Control Theory.
Assignmentn2.
Polina Turishcheva. Group 3. Variant B

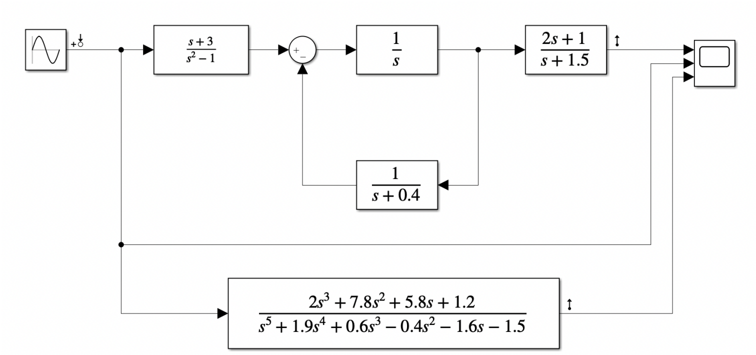
Task 2
2.1

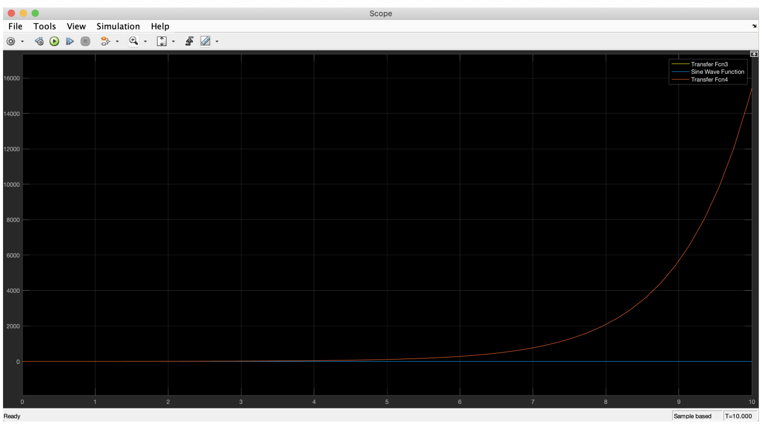
$$W = W_1 \frac{W_2}{1 + W_2 W_4} W_3$$

$$W = \frac{s+3}{s^2-1} * \frac{1/s}{1 + (1/s)(1/(s+0.4))} * \frac{2s+1}{s+1.5}$$

$$W = \frac{2s^3 + 7.8s^2 + 5.8s + 1.2}{s^5 + 1.9s^4 + 0.6s^3 - 0.4s^2 - 1.6s - 1.5}$$

2.2





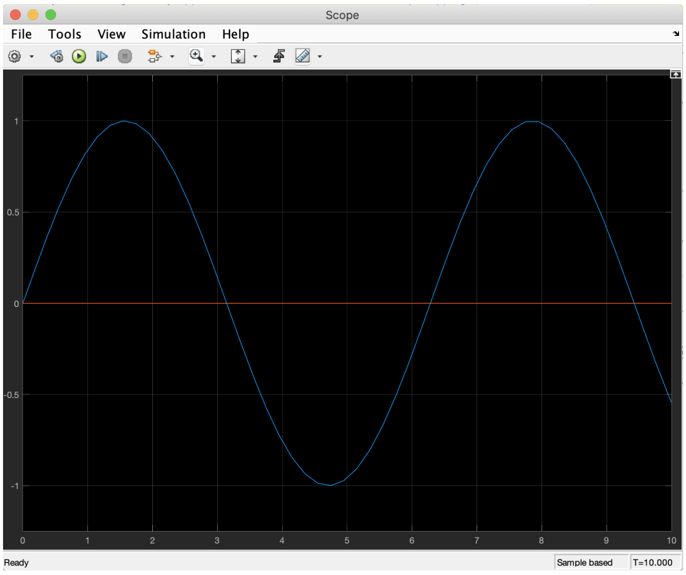


Figure 1: Frequency response describes the steady-state response of a system to sinusoidal inputs. Here input is $sin(x)$. Yellow and red lines are the same one. The blue one is from input. Because of scale its shape is not clear, hence, it is also drawn separately.

