A Study on Group Equivariant CNNs

Geometric Data Analysis, MVA

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December 19, 2023

Contents

Motivation: Comparision with Regular CNNs

CNNs

- are inherently equivariant to translations
- are not equivariant to other transformations: rotations, reflections, etc.
- use data augmentation to tackle such problems

Group Equivariant CNNs [Cohen, 2016]

- are designed to be equivariant to symmetry groups
- use specific filters to guarantee equivariance to group elements
- have enhanced capacity for group specific representation learning

Study & Contributions

1. Theoretical study of base paper's approaches
2. Testing and benchmarking on:
   - classification task: MNIST, MNIST-Rot
   - segmentation task on dermascopic images

Symmetry Groups

A group is a non-empty set G together with a binary operation "$\cdot$" $: G \times G \to G$ such that

- $\forall a, b, c \in G \, : \, a \cdot (b \cdot c) = (a \cdot b) \cdot c$
- $\exists e \in G \; \forall a \in G \, : \, a \cdot e = e \cdot a = a$
- $\forall a \in G \; \exists a^{-1} \in G \, : \, a \cdot a^{-1} = a^{-1} \cdot a = e$

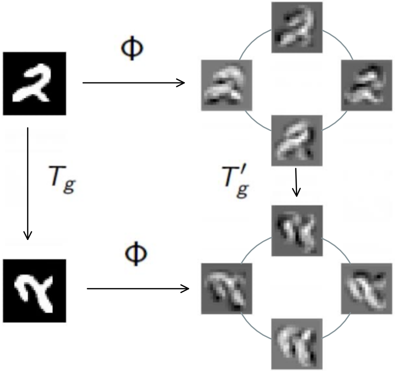
Examples

- $\mathbb{Z}^2$ – 2D integer translations
- $p4$ (pn) – compositions of translations and rotations by $\pi/2$ ($2\pi/n$)
- $p4m$ – compositions of translations, mirror reflections, and rotations by $\pi/2$

Equivariance

An operator $\Phi : X \to Y$ is G-equivariant if it commutes with the group action:

$$\Phi(T_g\, x) \;=\; T'_g\, \Phi(x)$$



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Properties

- invariance is a special case of equivariance with $T'_g \Phi = \Phi$
- composition preserves equivariance $\implies$ deep neural networks
- sum preserves equivariance $\implies$ skip-connections and residual blocks

G-equivariant convolutions

Regular convolution (correlation) transforms a stack of feature maps $f : \mathbb{Z}^2 \to \mathbb{R}^K$ by

$$[f \star \psi](x) = \sum_{y \in \mathbb{Z}^2} \sum_{k} f_k(y)\psi_k(x - y)$$

G-equivariant convolution transforms a stack of feature maps $f : H \to \mathbb{R}^K$ by

$$[f \star \psi](g) = \sum_{h \in H} \sum_{k} f_k(h)\psi_k(g^{-1}h) = \sum_{h \in H} \sum_{k} f_k(h) T_g[\psi_k](h)$$

Moreover, if g is a composition of a translation t and a transformation s, then

$$[f \star \psi](g) = \sum_{h \in H} \sum_{k} f_k(h) T_t[T_s \psi_k](h)$$

It can be implemented as a regular convolution with filters $T_s\psi_k$! But $\psi_k : G \to \mathbb{R}$.

Classification task: MNIST

CNN Model Architecture:

- 5 layers of 3×3 convolutions
- 8, 16, 32, 64 and 128 channels respectively
- ReLU activation, batch normalization and 3D max-pooling

Channel sizes of models to preserve the number of net trainable parameters:

- p4CNN: 8, 16, 32, 64, 128
- p6CNN: 8, 16, 32, 64, 72
- p8CNN: 4, 16, 16, 64, 64
- p4mCNN: 4, 16, 16, 64, 64

