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# Control Theory.

## Assignmentn2.

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### Task 2

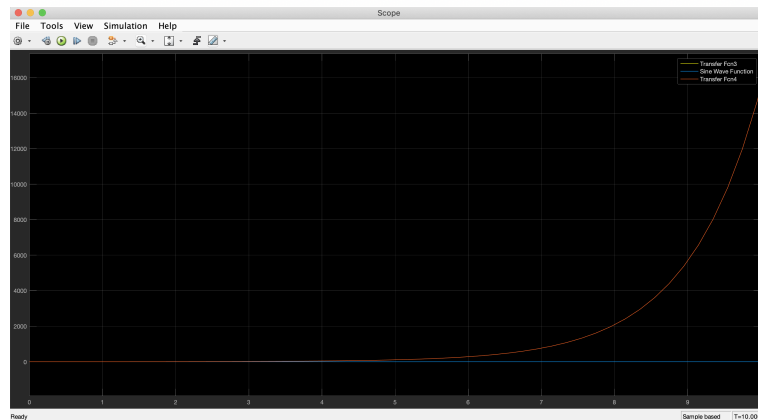
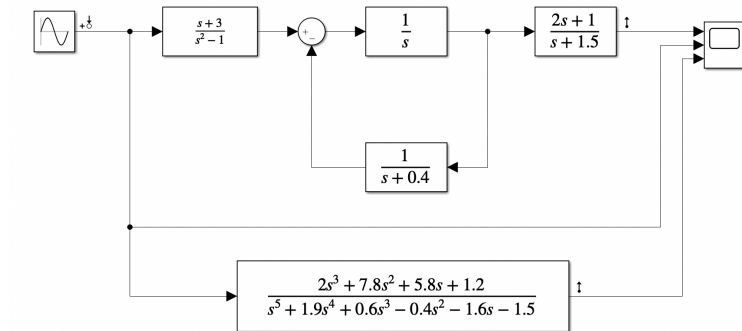
#### 2.1

$$W = W_1 \frac{W_2}{1 + W_2 W_4} W_3$$

$$W = \frac{s+3}{s^2-1} * \frac{1/s}{1 + (1/s)(1/(s+0.4))} * \frac{2s+1}{s+1.5}$$

$$W = \frac{2s^3 + 7.8s^2 + 5.8s + 1.2}{s^5 + 1.9s^4 + 0.6s^3 - 0.4s^2 - 1.6s - 1.5}$$

#### 2.2



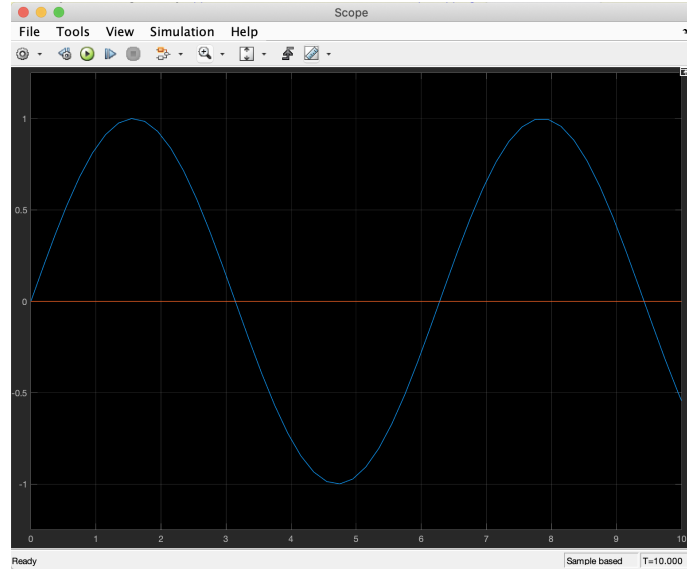


Figure 1: Frequency response describes the steady-state response of a system to sinusoidal inputs. Here input is  $\sin(x)$ . Yellow and red lines are the same one. The blue one is from input. Because of scale its shape is not clear, hence, it is also drawn separately.

