Chain in Standard Form (https://youtu.be/UuZU3LUBaIQ)](https://youtu.be/UuZU3LUBaIQ)

$$P=\begin{array}{c} \overbrace{\begin{array}{ccc} A & B & C \end{array}}^{P\text{:from}} \\ \left[\begin{array}{c|c|c} 0.3 & 0 & 0 \\ 0 & 1 & 0.6 \\ \underline{0.7} & \underline{0} & \underline{0.4} \end{array}\right] \\ \begin{array}{ccc} 1.0 & 1 & 1.0 \end{array} \end{array} \begin{array}{l} \text{to } A \\ \text{to } B \\ \text{to } C \end{array} \Rightarrow P=\begin{array}{c} \overbrace{\begin{array}{ccc} B & A & C \end{array}}^{P\text{:from}} \\ \left[\begin{array}{c|cc} 1 & 0 & 0.6 \\ 0 & 0.3 & 0 \\ \underline{0} & \underline{0.7} & \underline{0.4} \end{array}\right] \\ \begin{array}{ccc} 1 & 1.0 & 1.0 \end{array} \end{array} \begin{array}{l} \text{to } B \\ \text{to } A \\ \text{to } C \end{array}$$

STANDART Form

ABSORBING

$$P=\begin{array}{c} \overbrace{\begin{array}{cccc} A & B & C & D \end{array}}^{\text{FROM}} \\ \left[\begin{array}{c|c|c|c} 0.4 & 0 & 0.3 & 0 \\ 0.3 & 1 & 0.4 & 0 \\ 0.1 & 0 & 0.2 & 0 \\ \underline{0.2} & \underline{0} & \underline{0.1} & \underline{1} \end{array}\right] \\ \begin{array}{cccc} 1.0 & 1 & 1.0 & 1 \end{array} \end{array} \begin{array}{l} \text{to } A \\ \text{to } B \\ \text{to } C \\ \text{to } D \end{array} \Rightarrow P=\begin{array}{c} \overbrace{\begin{array}{cccc} B & D & A & C \end{array}}^{\text{FROM}} \\ \left[\begin{array}{c|c|cc} 1 & 0 & 0.3 & 0.4 \\ 0 & 1 & 0.2 & 0.1 \\ 0 & 0 & 0.4 & 0.3 \\ \underline{0} & \underline{0} & \underline{0.1} & \underline{0.2} \end{array}\right] \\ \begin{array}{cccc} 1 & 1 & 1.0 & 1.0 \end{array} \end{array} \begin{array}{l} \text{to } B \\ \text{to } D \\ \text{to } A \\ \text{to } C \end{array}$$

STANDART Form

ABSORBING

[Prob & Stats - Markov Chains (25 of 38) Absorbing Markov Chain: Stable Matrix=? (https://youtu.be/bj_O4edCwgc)](https://youtu.be/bj_O4edCwgc)

$$P=\begin{array}{c} \overbrace{\begin{array}{ccc} A & B & C \end{array}}^{\text{FROM}} \\ \left[\begin{array}{c|cc} 1 & 0.4 & 0.3 \\ 0 & 0.3 & 0.2 \\ \underline{0} & \underline{0.3} & \underline{0.5} \end{array}\right] \\ \begin{array}{ccc} 1 & 1.0 & 1.0 \end{array} \end{array} \begin{array}{l} \text{to } A \\ \text{to } B \\ \text{to } C \end{array} \Rightarrow \qquad \bar{P}=\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \bar{X}=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$\boxed{\bar{P}=\left[\begin{array}{c|c} I & S \\ \hdashline O & R \end{array}\right]^N=\left[\begin{array}{c|c} I & S(I-R)^{-1} \\ \hdashline O & O \end{array}\right] \qquad \text{STANDART} \qquad \text{Form}}$$

$$P_0=\left[\begin{array}{c|c} I & S \\ \hdashline O & R \end{array}\right]=\left[\begin{array}{c|cc} 1 & 0.4 & 0.3 \\ \hdashline 0 & 0.3 & 0.2 \\ 0 & 0.3 & 0.3 \end{array}\right]$$

$$P_0 = \left[\begin{array}{c|c} I & S \\ \hline O & R \end{array}\right] = \begin{array}{c} \overbrace{\begin{array}{cccc} A & B & C & D \end{array}}^{\text{FROM}} \\ \left[\begin{array}{cc|cc} 1 & 0 & 0.2 & 0.1 \\ 0 & 1 & 0.1 & 0.2 \\ \hline 0 & 0 & 0.5 & 0.1 \\ \underline{0} & \underline{0} & \underline{0.2} & \underline{0.6} \end{array}\right] \\ \begin{array}{cccc} 1 & 1 & 1.0 & 1.0 \end{array} \end{array} \begin{array}{l} \text{to } A \\ \text{to } B \\ \text{to } C \\ \text{to } D \end{array} \qquad \bar{X} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}$$

$$\bar{P} = \left[\begin{array}{c|c} I & S \\ \hline O & R \end{array}\right]^N = P_0^N = \left[\begin{array}{cc|cc} 1 & 0 & 0.2 & 0.1 \\ 0 & 1 & 0.1 & 0.2 \\ \hline 0 & 0 & 0.5 & 0.1 \\ 0 & 0 & 0.2 & 0.6 \end{array}\right]^N = \left[\begin{array}{c|c} I & S(I-R)^{-1} \\ \hline O & O \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 0 & ? & ? \\ 0 & 1 & ? & ? \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{P_0^1 = \left[\begin{array}{c|c} I & S \\ \hline O & R \end{array}\right]} = \left[\begin{array}{cc|cc} 1 & 0 & A & B \\ 0 & 1 & C & D \\ \hline 0 & 0 & v & w \\ 0 & 0 & x & z \end{array}\right]$$

$$P_0^2 = \left[\begin{array}{cc} I & S \\ \hline O & R \end{array}\right] \cdot \left[\begin{array}{c|c} I & S \\ O & R \end{array}\right] = \left[\begin{array}{c|c} I \cdot I + S \cdot O & I \cdot S + S \cdot R \\ \hline O \cdot I + R \cdot O & O \cdot S + R \cdot R \end{array}\right] = \left[\begin{array}{c|c} I & S + SR \\ \hline O & R^2 \end{array}\right]$$

$$\boxed{P_0^2 = \left[\begin{array}{c|c} I & S(I+R) \\ \hline O & R^2 \end{array}\right]}$$

$$P_0 \cdot P_0 = \left[\begin{array}{cccc} 1 & 0 & A & B \\ \hline 0 & 1 & C & D \\ \hline 0 & 0 & v & w \\ \hline 0 & 0 & x & z \end{array}\right] \left[\begin{array}{c|c|c|c} 1 & 0 & A & B \\ 0 & 1 & C & D \\ 0 & 0 & v & w \\ 0 & 0 & x & z \end{array}\right] =$$

$$= \left[\begin{array}{c|c|c|c} 1 \cdot 1 + 0 \cdot 0 + A \cdot 0 + B \cdot 0 & 1 \cdot 0 + 0 \cdot 1 + A \cdot 0 + B \cdot 0 & 1 \cdot A + 0 \cdot C + Av + Bx & 1 \cdot B + 0 \cdot D + Aw + Bz \\ \hline 0 \cdot 1 + 1 \cdot 0 + C \cdot 0 + D \cdot 0 & 0 \cdot 0 + 1 \cdot 1 + C \cdot 0 + D \cdot 0 & 0 \cdot A + 1 \cdot C + Cv + Dx & 0 \cdot B + 1 \cdot D + Cw + Dz \\ \hline 0 \cdot 1 + 0 \cdot 0 + v \cdot 0 + w \cdot 0 & 0 \cdot 0 + 0 \cdot 1 + v \cdot 0 + w \cdot 0 & 0 \cdot A + 0 \cdot C + vv + wx & 0 \cdot B + 0 \cdot D + vw + wz \\ \hline 0 \cdot 1 + 0 \cdot 0 + x \cdot 0 + z \cdot 0 & 0 \cdot 0 + 0 \cdot 1 + x \cdot 0 + z \cdot 0 & 0 \cdot A + 0 \cdot C + xv + zx & 0 \cdot B + 0 \cdot D + xw + zz \end{array}\right] =$$