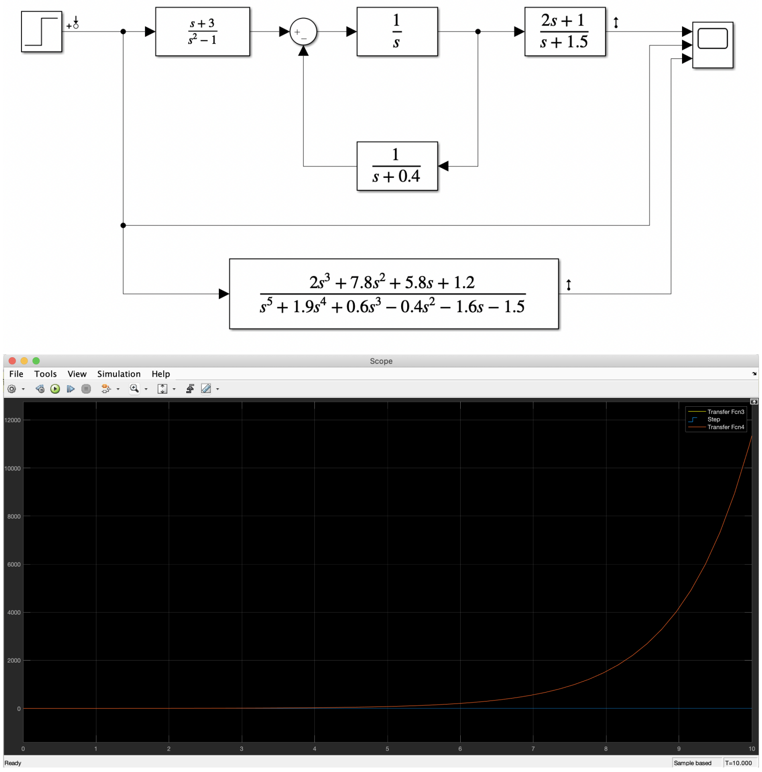


Figure 2: Step at time 1, amplitude 1.Again responses from initial and simplified systems are the same.

