Uniform Distribution

Eugene Politov

October 21, 2016

Uniform Distribution

Probability density function

Suppose $f(x)$ is the PDF of a continuous uniform distribution on the interval $[a, b]$. Since a uniform distribution requires that each value of the random variable be equally likely, $f(x)$ will be a constant function. Therefore, suppose $f(x) = c$ on the interval $[a, b]$. Since the definite integral of the PDF over the interval must be equal to 1, we can solve the equation $\int_a^b c \, \mathrm{d}x = 1$ for c. Working with the integral, we have

$\int_a^b c \, \mathrm{d}x = cx|_a^b = c(b - a) = 1$ which gives $c = \dfrac{1}{b - a}$

In other words, the PDF is $f(axb) = \frac{1}{b-a}$

Cumulative distribution function

The CDF is simply the integral of the PDF.

$\int_a^x \dfrac{1}{b-a} \mathrm{d}t = \dfrac{1}{b-a} t \bigg|_a^x = \dfrac{x-a}{b-a}$

$F(x) = \dfrac{x-a}{b-a}$

Moments

Mean

$E(X) = \int_a^b \dfrac{1}{b-a} x \, \mathrm{d}x = \dfrac{1}{2(b-a)} x^2 \bigg|_a^b = \dfrac{b^2 - a^2}{2(b-a)} = \dfrac{(b-a)(b+a)}{2(b-a)} = \dfrac{a+b}{2}$

$E(X) = \dfrac{a+b}{2}$

Variance

$Var(X) = E(X^2) - (E(X))^2$

$E(X^2) = \int_a^b \frac{1}{b-a} x^2 \mathrm{d}x = \left. \frac{1}{3(b-a)} x^3 \right|_a^b = \frac{b^3 - a^3}{3(b-a)} = \frac{(b-a)(b^2+ab+a^2)}{3(b-a)} = \frac{b^2+ab+a^2}{3}$

$(E(X))^2 = \left(\frac{a+b}{2}\right)^2$

$Var(X) = \frac{b^2+ab+a^2}{3} - \left(\frac{a+b}{2}\right)^2 =$

$= \frac{b^2+ab+a^2}{3} - \frac{a^2+2ab+b^2}{4} =$

$= \frac{4(b^2+ab+a^2)}{12} - \frac{3(a^2+2ab+b^2)}{12} =$

$= \frac{4a^2+4ab+4b^2-3a^2-6ab-3b^2}{12} =$

$= \frac{a^2-2ab+b^2}{12} = \frac{b^2-2ab+a^2}{12} =$

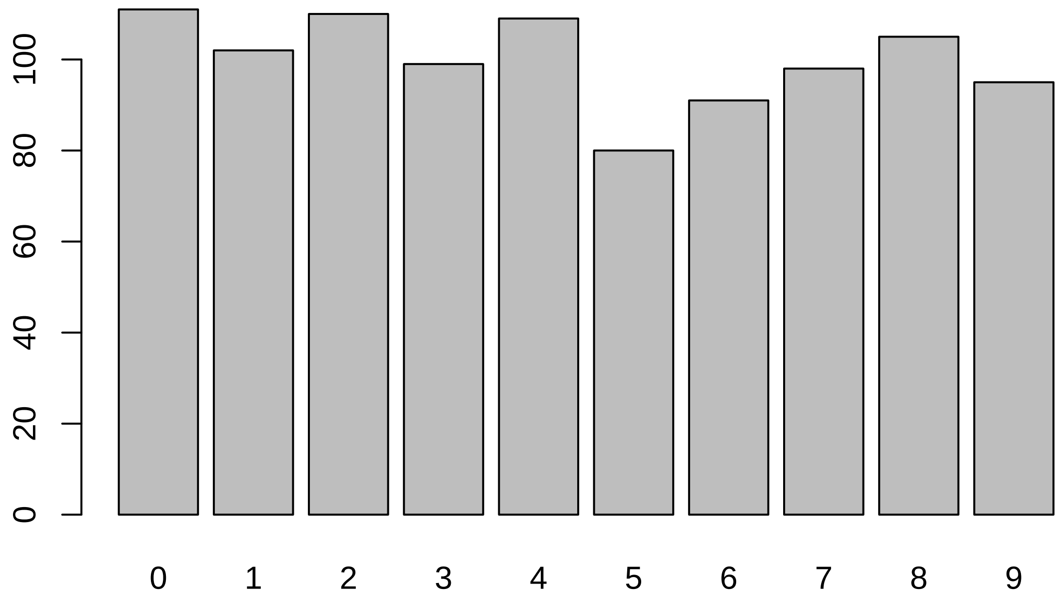
$= \frac{(b-a)^2}{12}$

$Var(X) = \frac{(b-a)^2}{12}$

Standard Deviation

$\sigma_X = \frac{b-a}{\sqrt{12}}$

```r
#dunif(x, min = 0, max = 1, log = FALSE)
#punif(q, min = 0, max = 1, lower.tail = TRUE, log.p = FALSE)
#qunif(p, min = 0, max = 1, lower.tail = TRUE, log.p = FALSE)
#runif(n, min = 0, max = 1)
n <- floor(runif(1000)*10)
t <- table(n)
barplot(t)
```


0
20
40
60
80
100
0
1
2
3
4
5
6
7
8
9