$$= \left[\begin{array}{c|c|c|c} 1 & 0 & A + Av + Bx & B + Aw + Bz \\ \hline 0 & 1 & C + Cv + Dx & D + Cw + Dz \\ \hline 0 & 0 & v^2 + wx & vw + wz \\ \hline 0 & 0 & xv + zx & xw + z^2 \end{array}\right]$$

$$S \cdot R = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \cdot \left[\begin{array}{c|c} v & w \\ x & z \end{array}\right] = \left[\begin{array}{c|c} Av + Bx & Aw + Bz \\ \hline Cv + Dx & Cw + Dz \end{array}\right]$$

$$S + S \cdot R = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] + \left[\begin{array}{c|c} Av + Bx & Aw + Bz \\ \hline Cv + Dx & Cw + Dz \end{array}\right] =$$

$$= \boxed{\left[\begin{array}{c|c} A + Av + Bx & B + Aw + Bz \\ \hline C + Cv + Dx & D + Cw + Dz \end{array}\right] = S + S \cdot R}$$

$$R \cdot R = \left[\begin{array}{cc} v & w \\ \hline x & z \end{array}\right] \cdot \left[\begin{array}{c|c} v & w \\ x & z \end{array}\right] = \left[\begin{array}{c|c} vv + wx & vw + wz \\ \hline xv + zx & xw + zz \end{array}\right] =$$

$$= \boxed{\left[\begin{array}{c|c} v^2 + wx & vw + wz \\ \hline xv + zx & xw + z^2 \end{array}\right] = R^2}$$

$$P_0^3 = P_0^2 \cdot \left[\begin{array}{c|c} I & S \\ \hline O & R \end{array}\right] = \left[\begin{array}{c|c} I & S + S \cdot R \\ \hline O & R^2 \end{array}\right] \cdot \left[\begin{array}{c|c} I & S \\ \hline O & R \end{array}\right] =$$

$$= \left[\begin{array}{c|c} I \cdot I + (S + SR) \cdot O & I \cdot S + (S + SR) \cdot R \\ \hline O \cdot I + R^2 \cdot O & O \cdot S + R^2 \cdot R \end{array}\right] = \left[\begin{array}{c|c} I & S + SR + SR^2 \\ \hline O & R^3 \end{array}\right] =$$

$$\boxed{P_0^3 = \left[\begin{array}{c|c} I & S(I + R + R^2) \\ \hline O & R^3 \end{array}\right]}$$

$$= \left[\begin{array}{c|c|c|c} 1 & 0 & (A + Av + Bx) & (B + Aw + Bz) \\ \hline 0 & 1 & (C + Cv + Dx) & (D + Cw + Dz) \\ \hline 0 & 0 & (v^2 + wx) & (vw + wz) \\ \hline 0 & 0 & (xv + zx) & (xw + z^2) \end{array}\right] \cdot \left[\begin{array}{c|c|c|c} 1 & 0 & A & B \\ 0 & 1 & C & D \\ 0 & 0 & v & w \\ 0 & 0 & x & z \end{array}\right] =$$

$$P_0^4 = P_0^3 \cdot \left[\begin{array}{c|c} I & S \\ \hline O & R \end{array}\right] = \left[\begin{array}{c|c} I & S + SR + SR^2 \\ \hline O & R^3 \end{array}\right] \cdot \left[\begin{array}{c|c} I & S \\ \hline O & R \end{array}\right] =$$

$$= \left[\begin{array}{c|c} I \cdot I + (S + SR + SR^2) \cdot O & I \cdot S + (S + SR + SR^2) \cdot R \\ \hline O \cdot I + R^3 \cdot O & O \cdot S + R^3 \cdot R \end{array}\right] = \left[\begin{array}{c|c} I & S + SR + SR^2 + SR^3 \\ \hline O & R^4 \end{array}\right] =$$

$$\boxed{P_0^4 = \left[\begin{array}{c|c} I & S(I + R + R^2 + R^3) \\ \hline O & R^4 \end{array}\right]}$$

$$\bar{P} = P_0^N = \left[\begin{array}{c:c} I & S \\ \hdashline O & R \end{array}\right]^N = \left[\begin{array}{c:c} I & S + S \cdot R + S \cdot R^2 + S \cdot R^3 + \cdots + S \cdot R^{N-1} \\ \hdashline O & R^N \end{array}\right] =$$

$$\boxed{\bar{P} = \left[\begin{array}{c:c} I & S(I + R + R^2 + R^3 + \cdots + R^{N-1}) \\ \hdashline O & R^N \end{array}\right] \Rightarrow \left[\begin{array}{c:c} I & S(I - R)^{-1} \\ \hdashline O & R^N \end{array}\right]}$$

[Геометрическая прогрессия (https://ru.wikipedia.org/wiki/%D0%93%D0%B5%D0%BE%D0%BC%D0%B5%D1%82%D1%%](https://ru.wikipedia.org/wiki/%D0%93%D0%B5%D0%BE%D0%BC%D0%B5%D1%82%D1%)

Геометри́ческая прогре́ссия — последовательность чисел $b_1, b_2, b_3, \ldots$ (называемых членами прогрессии), в которой каждое последующее число, начиная со второго, получается из предыдущего умножением его на определённое число q (называемое знаменателем прогрессии), где $b_1 \neq 0$, $q \neq 0$: $b_1, b_2 = b_1 q, b_3 = b_2 q, \ldots, b_n = b_{n-1} q$

Любой член геометрической прогрессии может быть вычислен по формуле

$$b_n = b_1 q^{n-1}$$

Если $b_1 > 0$ и $q > 1$, прогрессия является ***возрастающей*** последовательностью, если $0 < q < 1$, — ***убывающей*** последовательностью, а при $q < 0$ — ***знакочередующейся***, при $q = 1$ — ***стационарной***.

Сумма n первых членов геометрической прогрессии

$$S_n = \begin{cases} \sum_{i=1}^{n} b_i = \dfrac{b_1 - b_1 q^n}{1 - q} = \dfrac{b_1 (1 - q^n)}{1 - q}, & \text{if } q \neq 1 \\ n b_1, & \text{if } q = 1 \end{cases}$$

Сумма всех членов убывающей прогрессии:

$|q| < 1$ то то $b_n \to 0$ при $n \to +\infty$, и

$S_n \to \frac{b_1}{1-q}$ при $n \to +\infty$

$$\begin{array}{llllllll} S_n & = 1 & +q & +q^2 & +q^3 & +q^4 & +\cdots & +q^n \end{array}$$

$$\begin{array}{llllllll} q \cdot S_n & = 1 \cdot q & +q \cdot q & +q^2 \cdot q & +q^3 \cdot q & +q^4 \cdot q & +\cdots & +q^n \cdot q \\ qS_n & = q & +q^2 & +q^3 & +q^4 & +q^5 & +\cdots & +q^{n+1} \end{array}$$

Вычитаем:

$$\begin{array}{llllllll} S_n & = 1 & +q & +q^2 & +q^3 & +q^4 & +\cdots & +q^n \\ - & - & - & - & - & - & & - \\ qS_n & = q & +q^2 & +q^3 & +q^4 & +q^5 & +\cdots & +q^{n+1} \\ \hline S_n - qS_n & = 1 - q & +q - q^2 & +q^2 - q^3 & +q^3 - q^4 & +q^4 - q^5 & +\cdots & +q^n - q^{n+1} \end{array}$$

$$\begin{array}{llllllll} S_n(1-q) & = 1 & +0 & +0 & +0 & +0+0 & \cdots & +0 - q^{n+1} \\ S_n(1-q) & = 1 - q^{n+1} \end{array}$$

$$\boxed{S_n = \frac{1 - q^{n+1}}{1 - q}}$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1 - q^{n+1}}{1 - q} =$$