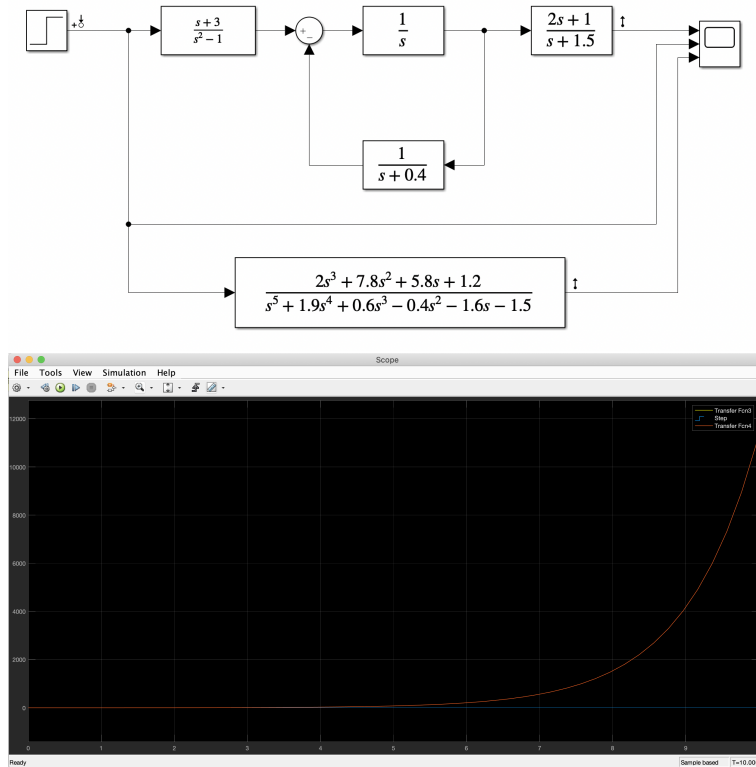


Figure 2: Step at time 1, amplitude 1. Again responses from initial and simplified systems are the same.

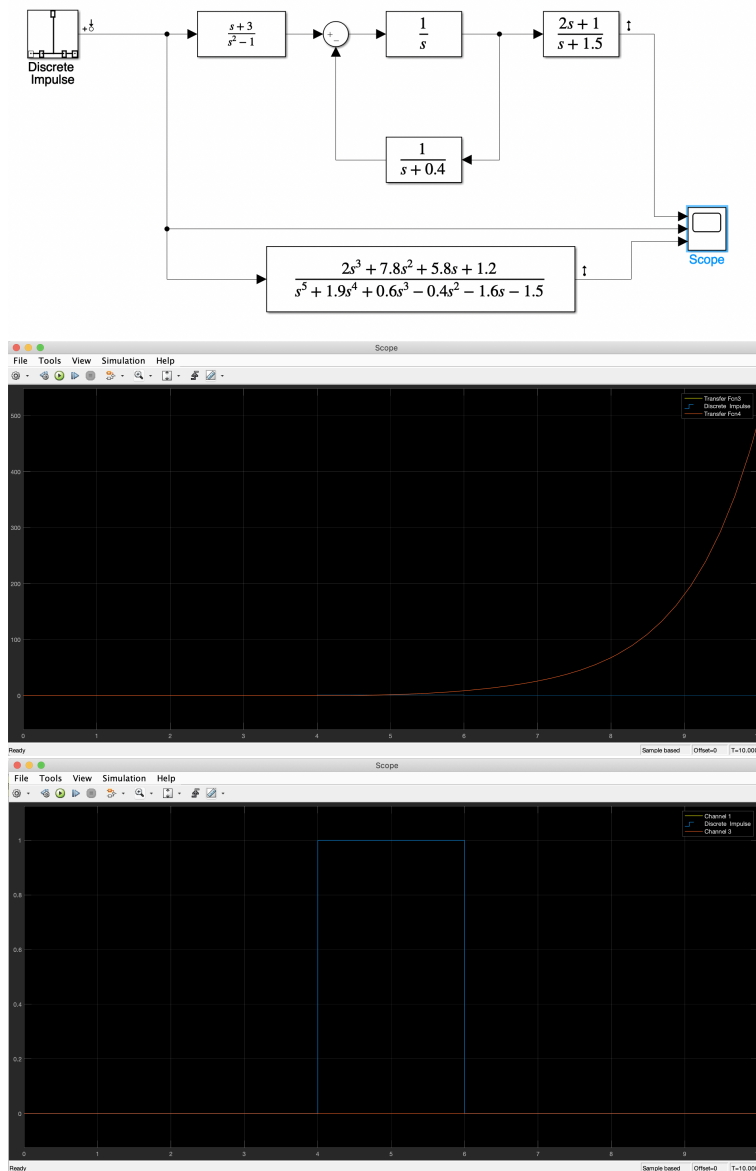


Figure 3: Here the amplitude of transfer function response is less and it is possible to notice the change in input. The amplitude of discrete impulse input is 1, duration= 2.