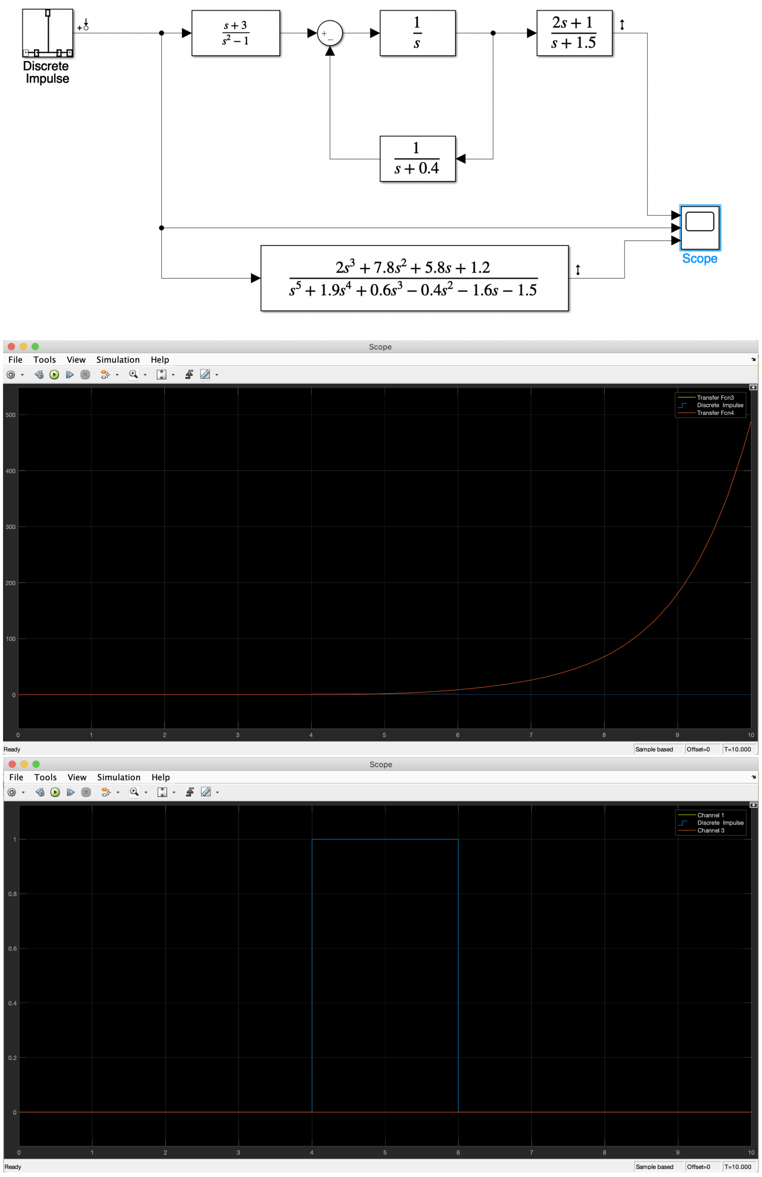


Figure 3: Here the amplitude of transfer function response is less and it is possible to notice the change in input. The amplitude of discrete impulse input is 1, duration= 2.

2.3 I have chosen a $sin(x)$ input:

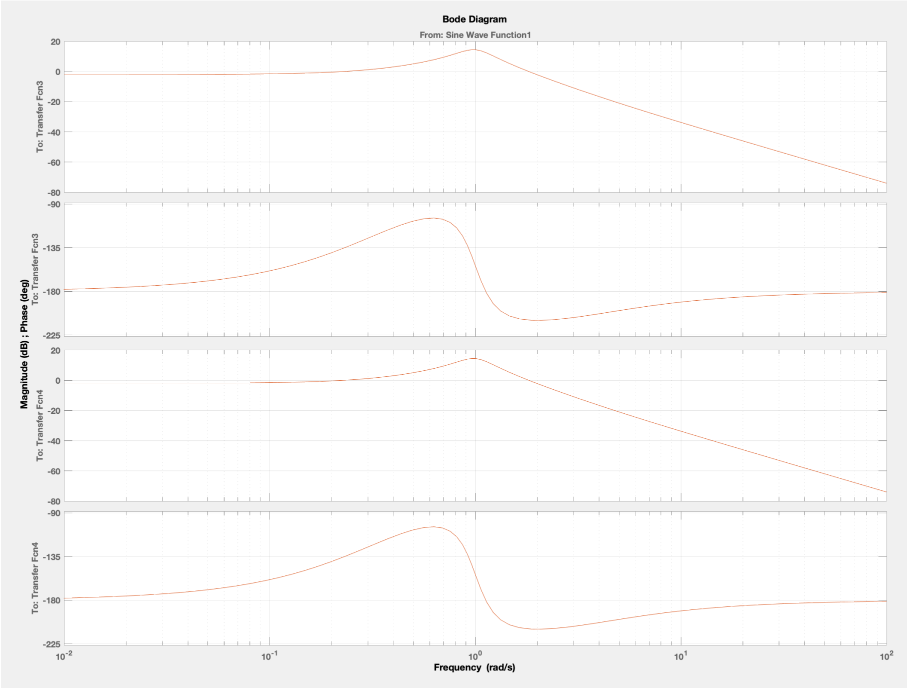


Figure 4: Bode Plot. Amplitude-Phase for non-simplified system and Amplitude-Phase for the simplified system are identical.