при $n \to \infty$ $q^{n+1} \to 0$:

$$\boxed{S = \frac{1}{1-q}}$$

$$I + R + R^2 + R^3 + \cdots + R^{N-1} = \frac{1}{I - R} = (I - R)^{-1}$$

$$\boxed{\bar{P} = \left[\begin{array}{c:c} I & S(I + R + R^2 + R^3 + \cdots + R^{N-1}) \\ \hdashline O & R^N \end{array}\right] = \left[\begin{array}{c:c} I & S(I - R)^{-1} \\ \hdashline O & R^N \end{array}\right]}$$

[Prob & Stats - Markov Chains (26 of 38) Absorbing Markov Chain: Stable Matrix=? Ex. 1 (https://youtu.be/72lpee3ueUs)](https://youtu.be/72lpee3ueUs)

$$\begin{array}{cc} & \overbrace{\begin{array}{ccc} A & B & C \end{array}}^{\text{FROM}} \\ P = & \left[\begin{array}{c:cc} 1 & 0.4 & 0.3 \\ \hdashline 0 & 0.3 & 0.2 \\ \underline{0} & \underline{0.3} & \underline{0.5} \end{array}\right] \\ & \begin{array}{ccc} 1 & 1.0 & 1.0 \end{array} \end{array} \begin{array}{l} \\ \text{to } A \\ \text{to } B \\ \text{to } C \\ \\ \end{array}$$

$$\bar{P} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \bar{X} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$I-R=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}0.3 & 0.2\\ 0.3 & 0.5\end{bmatrix}=\begin{bmatrix}1-0.3 & 0-0.2\\ 0-0.3 & 1-0.5\end{bmatrix}=\begin{bmatrix}0.7 & -0.2\\ -0.3 & 0.5\end{bmatrix}$$

Determinant $D=(0.7)\cdot(0.5)-(-0.2)\cdot(-0.3)=0.35-0.06=0.29$

$$(I-R)^{-1}=\frac{1}{D}\left[C^{*}\right]=\frac{1}{0.29}\cdot\begin{bmatrix}0.5 & 0.2\\ 0.3 & 0.7\end{bmatrix}$$

```
Determinant == 0.29 == 29/100
[1 0][0.3 0.2][0.7  -0.2]
[0 1][0.3 0.5][-0.3 0.5 ]
cofactor:
 [[0.5 0.3]
 [0.2 0.7]]
cofactor.T
 [[0.5 0.2]
 [0.3 0.7]]
A Inverted ==
[1.72414 0.689655]
[1.03448 2.41379 ]
Checking: A*cofactor' A* A'(Inverted)   A' - A'(fraction)
[  0.29  -0.00][  1.00  -0.00]
[  0.00   0.29][  0.00   1.00]
[1.72414 0.689655][1.72414 0.689655][0 0.000000]
[1.03448 2.41379 ][1.03448 2.41379 ][0 0.000000]
```

$$\boxed{\begin{gathered}\bar{P}=\left[\begin{array}{c:c}I & S\\ \hdashline O & R\end{array}\right]^{N}=\left[\begin{array}{c:cc}1 & 0.4 & 0.3\\ \hdashline 0 & 0.3 & 0.2\\ 0 & 0.3 & 0.3\end{array}\right]^{N}=\left[\begin{array}{c:c}I & S(I-R)^{-1}\\ \hdashline O & O\end{array}\right]\\ I=\begin{bmatrix}1\end{bmatrix}\qquad O=\begin{bmatrix}0\\ 0\end{bmatrix}\qquad S=\begin{bmatrix}0.4 & 0.3\end{bmatrix}\qquad R=\begin{bmatrix}0.3 & 0.2\\ 0.3 & 0.3\end{bmatrix}\end{gathered}}$$

через десятичные дроби :

$$S\cdot(I-R)^{-1}=S\cdot\frac{1}{D}\left[C^{*}\right]=\begin{bmatrix}0.4 & 0.3\end{bmatrix}\cdot\frac{1}{0.29}\begin{bmatrix}0.5 & 0.2\\ 0.3 & 0.7\end{bmatrix}=\frac{1}{0.29}\begin{bmatrix}0.4 & 0.3\end{bmatrix}\cdot\begin{bmatrix}0.5 & 0.2\\ 0.3 & 0.7\end{bmatrix}=$$

$$=\frac{1}{0.29}\cdot\left[\begin{array}{c:c}0.4\cdot0.5+0.3\cdot0.3 & 0.4\cdot0.2+0.3\cdot0.7\end{array}\right]=\frac{1}{0.29}\cdot\left[\begin{array}{c:c}0.20+0.09 & 0.08+0.21\end{array}\right]=$$

$$=\frac{1}{0.29}\cdot\left[\begin{array}{c:c}0.29 & 0.29\end{array}\right]=\begin{bmatrix}1 & 1\end{bmatrix}$$

$$\boxed{S\cdot(I-R)^{-1}=\left[\begin{array}{c:c}1 & 1\end{array}\right]}$$

через обычные дроби :

$$S\cdot(I-R)^{-1}=S\cdot\frac{1}{D}\left[C^{*}\right]=\begin{bmatrix}0.4 & 0.3\end{bmatrix}\cdot\frac{1}{0.29}\begin{bmatrix}0.5 & 0.2\\ 0.3 & 0.7\end{bmatrix}=$$

$$= \frac{1}{\frac{29}{100}} \cdot \left[\frac{4}{10} \quad \frac{3}{10} \right] \cdot \left[\begin{array}{c:c} \frac{5}{10} & \frac{2}{10} \\ \frac{3}{10} & \frac{7}{10} \end{array} \right] = \frac{1}{\frac{29}{100}} \cdot \left[\begin{array}{c:c} \frac{4}{10}\frac{5}{10} + \frac{3}{10}\frac{3}{10} & \frac{4}{10}\frac{2}{10} + \frac{3}{10}\frac{7}{10} \end{array} \right] =$$

$$\frac{1}{\frac{29}{100}} \cdot \left[\begin{array}{c:c} \frac{20+9}{100} & \frac{8+21}{100} \end{array} \right] = \frac{1}{\frac{29}{100}} \cdot \left[\begin{array}{c:c} \frac{29}{100} & \frac{29}{100} \end{array} \right] = \left[\begin{array}{c:c} 1 & 1 \end{array} \right]$$

$$\boxed{S \cdot (I - R)^{-1} = \left[\begin{array}{c:c} 1 & 1 \end{array} \right]}$$

$$\bar{P} = \left[\begin{array}{c:c} I & S \\ \hdashline O & R \end{array} \right]^N = P_0^N = \left[\begin{array}{cc:cc} 1 & 0 & 0.2 & 0.1 \\ 0 & 1 & 0.1 & 0.2 \\ \hdashline 0 & 0 & 0.5 & 0.1 \\ 0 & 0 & 0.2 & 0.6 \end{array} \right]^N = \left[\begin{array}{c:c} I & S(I-R)^{-1} \\ \hdashline O & O \end{array} \right] = \left[\begin{array}{cc:cc} 1 & 0 & ? & ? \\ 0 & 1 & ? & ? \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

```
[1 0 0.2 0.1][1 0 0.32 0.18][1 0 0.396 0.24 ][1 0 0.446 0.284]
[0 1 0.1 0.2][0 1 0.19 0.33][0 1 0.261 0.417][0 1 0.314 0.476]
[0 0 0.5 0.1][0 0 0.27 0.11][0 0 0.157 0.093][0 0 0.097 0.071]
[0 0 0.2 0.6][0 0 0.22 0.38][0 0 0.186 0.25 ][0 0 0.143 0.169]
P(30):
[1 0 0.555549 0.388882]
[0 1 0.444436 0.611102]
[0 0 0.000005 0.000005]
[0 0 0.000011 0.000011]
[5.55548543e-01 4.44435679e-01 5.25917971e-06 1.05183585e-05]
P(100):
[1 0 0.555556 0.388889]
[0 1 0.444444 0.611111]
[0 0 0.000000 0.000000]
[0 0 0.000000 0.000000]
1  0  5/9 7/18
0  1  4/9 11/18
0  0  0   0
0  0  0   0
```

$$(I - R) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.6 \end{bmatrix} = \begin{bmatrix} 1-0.5 & 0-0.1 \\ 0-0.2 & 1-0.6 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.1 \\ -0.2 & 0.4 \end{bmatrix}$$