### 2.3 I have chosen a $\sin(x)$ input:

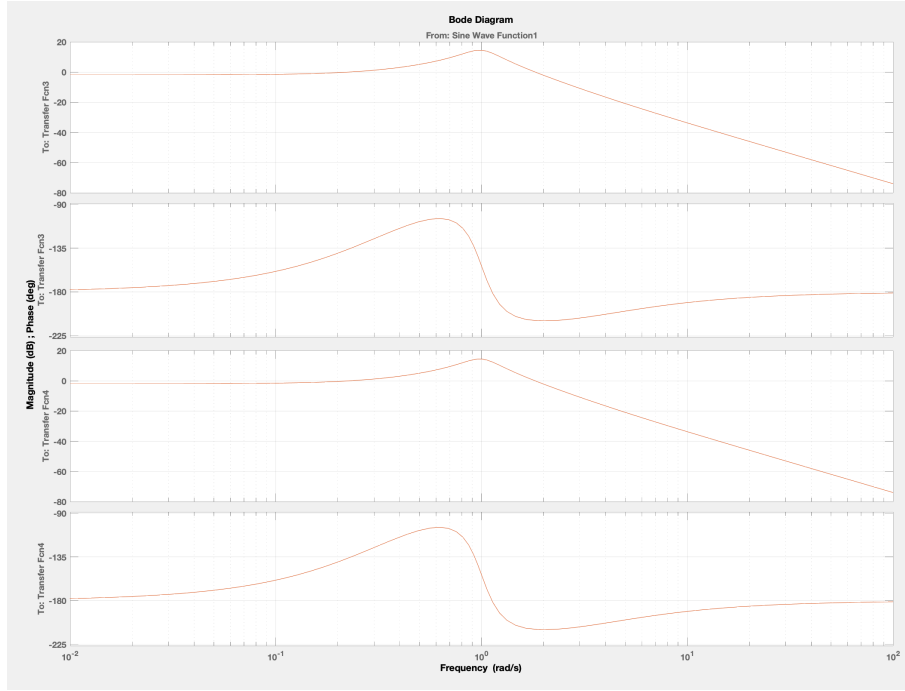


Figure 4: Bode Plot. Amplitude-Phase for non-simplified system and Amplitude-Phase for the simplified system are identical.

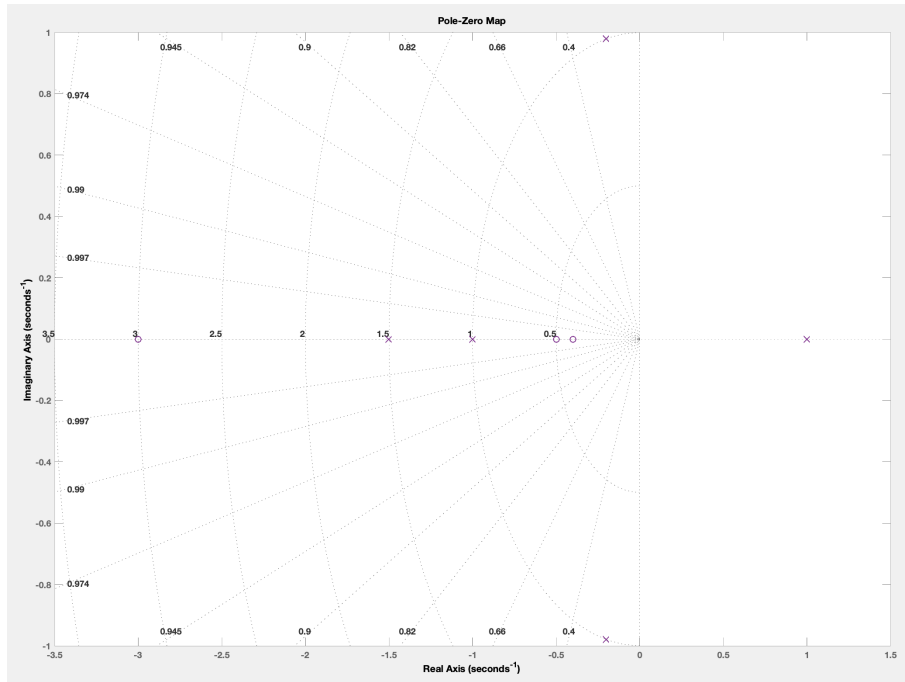


Figure 5: Pole-Zero: Poles at  $(-0.2, \pm 0.98i)$ ,  $(-1, 0)$ ,  $(-1.5, 0)$ ,  $(1, 0)$ . Zeros at  $(-3, 0)$ ,  $(-0.5, 0)$ ,  $(-0.4, 0)$ . If any pole has a positive real part there is a component in the output that increases without bound, causing the system to be unstable. We have a pole  $(1, 0)$ , hence, system is unstable.