MNIST: Experiments & Analysis

Model	MNIST test accuracy	MNIST (+transforms) test accuracy
CNN	98.2%	34.1%
p4CNN	95.8%	62.9%
p6CNN	96.0%	42.0%
p8CNN	95.9%	63.5%
p4mCNN	94.2%	79.6%

Table: Accuracy of models trained on MNIST and tested on MNIST and on randomly rotated and flipped MNIST images

Key Points:

- CNN model (original accuracy: 98.2%) is highly unsuccessful at classifying rotated numbers
- Poor performance of p6CNN might be due to the lack of right angles among it's group elements - interpolation of pixel values affects accuracy

Rotated MNIST: Experiments & Analysis

Model	Test accuracy
CNN	92.43%
p4CNN	96.17%
p6CNN	94.82%
p8CNN	96.01%
p4mCNN	93.95%

Table: Accuracy of models trained and tested on Rotated MNIST

Key Points:

- p8CNN performs poorer than p4CNN; probably due to lower capacity of the model (reduced channels)
- p4mCNN lacks promising results as test set has no reflected images; mirroring parameters are unused.

Segmentation Models

SegNet Model Architecture: [Badrinarayanan, 2017]

- 4 encoder blocks followed by max-pooling
- 4 decoder blocks preceded by upsampling (or transposed convolution)
- ReLU activation and batch normalization
- last layer: max-pooling over stabilizer dimension

Analysis

- max-pooling in spatial dimension is H-equivariant $\iff$ subsample on a subgroup $H \in G$
- last max-pooling is coset pooling $\implies$ it is G-equivariant
- transposed G-convolutions with stride are G-equivariant $\implies$ upsampling is G-equivariant

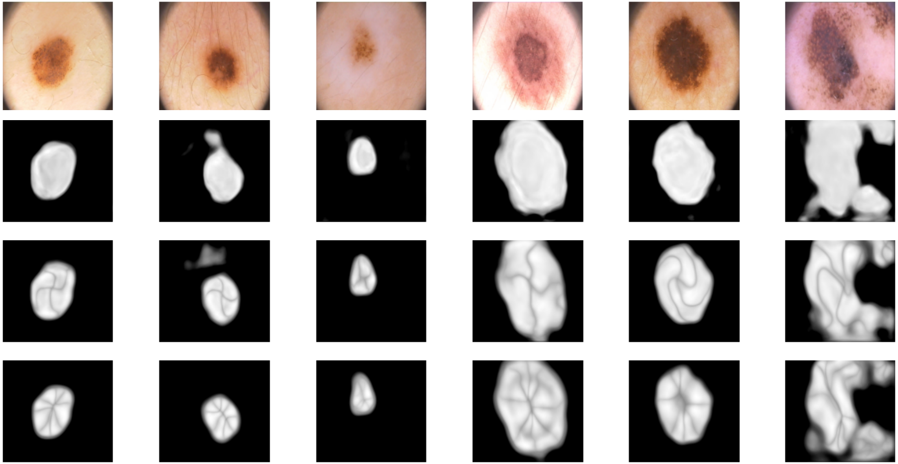
Predictions

Input

SegNet

p4SegNet

p4mSegNet



Segmentation Scores

Model	train IoU	test IoU	rotated test IoU
SegNet	0.885	0.718	0.646
p4SegNet	0.87	**0.786**	**0.786**
p4mSegNet	0.848	0.745	0.745
SegNet+	0.859	0.767	0.771
p4SegNet+	0.894	**0.788**	**0.788**

Table: Performance of segmentation models

Overview of G-equivariant CNNs

Advantages

- Equivariance guaranties
- Weight sharing
- Efficient implementation for *split* groups
- Better generalization properties

Limitations

- Only discrete groups are supported
- Matrix operations cannot perfectly model actions of some discrete groups (pn, $n > 4$)
- Computational time depends on the size of the group
- Number of weights per channel grows with the size of the group

Steerable CNNs

- Convolution kernels parameterised as steerable functions [Cohen, 2017, Weiler, 2018]
- Feature maps encode directional information by convolving with basis functions
- Fourier basis functions produce a vector field feature map of Fourier coefficients [Fageot & Uhlmann, 2021]

Advantage: These feature maps can recover continuous transformed signals as opposed to distinct discrete elements sampled explicitly.