Determinant $D = (0.5) \cdot (0.4) - (-0.1) \cdot (-0.2) = 0.20 - 0.02 = 0.18$

$$(I - R)^{-1} = \frac{1}{D}\left[C^* \right] = \frac{1}{0.18} \cdot \begin{bmatrix} 0.4 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}$$

```
[1 0][0.5 0.1][0.5  -0.1]
[0 1][0.2 0.6][-0.2 0.4 ]
Determinant == 0.18 == 9/50
cofactor => cofactor.T
[0.4 0.2][0.4 0.1]
[0.1 0.5][0.2 0.5]
A Inverted ==
[2.22222 0.555556]
[1.11111 2.77778 ]
S(I-R)^-1==
[0.555556 0.388889]
[0.444444 0.611111]
5/9 7/18
4/9 11/18
```

$$\boxed{\begin{array}{l}\bar{P} = \left[\begin{array}{c:c} I & S \\ \hdashline O & R \end{array}\right]^N = P_0^N = \left[\begin{array}{cc:cc} 1 & 0 & 0.2 & 0.1 \\ 0 & 1 & 0.1 & 0.2 \\ \hdashline 0 & 0 & 0.5 & 0.1 \\ 0 & 0 & 0.2 & 0.6 \end{array}\right]^N = \left[\begin{array}{c:c} I & S(I-R)^{-1} \\ \hdashline O & O \end{array}\right] \\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad S = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix} \quad R = \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.6 \end{bmatrix}\end{array}}$$

$$(I-R)^{-1} = \frac{1}{D}\left[C^*\right] = \frac{1}{0.18} \cdot \begin{bmatrix} 0.4 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}$$

через десятичные дроби :

$$S \cdot (I-R)^{-1} = S \cdot \frac{1}{D}\left[C^*\right] = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix} \cdot \frac{1}{0.18}\begin{bmatrix} 0.4 & 0.1 \\ 0.2 & 0.5 \end{bmatrix} = \frac{1}{0.18}\left[\begin{array}{cc} 0.2 & 0.1 \\ \hdashline 0.1 & 0.2 \end{array}\right] \cdot \left[\begin{array}{c:c} 0.4 & 0.1 \\ 0.2 & 0.5 \end{array}\right] =$$

$$= \frac{1}{0.18} \cdot \left[\begin{array}{c:c} 0.2 \cdot 0.4 + 0.1 \cdot 0.2 & 0.2 \cdot 0.1 + 0.1 \cdot 0.5 \\ 0.1 \cdot 0.4 + 0.2 \cdot 0.2 & 0.1 \cdot 0.1 + 0.2 \cdot 0.5 \end{array}\right] = \frac{1}{0.18} \cdot \left[\begin{array}{c:c} 0.08 + 0.02 & 0.02 + 0.05 \\ 0.04 + 0.04 & 0.01 + 0.10 \end{array}\right] =$$

$$= \frac{1}{0.18} \cdot \begin{bmatrix} 0.10 & 0.07 \\ 0.08 & 0.11 \end{bmatrix} = \begin{bmatrix} 0.555556 & 0.388889 \\ 0.444444 & 0.611111 \end{bmatrix}$$

$$\boxed{S \cdot (I-R)^{-1} = \begin{bmatrix} 0.555556 & 0.388889 \\ 0.444444 & 0.611111 \end{bmatrix}}$$

через обычные дроби :

$$S \cdot (I-R)^{-1} = S \cdot \frac{1}{D}\left[C^*\right] = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix} \cdot \frac{1}{0.18}\begin{bmatrix} 0.4 & 0.1 \\ 0.2 & 0.5 \end{bmatrix} = \frac{1}{0.18}\left[\begin{array}{cc} 0.2 & 0.1 \\ \hdashline 0.1 & 0.2 \end{array}\right] \cdot \left[\begin{array}{c:c} 0.4 & 0.1 \\ 0.2 & 0.5 \end{array}\right] =$$

$$= \frac{1}{\frac{18}{100}} \cdot \left[\begin{array}{cc} \frac{2}{10} & \frac{1}{10} \\ \hdashline \frac{1}{10} & \frac{2}{10} \end{array}\right] \cdot \left[\begin{array}{c:c} \frac{4}{10} & \frac{1}{10} \\ \frac{2}{10} & \frac{5}{10} \end{array}\right] = \frac{100}{18} \cdot \left[\begin{array}{c:c} \frac{2}{10}\frac{4}{10} + \frac{1}{10}\frac{2}{10} & \frac{2}{10}\frac{1}{10} + \frac{1}{10}\frac{5}{10} \\ \hdashline \frac{1}{10}\frac{4}{10} + \frac{2}{10}\frac{2}{10} & \frac{1}{10}\frac{1}{10} + \frac{2}{10}\frac{5}{10} \end{array}\right] =$$

$$= \frac{100}{18} \cdot \left[\begin{array}{c:c} \frac{8}{100} + \frac{2}{100} & \frac{2}{100} + \frac{5}{100} \\ \hdashline \frac{4}{100} + \frac{4}{100} & \frac{1}{100} + \frac{10}{100} \end{array}\right] = \frac{100}{18} \cdot \left[\begin{array}{c:c} \frac{10}{100} & \frac{7}{100} \\ \hdashline \frac{8}{100} & \frac{11}{100} \end{array}\right] = \left[\begin{array}{c:c} \frac{10}{18} & \frac{7}{18} \\ \hdashline \frac{8}{18} & \frac{11}{18} \end{array}\right]$$

$$\boxed{S \cdot (I - R)^{-1} = \left[\begin{array}{c:c} \frac{10}{18} & \frac{7}{18} \\ \hdashline \frac{8}{18} & \frac{11}{18} \end{array}\right]}$$

```
P(100):
[1 0 0.5556 0.3889]
[0 1 0.4444 0.6111]
[0 0 0.0000 0.0000]
[0 0 0.0000 0.0000]
1  0  5/9 7/18
0  1  4/9 11/18
0  0  0   0
0  0  0   0
[5.55555556e-01 4.44444444e-01 7.54711186e-17 1.50942237e-16]
```

$$\bar{P} = \left[\begin{array}{c:c} I & S \\ \hdashline O & R \end{array}\right]^N = P_0^N = \left[\begin{array}{cc:cc} 1 & 0 & 0.2 & 0.1 \\ 0 & 1 & 0.1 & 0.2 \\ \hdashline 0 & 0 & 0.5 & 0.1 \\ 0 & 0 & 0.2 & 0.6 \end{array}\right]^N = \left[\begin{array}{c:c} I & S(I-R)^{-1} \\ \hdashline O & O \end{array}\right]$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad S = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix} \qquad R = \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.6 \end{bmatrix}$$

$$\bar{P} = \left[\begin{array}{cc:cc} 1 & 0 & 0.5556 & 0.3889 \\ 0 & 1 & 0.4444 & 0.6111 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{cc:cc} 1 & 0 & \frac{10}{18} & \frac{7}{18} \\ 0 & 1 & \frac{8}{18} & \frac{11}{18} \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

[Prob & Stats - Markov Chains (27 of 38) Absorbing Markov Chain: Stable Matrix=? Ex. 2 (https://youtu.be/TWq0CvkAWVg)](https://youtu.be/TWq0CvkAWVg)

$$\bar{P} = \left[\begin{array}{c|c} I & S \\ \hline O & R \end{array}\right]^N = P_0^N = \left[\begin{array}{cc|cc} 1 & 0 & 0.2 & 0.3 \\ 0 & 1 & 0.2 & 0.1 \\ \hline 0 & 0 & 0.5 & 0.2 \\ 0 & 0 & 0.1 & 0.4 \end{array}\right]^N = \left[\begin{array}{c|c} I & S(I-R)^{-1} \\ \hline O & O \end{array}\right]$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad S = \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0.4 \end{bmatrix} \qquad R = \begin{bmatrix} 0.2 & 0.3 \\ 0.2 & 0.1 \end{bmatrix}$$

```
[1 0][0.5 0.2][0.5  -0.2]
[0 1][0.1 0.4][-0.1 0.6 ]
Determinant == 0.28 == 7/25
cofactor => cofactor.T
[0.6 0.1][0.6 0.2]
[0.2 0.5][0.1 0.5]
A Inverted ==
[2.14286  0.714286]
[0.357143 1.78571 ]
S(I-R)^-1==
[0.535714 0.678571]
[0.464286 0.321429]
15/28 19/28
13/28 9/28
```

$$(I - R) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0.4 \end{bmatrix} = \begin{bmatrix} 1-0.5 & 0-0.2 \\ 0-0.1 & 1-0.4 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.2 \\ -0.1 & 0.6 \end{bmatrix}$$