**2.4** We know poles and zeroes from the previous step. Actually, they are As for constant,  $c_1 = 1.2$  in numerator,  $c_2 = -1.5$  in denominator, in final simplified version of the transfer function.. Hence , we have a line at  $20\log_{10}|\frac{1.2}{-1.5}| = -1.93$ .

For real poles (real zeros in denominator, clear from part 2.1), we have -20dB for a decade (e.g. -20 for 10, -40 for 100), we start subtraction after  $w_0$  frequency, which is exactly the value of the current pole. For real zeroes (zeroes in the numerator), we have +20dB for a decade, starting exactly from current zero. For both real poles and zeros before the  $w_0$  they are assumed to have 0 amplitude. Note that we take absolute values of real poles and zeroes. For complex poles we have  $s^2 - 2\epsilon w_0 s + w_0^2 = s^2 + 0.4s + 1$ , hence,  $w_0 = 1$ ,  $\epsilon = 0.2$ , therefore, we have resonance peak at  $w_r = w_0 * \sqrt{1 - 2\epsilon^2} = 0.96$  with height  $h = -20\log_{10}(2\epsilon\sqrt{1 - \epsilon^2}) = 8.14$ , than it starts decreasing from this point with -40dB for a decade. There are no complex zeroes in this case. Summing all of those we get the following:

$$A(f) = \begin{cases} -1.93, & \text{if } f < 0.4. \\ -1.93 + 20\log_{10}(\frac{w}{0.4}), & \text{if } 0.4 \leq f < 0.5. \\ -1.93 + 20\log_{10}(\frac{w}{0.4}) + 20\log_{10}(\frac{w}{0.5}), & \text{if } 0.5 \leq f < 0.96. \\ -1.93 + 20\log_{10}(\frac{w}{0.4}) + 20\log_{10}(\frac{w}{0.5}) + 8.14 = 19.48, & \text{if } f = 0.96. \\ -1.93 + 20\log_{10}(\frac{w}{0.4}) + 20\log_{10}(\frac{w}{0.5}) - 40\log_{10}(\frac{w}{0.96}), & \text{if } 0.96 \leq f < 1. \\ -1.93 + 20\log_{10}(\frac{w}{0.4}) + 20\log_{10}(\frac{w}{0.5}) - 40\log_{10}(\frac{w}{0.96}) - & \text{if } 1 \leq f < 1.5. \\ -2*20\log_{10}(\frac{w}{1}) = -19.3 + 20\log_{10}\frac{1}{0.2} - 40\log_{10}(\frac{w}{0.96}) & \\ = -1.93 + 13.98 - 40\log_{10}(\frac{w}{0.96}) = 12.05 - 40\log_{10}(\frac{w}{0.96}) & \\ \text{we have 2 here as there are 1 and -1 frequencies.} & \\ 12.05 - 40\log_{10}(\frac{w}{0.96}) - 20\log_{10}(\frac{w}{1.5}), & \text{if } 1.5 \leq f < 3. \\ 12.05 - 40\log_{10}(\frac{w}{0.96}) - 20\log_{10}(\frac{w}{1.5}) + 20\log_{10}(\frac{w}{3}), & \text{otherwise.} \end{cases} \quad (1)$$

From the function definition we can find approximate axis intersection and max value. They are:  $A = 0 \Rightarrow f = 0.40$  or  $f = 1.39$ ,  $f = 0 \Rightarrow A = -1.93$ , peak value (at  $x = 0.96$ ) is 19.48 -  $f = 0.96$  is also the break frequency value.

Real values from the matlab bode plot are  $A = 0 \Rightarrow f \approx 0.2$  or  $f \approx 1.8$ ,  $f = 0 \Rightarrow A \approx -1.93$ , peak value (at  $x \approx 0.96$ ) is  $\approx 14.5$ .