Limitation: The number of basis functions remains a hyperparameter. This imposes an approximation on other information encoded by higher order frequencies.

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Robustness

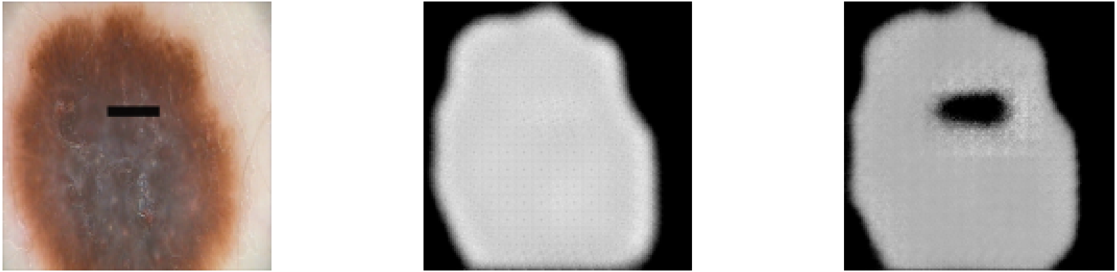


Figure: Left: Corrupted input. Middle: *p4SegNet*$_T$ prediction. Right: *SegNet*$_T$ prediction.

Feature Maps

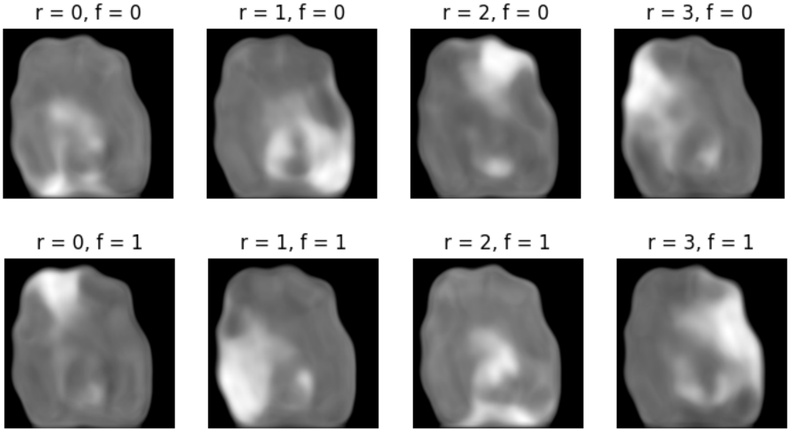
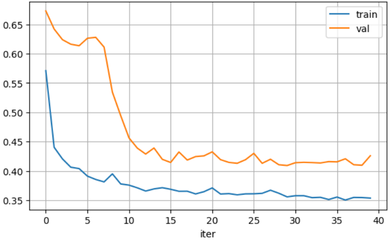


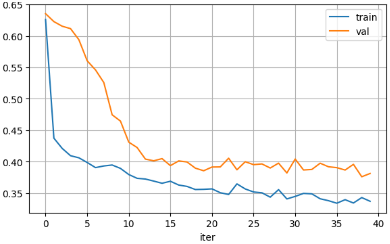
Figure: Feature maps taken from the last layer of p4mSegNet before pooling across the stabilizer dimension

Training plots

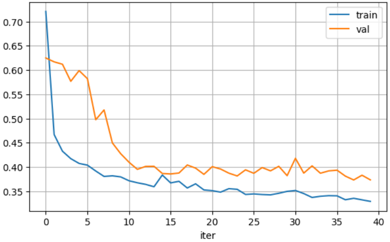
SegNet



p4SegNet



p4mSegNet



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