Determinant $D = (0.5)\cdot(0.6) - (-0.2)\cdot(-0.1) = 0.30 - 0.02 = 0.28$

$$(I - R)^{-1} = \frac{1}{D}\left[C^*\right] = \frac{1}{0.28}\cdot\begin{bmatrix} 0.6 & 0.2 \\ 0.1 & 0.5 \end{bmatrix}$$

через десятичные дроби :

$$S\cdot(I-R)^{-1} = S\cdot\frac{1}{D}\left[C^*\right] = \left[\begin{array}{cc} 0.2 & 0.3 \\ \hline 0.2 & 0.1 \end{array}\right]\cdot\frac{1}{0.28}\cdot\left[\begin{array}{c|c} 0.6 & 0.2 \\ 0.1 & 0.5 \end{array}\right] = \frac{1}{0.28}\left[\begin{array}{cc} 0.2 & 0.3 \\ \hline 0.2 & 0.1 \end{array}\right]\cdot\left[\begin{array}{c|c} 0.6 & 0.2 \\ 0.1 & 0.5 \end{array}\right]$$

$$= \frac{1}{0.28}\cdot\left[\begin{array}{c|c} 0.2\cdot0.6+0.3\cdot0.1 & 0.2\cdot0.2+0.3\cdot0.5 \\ 0.2\cdot0.6+0.1\cdot0.1 & 0.2\cdot0.2+0.1\cdot0.5 \end{array}\right] = \frac{1}{0.28}\cdot\left[\begin{array}{c|c} 0.12+0.03 & 0.04+0.15 \\ 0.12+0.01 & 0.04+0.05 \end{array}\right] =$$

$$= \frac{1}{0.28}\cdot\begin{bmatrix} 0.15 & 0.19 \\ 0.13 & 0.09 \end{bmatrix} = \begin{bmatrix} 0.535714 & 0.678571 \\ 0.464286 & 0.321429 \end{bmatrix}$$

$$\boxed{S\cdot(I-R)^{-1} = \begin{bmatrix} 0.535714 & 0.678571 \\ 0.464286 & 0.321429 \end{bmatrix}}$$

через обычные дроби :

$$S \cdot (I-R)^{-1} = S \cdot \frac{1}{D}\left[C^*\right] = \left[\begin{array}{cc} 0.2 & 0.3 \\ \hline 0.2 & 0.1 \end{array}\right] \cdot \frac{1}{0.28} \cdot \left[\begin{array}{c:c} 0.6 & 0.2 \\ 0.1 & 0.5 \end{array}\right] = \frac{1}{0.28}\left[\begin{array}{cc} 0.2 & 0.3 \\ \hline 0.2 & 0.1 \end{array}\right] \cdot \left[\begin{array}{c:c} 0.6 & 0.2 \\ 0.1 & 0.5 \end{array}\right]$$

$=$

$$= \frac{1}{\frac{28}{100}} \cdot \left[\begin{array}{cc} \frac{2}{10} & \frac{3}{10} \\ \hline \frac{2}{10} & \frac{1}{10} \end{array}\right] \cdot \left[\begin{array}{c:c} \frac{6}{10} & \frac{2}{10} \\ \frac{1}{10} & \frac{5}{10} \end{array}\right] = \frac{100}{28} \cdot \left[\begin{array}{c:c} \frac{2}{10}\frac{6}{10} + \frac{3}{10}\frac{1}{10} & \frac{2}{10}\frac{2}{10} + \frac{3}{10}\frac{5}{10} \\ \hline \frac{2}{10}\frac{6}{10} + \frac{1}{10}\frac{1}{10} & \frac{2}{10}\frac{2}{10} + \frac{1}{10}\frac{5}{10} \end{array}\right] =$$

$$\frac{100}{28} \cdot \left[\begin{array}{c:c} \frac{12}{100} + \frac{3}{100} & \frac{4}{100} + \frac{15}{100} \\ \hline \frac{12}{100} + \frac{1}{100} & \frac{4}{100} + \frac{5}{100} \end{array}\right] = \frac{100}{28} \cdot \left[\begin{array}{c:c} \frac{15}{100} & \frac{19}{100} \\ \hline \frac{13}{100} & \frac{9}{100} \end{array}\right] = \left[\begin{array}{c:c} \frac{15}{28} & \frac{19}{28} \\ \hline \frac{13}{28} & \frac{9}{28} \end{array}\right]$$

$$\boxed{S \cdot (I-R)^{-1} = \left[\begin{array}{c:c} \frac{15}{28} & \frac{19}{28} \\ \hline \frac{13}{28} & \frac{9}{28} \end{array}\right]}$$

```
P(100):
[1 0 0.535714 0.678571]
[0 1 0.464286 0.321429]
[0 0 0.000000 0.000000]
[0 0 0.000000 0.000000]
1  0  15/28 19/28
0  1  13/28 9/28
0  0  0     0
0  0  0     0
[5.35714286e-01 4.64285714e-01 2.61327449e-23 1.30663725e-23]
```

$$\bar{P} = \left[\begin{array}{c:c} I & S \\ \hline O & R \end{array}\right]^N = P_0^N = \left[\begin{array}{cc:cc} 1 & 0 & 0.2 & 0.3 \\ 0 & 1 & 0.2 & 0.1 \\ \hline 0 & 0 & 0.5 & 0.2 \\ 0 & 0 & 0.1 & 0.4 \end{array}\right]^N = \left[\begin{array}{c:c} I & S(I-R)^{-1} \\ \hline O & O \end{array}\right]$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad S = \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0.4 \end{bmatrix} \qquad R = \begin{bmatrix} 0.2 & 0.3 \\ 0.2 & 0.1 \end{bmatrix}$$

$$\bar{P} = \left[\begin{array}{cc:cc} 1 & 0 & 0.535714 & 0.678571 \\ 0 & 1 & 0.464286 & 0.321429 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{cc:cc} 1 & 0 & \frac{15}{28} & \frac{19}{28} \\ 0 & 1 & \frac{13}{28} & \frac{9}{28} \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

[Prob & Stats - Markov Chains (28 of 38) Absorbing Markov Chain: Stable Distribution]()

[Matrix=? (https://youtu.be/u89Sd514EDI)](https://youtu.be/u89Sd514EDI)

$$\bar{P} = \left[\begin{array}{c:c} I & S \\ \hdashline O & R \end{array}\right]^N = P_0^N = \left[\begin{array}{cc:cc} 1 & 0 & 0.2 & 0.3 \\ 0 & 1 & 0.2 & 0.1 \\ \hdashline 0 & 0 & 0.5 & 0.2 \\ 0 & 0 & 0.1 & 0.4 \end{array}\right]^N = \left[\begin{array}{c:c} I & S(I-R)^{-1} \\ \hdashline O & O \end{array}\right]$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad S = \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0.4 \end{bmatrix} \qquad R = \begin{bmatrix} 0.2 & 0.3 \\ 0.2 & 0.1 \end{bmatrix}$$

$$\bar{P} = \left[\begin{array}{cc:cc} 1 & 0 & 0.535714 & 0.678571 \\ 0 & 1 & 0.464286 & 0.321429 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{cc:cc} 1 & 0 & \frac{15}{28} & \frac{19}{28} \\ 0 & 1 & \frac{13}{28} & \frac{9}{28} \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Example 1:

$$\bar{X} = \bar{P} \cdot X_0$$

$\bar{P} \cdot X_0 = \bar{X}$ Let $X_0 = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$:

```
P(100):
[1 0 0.535714 0.678571]
[0 1 0.464286 0.321429]
[0 0 0.000000 0.000000]
[0 0 0.000000 0.000000]
1  0  15/28 19/28
0  1  13/28 9/28
0  0  0     0
0  0  0     0
[0.553571]
[0.446429]
[0.000000]
[0.000000]
31/56
25/56
0
0
```

$$\bar{P} \cdot X_0 = \left[\begin{array}{cc:cc} 1 & 0 & 0.535714 & 0.678571 \\ 0 & 1 & 0.464286 & 0.321429 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix} = \left[\begin{array}{cc:cc} 1 & 0 & \frac{15}{28} & \frac{19}{28} \\ 0 & 1 & \frac{13}{28} & \frac{9}{28} \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{bmatrix} = \begin{bmatrix} 0.553571 \\ 0.446429 \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{P} \cdot X_0 = \bar{X} = \begin{bmatrix} 0.553571 \\ 0.446429 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{31}{56} \\ \frac{25}{56} \\ 0 \\ 0 \end{bmatrix}$$