### Task 3

From lab we have the formula for Total Transfer Function of the same system:

$$T(s) = \frac{W(s)}{1 + W(s)}G + \frac{M(s)}{1 + W(s)}F$$

$$\text{In this case } W(s) = \frac{s^2 + 4s + 1}{2s^2 + 5s}, M(s) = \frac{3}{s - 3}$$

$$\text{Hence, } \frac{W(s)}{1 + W(s)} = \frac{(s^2 + 4s + 1)/(2s^2 + 5s)}{(3s^2 + 9s + 1)/(2s^2 + 5s)} = \frac{s^2 + 4s + 1}{3s^2 + 9s + 1}$$

$$\frac{M(s)}{1 + W(s)} = \frac{(3)(s - 3)}{(3s^2 + 9s + 1)/(2s^2 + 5s)} = \frac{3(2s^2 + 5s)}{3(s^3 - 2s^2 - 2s - 1)} = \frac{2s^2 + 5s}{s^3 - 2s^2 - 2s - 1}$$

Therefore, our total transfer function looks like:

$$T(s) = \frac{s^2 + 4s + 1}{3s^2 + 9s + 1}G + \frac{2s^2 + 5s}{s^3 - 2s^2 - 2s - 1}F$$

### Task 4

System:

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad C = (3 \quad 0) \quad D = (0)$$

Solution:

$$\text{Transfer function} = C(sI - A)^{-1}B + D$$

$$(sI - A) = \begin{pmatrix} s - 1 & 1 \\ -2 & s - 1 \end{pmatrix} \Rightarrow (sI - A)^{-1} = \frac{1}{(s - 1)^2 + 2} \begin{pmatrix} s - 1 & -1 \\ 2 & s - 1 \end{pmatrix}$$

$$C(sI - A)^{-1}B = \frac{1}{(s - 1)^2 + 2} (3 \quad 0) \begin{pmatrix} s - 1 & -1 \\ 2 & s - 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} =$$

$$= \frac{1}{(s - 1)^2 + 2} (3s - 3 \quad -3) \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{6s - 9}{(s - 1)^2 + 2}$$

$$\text{Transfer function} = C(sI - A)^{-1}B + D = \frac{6s - 9}{(s - 1)^2 + 2}$$

### Task 5

System:

$$A = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 3 \\ 2 & 0 \end{pmatrix} \quad C = (-1 \quad 4) \quad D = (2 \quad 1)$$

Solution:

$$\text{Transfer function} = C(sI - A)^{-1}B + D$$

$$(sI - A) = \begin{pmatrix} s-1 & 2 \\ -2 & s+1 \end{pmatrix} \Rightarrow (sI - A)^{-1} = \frac{1}{s^2 + 3} \begin{pmatrix} s-1 & -2 \\ 2 & s+1 \end{pmatrix}$$

$$\begin{aligned} C(sI - A)^{-1}B &= \frac{1}{s^2 + 3} (-1 \quad 4) \begin{pmatrix} s-1 & 2 \\ -2 & s+1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 2 & 0 \end{pmatrix} = \\ &= \frac{1}{s^2 + 3} (-7 - s \quad 4s - 4) \begin{pmatrix} 2 & 3 \\ 2 & 0 \end{pmatrix} = \frac{1}{s^2 + 3} (-22 + 6s \quad -21 - 3s) \end{aligned}$$

$$\text{Transfer function} = C(sI - A)^{-1}B + D = \frac{1}{s^2 + 3} (-20 + 6s, \quad 20 - 3s)$$

### Task 6

$$X = 0 \Rightarrow \Phi_f(x) = \frac{W_5 W_6 W_7}{1 - W_4 W_6} * \frac{W_3}{1 - \frac{W_3 W_5 W_6 W_8}{(1 - W_4 W_6)}}$$

$$F = 0 \Rightarrow \Phi(x) = \frac{W_5 W_6 W_7}{1 - W_4 W_6} * \frac{W_3}{1 - \frac{W_3 W_5 W_6 W_8}{(1 - W_4 W_6)}} * \frac{W_2}{1 - \frac{W_1 W_2}{W_3}}$$

