Problem Set 1

Problem 1. Consider the following events in the roll of a single fair die:
A: Observe an odd number.
B: Observe an even number.
C: Observe a 1 or 2.

- Define the probability of each event
- Are A and B independent events?
- Are A and C independent events?

Problem 2. A fair die is rolled twice. Let A and B denote the following events: «the sum of the two results is greater than 9» and «the outcome on the second roll is odd», respectively.

- List the sample space for this experiment
- Find $P(A \cup B)$
- Find $P(A|B)$
- Find $P(A \cap B)$
- Define whether the events A and B are independent

Problem 3. A fair die is rolled twice. As a result, we obtain two numbers X = result of the first roll, and Y = result of the second roll. Find the probability of the following events:

- A defined as «X is less than Y». List the corresponding sample points.
- B defined as «You observe a 6 at least once». List the corresponding sample points.

Problem 4. There are two locks on the door. The probability that the first lock is closed is equal to 0.9. The probability that the second lock is closed is 0.8. It is also known that the probability of the two locks being closed simultaneously is 0.72.

- Can the events A = the first lock is closed and B = the second lock is closed be considered as independent? Justify your answer.
- Find the probability that at least one lock is closed

- Find the probability that only the first lock is closed
- Find the probability that only the second lock is closed
- Find the probability that only one of the two locks is closed

Problem 5. I have three bags that each contain 100 marbles. It is also known that the first bag has 75 red and 25 blue marbles; the second bag 2 has 60 red and 40 blue marbles, and the third one has 45 red and 55 blue marbles. I choose one of the bags at random and then pick a marble from the chosen bag, also at random. What is the probability that the chosen marble is red?

Problem 6.

The table below shows the joint distribution between the scores obtained for the first and the second test questions (X and Y stand for the number of scores obtained for the first and the second questions, respectively):

$X\backslash Y$	0	1	2
0	0.2	0.2	0.1
1	0.2		0.2

1. Fill in the gap in the table
2. Are X and Y independent?
3. Find $P(Y = 1|X = 1)$
4. Find $P(X = 1|Y = 2)$

Problem 7. Of the travelers arriving at a small airport, 60% fly on major airlines, 30% fly on privately owned planes, and the remainder fly on commercially owned planes not belonging to a major airline. Of those traveling on major airlines, 50% are traveling for business reasons, whereas 60% of those arriving on private planes and 90% of those arriving on other commercially owned planes are traveling for business reasons. Suppose that we randomly select one person arriving at this airport. What is the probability that the person

- is traveling on business?
- is traveling for business on a privately owned plane?

- arrived on a privately owned plane, given that the person is traveling for business reasons?

Problem 8. Two methods, A and B, are available for teaching a certain industrial skill. The failure rate is 20% for A and 10% for B. However, B is more expensive and hence is used only 30% of the time. (A is used the other 70%.) A worker was taught the skill by one of the methods but failed to learn it correctly. What is the probability that she was taught by method A?

Problem 9. Males and females are observed to react differently to a given set of circumstances. It has been observed that 70% of the females react positively to these circumstances, whereas only 40% of males react positively. A group of 20 people, 15 female and 5 male, was subjected to these circumstances, and the subjects were asked to describe their reactions on a written questionnaire. A response picked at random from the 20 was negative. What is the probability that it was that of a male?