Example 2:

$$\bar{X} = \bar{P} \cdot X_0$$

Let $X_0 = \begin{bmatrix} 0.1 \\ 0.5 \\ 0.1 \\ 0.3 \end{bmatrix}$:

```
P(100):
[1 0 0.535714 0.678571]
[0 1 0.464286 0.321429]
[0 0 0.000000 0.000000]
[0 0 0.000000 0.000000]
1  0  15/28 19/28
0  1  13/28 9/28
0  0  0     0
0  0  0     0
[0.357143]
[0.642857]
[0.000000]
[0.000000]
5/14
9/14
0
0
```

$$\bar{P} \cdot X_0 = \left[\begin{array}{cc:cc} 1 & 0 & 0.535714 & 0.678571 \\ 0 & 1 & 0.464286 & 0.321429 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} 0.1 \\ 0.5 \\ 0.2 \\ 0.3 \end{bmatrix} = \left[\begin{array}{cc:cc} 1 & 0 & \frac{15}{28} & \frac{19}{28} \\ 0 & 1 & \frac{13}{28} & \frac{9}{28} \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} \frac{1}{10} \\ \frac{5}{10} \\ \frac{2}{10} \\ \frac{3}{10} \end{bmatrix} = \begin{bmatrix} 0.357143 \\ 0.642857 \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{P} \cdot X_0 = \bar{X} = \begin{bmatrix} 0.357143 \\ 0.642857 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{14} \\ \frac{9}{14} \\ 0 \\ 0 \end{bmatrix}$$

[Prob & Stats - Markov Chains (29 of 38) Absorbing Markov Chain: Stable Distribution Matrix=? (https://youtu.be/0iNR2_gCM7I)](https://youtu.be/0iNR2_gCM7I)

$$\bar{P} = \left[\begin{array}{c:c} I & S \\ \hdashline O & R \end{array}\right]^N = P_0^N = \left[\begin{array}{c:ccc} 1 & 0.2 & 0.2 & 0.3 \\ \hdashline 0 & 0.5 & 0.2 & 0.1 \\ 0 & 0.2 & 0.5 & 0.2 \\ 0 & 0.1 & 0.1 & 0.4 \end{array}\right]^N = \left[\begin{array}{c:c} I & S(I-R)^{-1} \\ \hdashline O & O \end{array}\right]$$

$$I = \begin{bmatrix} 1 \end{bmatrix} \qquad O = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad S = \begin{bmatrix} 0.2 & 0.2 & 0.3 \end{bmatrix} \qquad R = \begin{bmatrix} 0.5 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.2 \\ 0.1 & 0.1 & 0.4 \end{bmatrix}$$

$$\bar{P} = \left[\begin{array}{c:ccc} 1 & 1 & 1 & 1 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

```
[1 0 0][0.5 0.2 0.1][0.5  -0.2 -0.1]
[0 1 0][0.2 0.5 0.2][-0.2 0.5  -0.2]
[0 0 1][0.1 0.1 0.4][-0.1 -0.1 0.6 ]
Determinant == 0.105 == 21/200
cofactor => cofactor.T==C^*
[0.28 0.14 0.07][0.28 0.13 0.09]
[0.13 0.29 0.07][0.14 0.29 0.12]
[0.09 0.12 0.21][0.07 0.07 0.21]
(I-R)^-1 == A Inverted == C*/det
[2.66667  1.2381   0.857143]
[1.33333  2.7619   1.14286 ]
[0.666667 0.666667 2       ]
S==
 [[0.2 0.2 0.3]]
S * C^*(==I_R_cofactor_T)
[0.105 0.105 0.105]
S(I-R)^-1==
[1 1 1]
```

$$(I-R)^{-1} = \begin{bmatrix} 2.66667 & 1.2381 & 0.857143 \\ 1.33333 & 2.7619 & 1.14286 \\ 0.666667 & 0.666667 & 2 \end{bmatrix} = \begin{bmatrix} \frac{8}{3} & \frac{26}{21} & \frac{6}{7} \\ \frac{4}{3} & \frac{58}{21} & \frac{8}{7} \\ \frac{2}{3} & \frac{2}{3} & 2 \end{bmatrix}$$

через Десятичные дроби :

$$S \cdot (I-R)^{-1} = S \cdot \frac{1}{D}\left[C^*\right] = \begin{bmatrix} 0.2 & 0.2 & 0.3 \end{bmatrix} \cdot \frac{1}{0.105} \cdot \left[\begin{array}{c:c:c} 0.28 & 0.13 & 0.09 \\ 0.14 & 0.29 & 0.12 \\ 0.07 & 0.07 & 0.21 \end{array}\right] =$$

$$= \frac{1}{0.105} \cdot \begin{bmatrix} 0.2 & 0.2 & 0.3 \end{bmatrix} \cdot \left[\begin{array}{c:c:c} 0.28 & 0.13 & 0.09 \\ 0.14 & 0.29 & 0.12 \\ 0.07 & 0.07 & 0.21 \end{array}\right] = \frac{1}{0.105} \cdot \begin{bmatrix} 0.105 & 0.105 & 0.105 \end{bmatrix}$$

$$\boxed{S \cdot (I-R)^{-1} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}}$$

```
[1 0 0][0.5 0.2 0.1][0.5  -0.2 -0.1]
[0 1 0][0.2 0.5 0.2][-0.2 0.5  -0.2]
[0 0 1][0.1 0.1 0.4][-0.1 -0.1 0.6 ]
Determinant == 0.105 == 21/200
cofactor => cofactor.T==C^*
7/25   7/50   7/100
13/100 29/100 7/100
9/100  3/25   21/100
(I-R)^-1 == A Inverted == C*/det
8/3 26/21 6/7
4/3 58/21 8/7
2/3 2/3   2
S==
 [[0.2 0.2 0.3]]
S * C^*(==I_R_cofactor_T)
21/200 21/200 21/200
S(I-R)^-1==
1     1     1
```

через Простые дроби :

$$S \cdot (I-R)^{-1} = S \cdot \frac{1}{D}\left[C^*\right] = \begin{bmatrix} \frac{2}{10} & \frac{2}{10} & \frac{3}{10} \end{bmatrix} \cdot \frac{1}{\frac{105}{1000}} \cdot \begin{bmatrix} \frac{7}{25} & \frac{7}{50} & \frac{7}{100} \\ \frac{13}{100} & \frac{29}{100} & \frac{7}{100} \\ \frac{9}{100} & \frac{3}{25} & \frac{21}{100} \end{bmatrix} =$$

$$= \frac{1000}{105} \cdot \begin{bmatrix} \frac{2}{10} & \frac{2}{10} & \frac{3}{10} \end{bmatrix} \cdot \begin{bmatrix} \frac{7}{25} & \frac{7}{50} & \frac{7}{100} \\ \frac{13}{100} & \frac{29}{100} & \frac{7}{100} \\ \frac{9}{100} & \frac{3}{25} & \frac{21}{100} \end{bmatrix} = \frac{200}{21} \cdot \begin{bmatrix} \frac{21}{200} & \frac{21}{200} & \frac{21}{200} \end{bmatrix}$$

$$\boxed{S \cdot (I-R)^{-1} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}}$$