$$\text{Total Transfer Function} = \Phi(s)G + \Phi_f(s)F$$

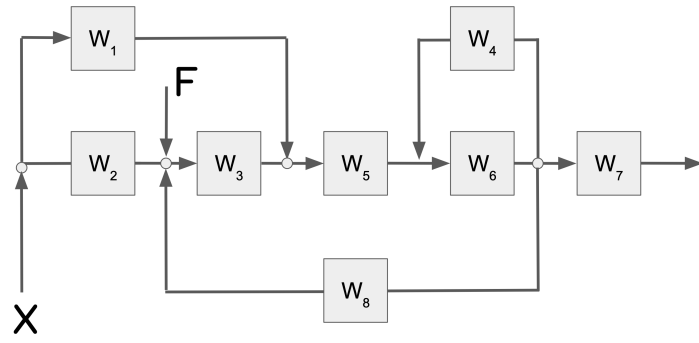


Figure 6: Initial scheme

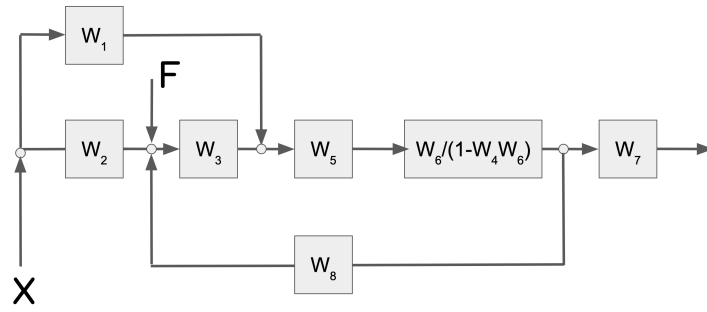


Figure 7: Simplified loop with  $W_4$  and  $W_6$



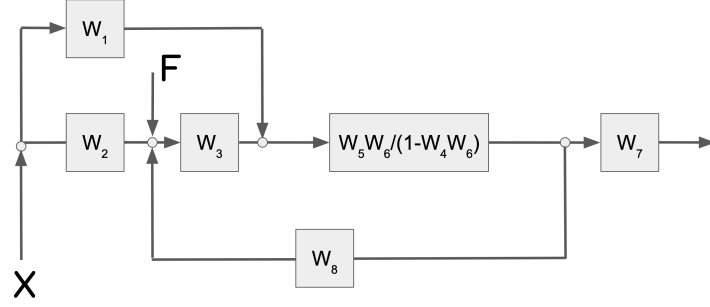


Figure 8: Combined  $W_5$  with former loop

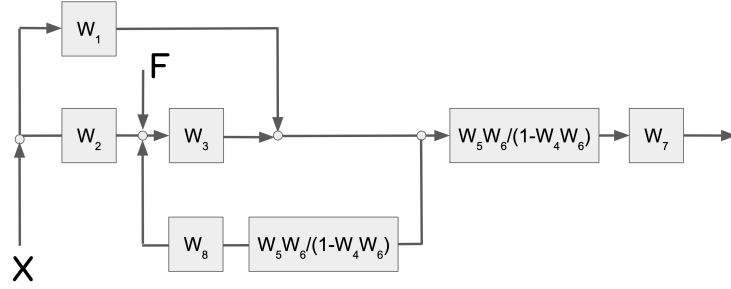


Figure 9: Moving take-off point before a block

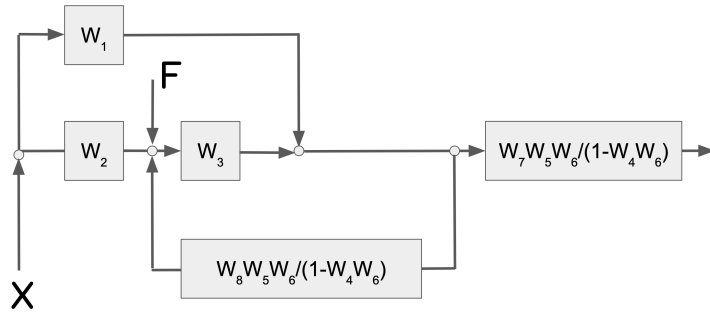


Figure 10: Combined  $W_8$  and  $W_7$  with their branches

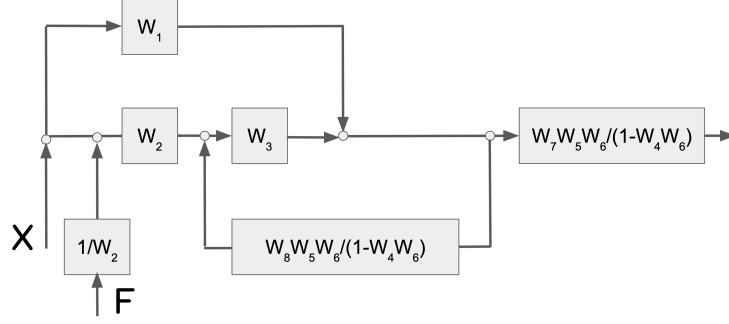


Figure 11: Moving summing point with  $F$  before  $W_2$  block