[Prob & Stats - Markov Chains: Method 2 (30 of 38) Basics*** (https://youtu.be/EAZ4K1Z7qws)](https://youtu.be/EAZ4K1Z7qws)

[Prob & Stats - Markov Chains: Method 2 (31 of 38) Powers of a Transition Matrix (https://youtu.be/phSQdD39qhE)](https://youtu.be/phSQdD39qhE)

[Prob & Stats - Markov Chains: Method 2 (32 of 38) Finding Stable State Matrix (https://youtu.be/SSeoW9IrVaw)](https://youtu.be/SSeoW9IrVaw)

[Prob & Stats - Markov Chains: Method 2 (33 of 38) What is an Absorbing Markov Chain (https://youtu.be/5_Hb0lvlbH4)](https://youtu.be/5_Hb0lvlbH4)

[Prob & Stats - Markov Chains: Method 2 (34 of 38) Finding the Stable State Matrix (https://youtu.be/p_6poNVikn8)](https://youtu.be/p_6poNVikn8)

[Prob & Stats - Markov Chains: Method 2 (35 of 38) Finding the Stable State & Transition Matrices (https://youtu.be/uferdSI_e5E)](https://youtu.be/uferdSI_e5E)

[Prob & Stats - Markov Chains: Method 2 (36 of 38) Absorbing Markov Chain: Standard Form - Ex. (https://youtu.be/MrmMyK5CuWs)](https://youtu.be/MrmMyK5CuWs)

[Prob & Stats - Markov Chains: Method 2 (37 of 38) Absorbing Markov Chain: Changing to Standard Form (https://youtu.be/gPOiDeHZX4E)](https://youtu.be/gPOiDeHZX4E)

[Prob & Stats - Markov Chains: Method 2 (38 of 38) Absorbing Markov Chain: Standard Form - Ex. (https://youtu.be/LUtqqJ9VFhU)](https://youtu.be/LUtqqJ9VFhU)

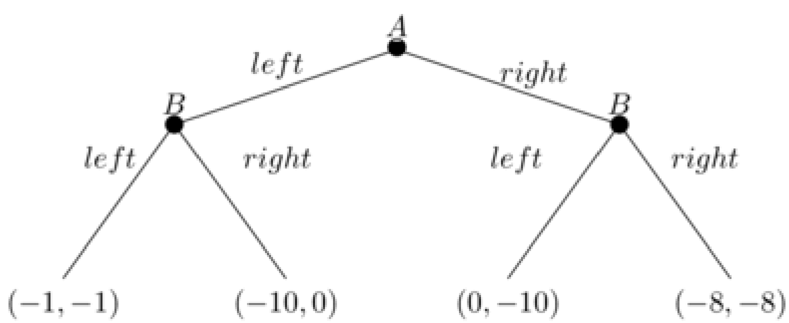
$\mathrm{km\ s^{-1}}$

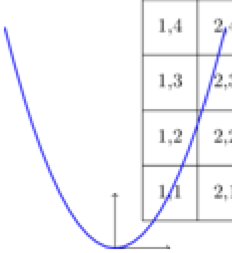
hll

$$\langle \Psi^* | \Psi \rangle$$

$$\left\langle \frac{\Psi^*}{2} \middle| \Psi \right\rangle$$

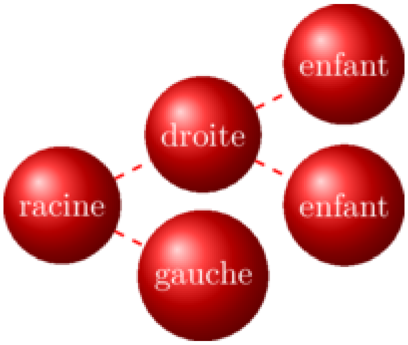
$$\langle \cdot | \Psi \rangle$$

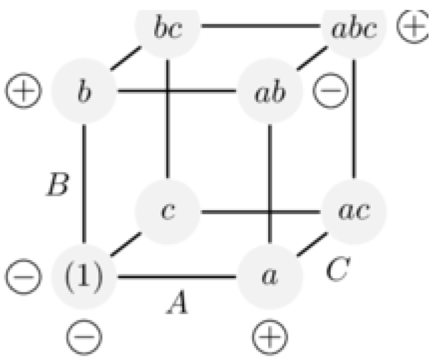


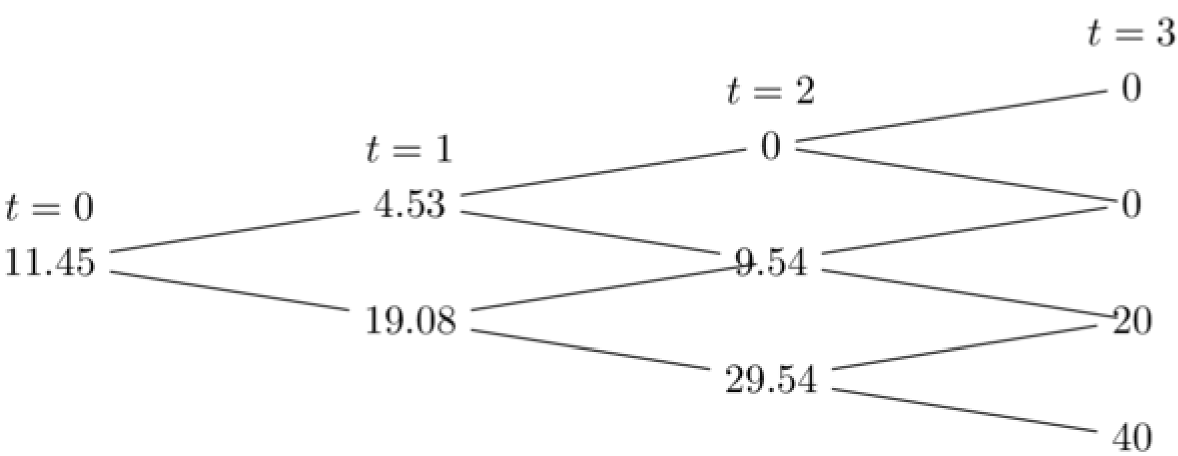


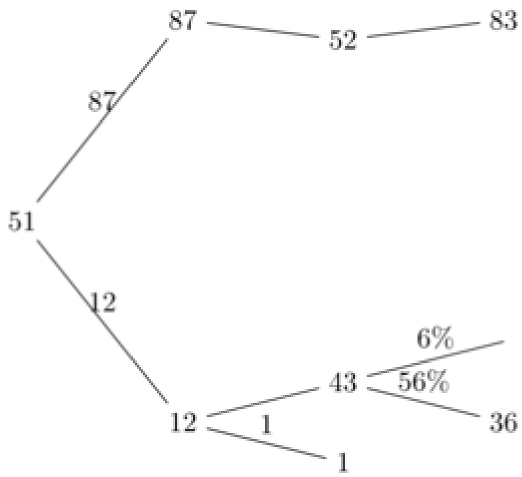
1,5	2,5	3,5	4,5	5,5
1,4	2,4	3,4	4,4	5,4
1,3	2,3	3,3	4,3	5,3
1,2	2,2	3,2	4,2	5,2
1,1	2,1	3,1	4,1	5,1

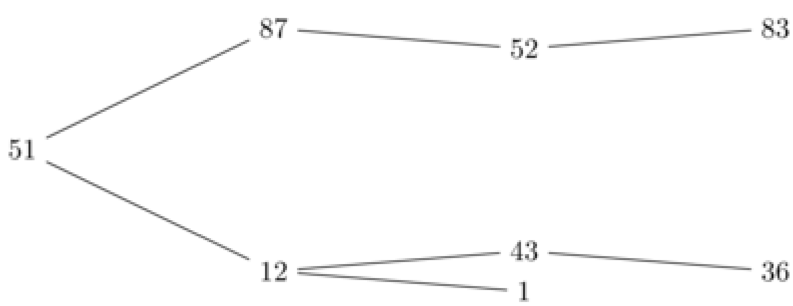
7,5	8,5	9,5	10,5	11,5	12,5
7,4	8,4	9,4	10,4	11,4	12,4
7,3	8,3	9,3	10,3	11,3	12,3
7,2	8,2	9,2	10,2	11,2	12,2
7,1	8,1	9,1	10,1	11,1	12,1

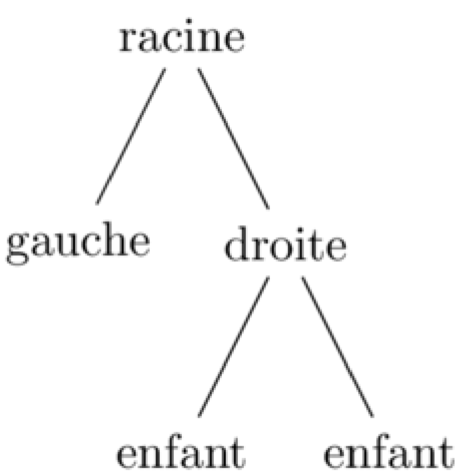

enfant
droite
racine
enfant
gauche

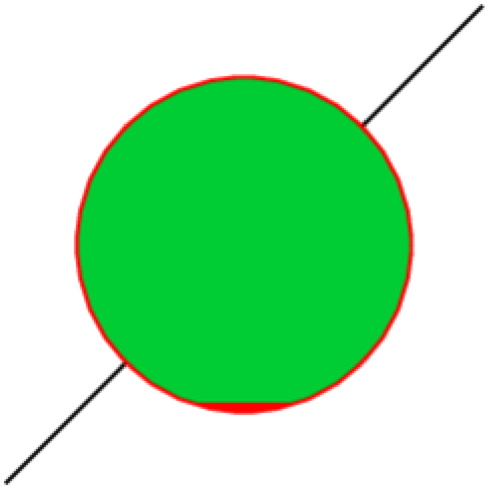

bc
abc
b
ab
B
c
ac
(1)
a
C
A

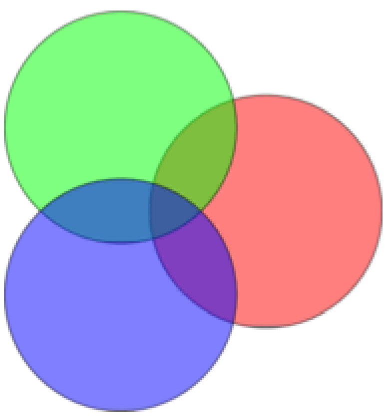

t = 3
0
t = 2
0
t = 1
t = 0
4.53
0
11.45
9.54
19.08
20
29.54
40


87
52
83
87
51
12
6%
43
56%
12
1
36
1

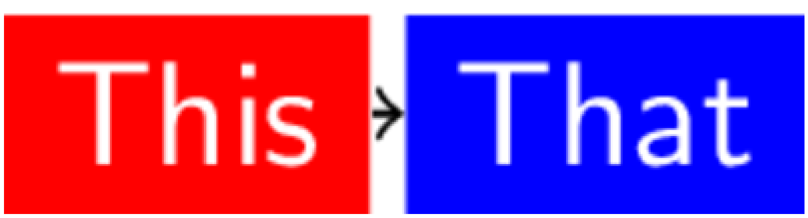

87
52
83
51
43
12
36
1


racine
gauche
droite
enfant
enfant

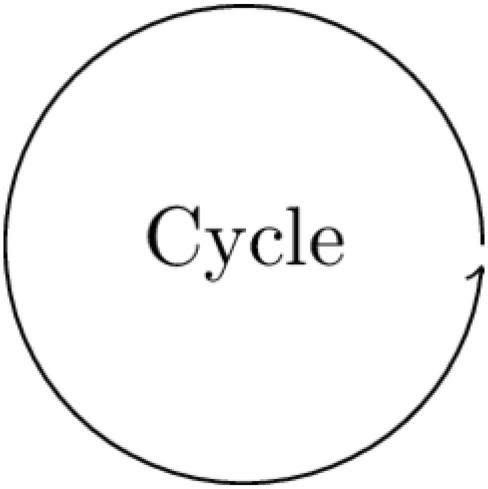


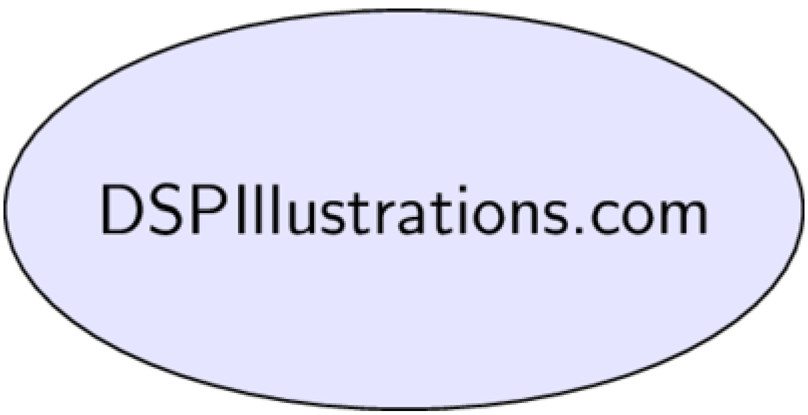


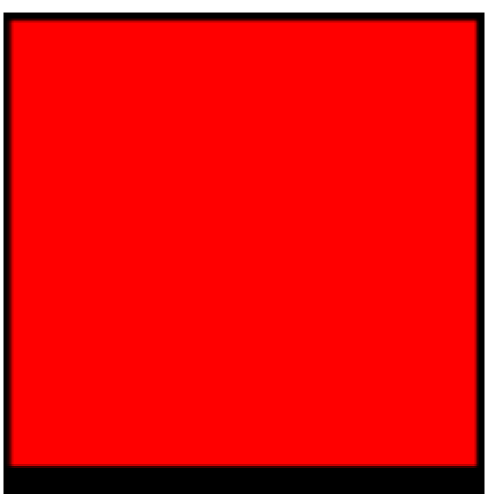

This
That


This
That


This
That







collapsible markdown?

▶ CLICK ME

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{2\pi i k} \, dx$$

$$\rho(x,y) \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array} \right]$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} = \frac{1}{\det(X)} \begin{bmatrix} X_{22}Y_{11} - X_{12}Y_{21} & X_{22}Y_{12} - X_{12}Y_{22} \\ X_{11}Y_{21} - X_{21}Y_{11} & X_{11}Y_{22} - X_{21}Y_{12} \end{bmatrix}$$

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} = \frac{1}{\det(X)} \begin{bmatrix} X_{22}Y_{11} - X_{12}Y_{21} & X_{22}Y_{12} - X_{12}Y_{22} \\ X_{11}Y_{21} - X_{21}Y_{11} & X_{11}Y_{22} - X_{21}Y_{12} \end{bmatrix}$$

$$\begin{array}{c} \\ A \\ C \\ G \\ T \end{array} \begin{array}{c} \begin{array}{cccc} A & C & G & T \end{array} \\ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 \end{pmatrix} \end{array}$$

$$\begin{array}{c} \begin{array}{cc} H_1 & H_2 \end{array} \\ \left[\begin{array}{cccc|cc} A & 0 & 0 & 0 & -1 & B \\ 0 & 0 & -1 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & C & D & 0 & 0 \end{array} \right] \\ \begin{array}{cc} H_3 & H_4 \end{array} \end{array} \left[\begin{array}{c} x_1 \\ x_{10} \\ x_5 \\ x_7 \\ \hline x_9 \\ x_3 \end{array} \right] \begin{array}{c} X_1 \\ \\ X_2 \end{array} = 0$$

$$\text{Middle two rows} \left\{ \begin{pmatrix} a & \overbrace{b \quad c \quad d}^{\text{Final three columns}} \\ e & f \quad g \quad h \\ i & j \quad k \quad l \\ m & n \quad o \quad p \end{pmatrix} \right.$$

Probability density function:

$$\begin{cases} \frac{1}{b-a} & \text{for } x \in [a,b] \\ 0 & \text{otherwise} \end{cases}$$

Cumulative distribution function:

$$\begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } x \in [a,b) \\ 1 & \text{for } x \ge b \end{cases}$$

Everything hide

`Markdown not work`

Click here to toggle on/off the raw code.