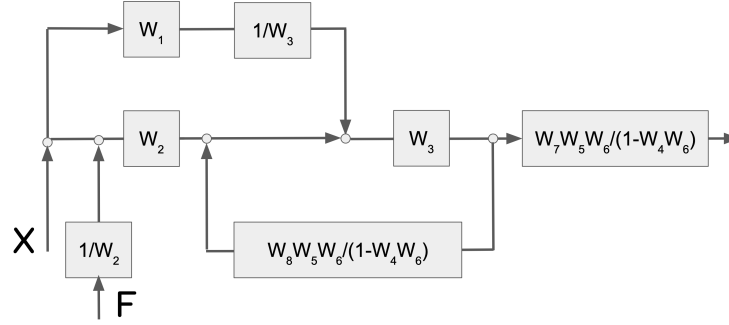


Figure 12: Moving summing point from bottom branch before  $W_3$  block

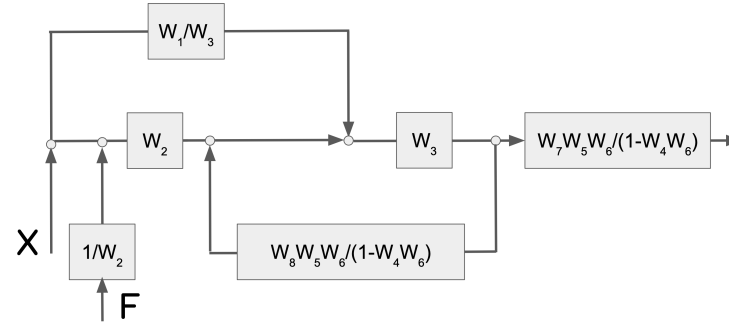


Figure 13: Combined  $W_1$  with its branch

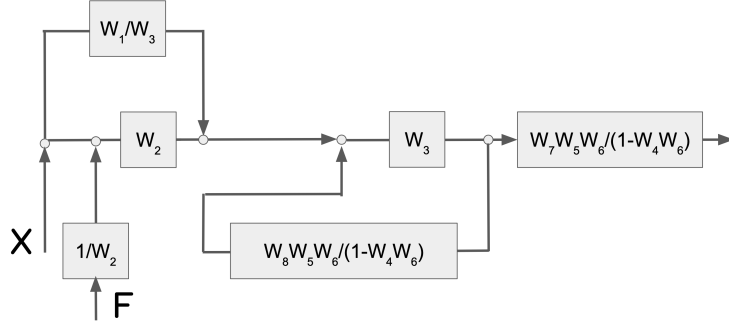


Figure 14: Associative Law - changed the order of summing points without a block between them

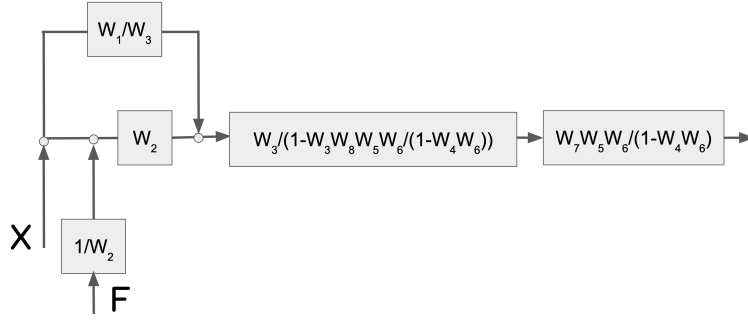


Figure 15: Simplified loop with  $W_3$

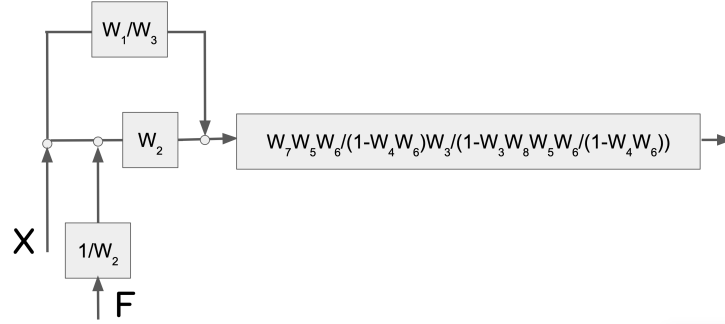


Figure 16: Combined two last blocks before output

$$\frac{W_5W_6W_7}{1 - W_4W_6} * \frac{W_3}{1 - \frac{W_3W_5W_6W_8}{(1-W_4W_6)}}$$

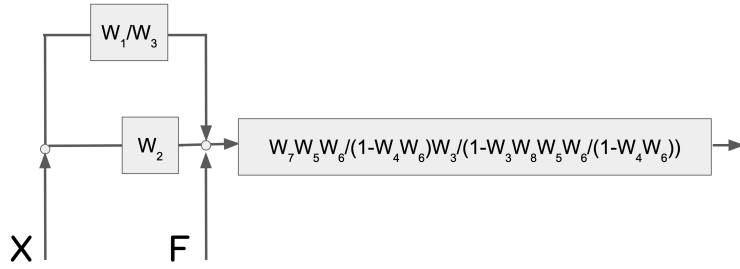


Figure 17: Move summing point after a block.

As  $W_2 * \frac{1}{W_2} = 1$  it becomes just  $F$

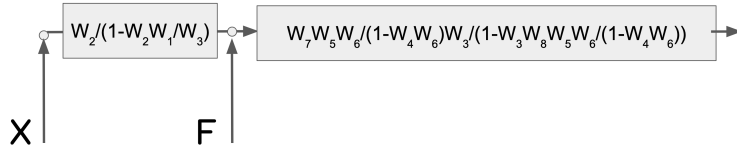


Figure 18: Simplified loop with  $W_2$  and  $\frac{W_1}{W_3}$

### 5.3. Rules for reduction of Block Diagram model:

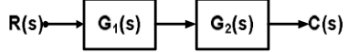
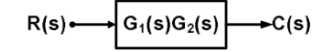

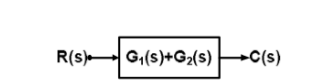
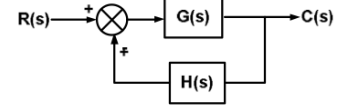
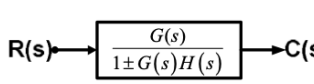
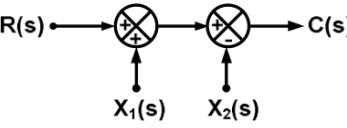
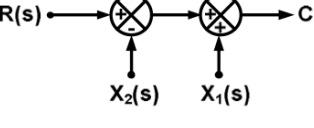
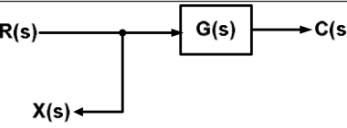
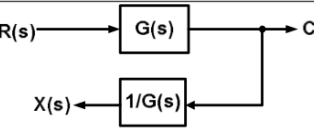
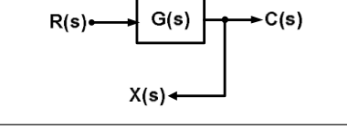

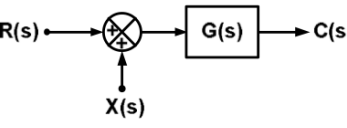
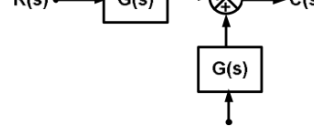
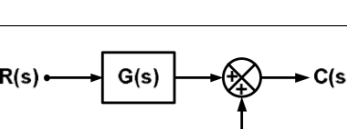
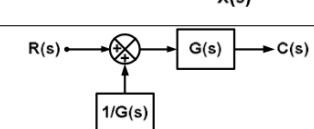
Sl. No.	Rule No.	Configuration	Equivalent	Name
1	Rule 1			Cascade
2	Rule 2			Parallel
3	Rule 3			Loop
4	Rule 4			Associative Law
5	Rule 5			Move take-off point after a block
6	Rule 6			Move take-off point before a block
7	Rule 7			Move summing-point after a block
8	Rule 8			Move summing-point before a block

Figure 19: **P.S.** Here is the list of rules used during simplification process.