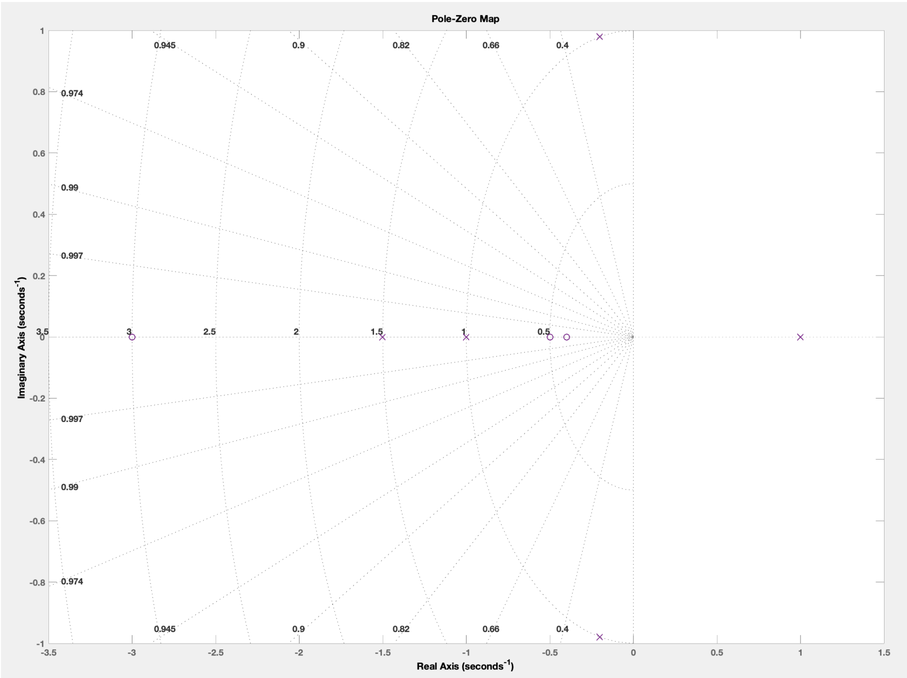


Figure 5: Pole-Zero: Poles at $(-0.2, \pm 0.98i), (-1, 0), (-1.5, 0), (1, 0)$. Zeroes at $(-3, 0), (-0.5, 0), (-0.4, 0)$. If any pole has a positive real part there is a component in the output that increases without bound, causing the system to be unstable. We have a pole (1, 0), hence, system is unstable.

2.4 We know poles and zeroes from the previous step. Actually, they are As for constant, $c_1 = 1.2$ in numerator, $c_2 = -1.5$ in denominator, in final simplified version of the transfer function.. Hence , we have a line at $20log_{10}|\frac{1.2}{-1.5}| = -1.93$.
For real poles (real zeros in denominator, clear from part 2.1), we have -20dB for a decade (e.g. -20 for 10, -40 for 100), we start subtraction after w_0 frequency, which is exactly the value of the current pole. For real zeroes (zeroes in the numerator), we have +20dB for a decade, starting exactly from current zero. For both real poles and zeros before the w_0 they are assumed to have 0 amplitude. Note that we take absolute values of real poles and zeroes. For complex poles we have $s^2 - 2\epsilon w_0 s + w_0^2 = s^2 + 0.4s + 1$, hence, $w_0 = 1$, $\epsilon = 0.2$, therefore, we have resonance peak at $w_r = w_0 * \sqrt{1 - 2\epsilon^2} = 0.96$ with height $h = -20log_{10}(2\epsilon\sqrt{1 - \epsilon^2}) = 8.14$, than it starts decreasing from this point with -40dB for a decade. There are no complex zeroes in this case.Summing all of those we get the following:

$$A(f) = \begin{cases} -1.93, & \text{if } f < 0.4. \\ \text{-1.93+20log}_{10}(\frac{w}{0.4}), & \text{if } 0.4 \leq f < 0.5. \\ \text{-1.93+20log}_{10}(\frac{w}{0.4}) + 20log_{10}(\frac{w}{0.5}), & \text{if } 0.5 \leq f < 0.96. \\ \text{-1.93+20log}_{10}(\frac{w}{0.4}) + 20log_{10}(\frac{w}{0.5}) + 8.14 = 19.48, & \text{if } f = 0.96. \\ \text{-1.93+20log}_{10}(\frac{w}{0.4}) + 20log_{10}(\frac{w}{0.5}) - 40log_{10}(\frac{w}{0.96}), & \text{if } 0.96 \leq f < 1. \\ \text{-1.93+20log}_{10}(\frac{w}{0.4}) + 20log_{10}(\frac{w}{0.5}) - 40log_{10}(\frac{w}{0.96}) - & \text{if } 1 \leq f < 1.5. \\ \text{-2*20log}_{10}(\frac{w}{1}) = -19.3 + 20log_{10}\frac{1}{0.2} - 40log_{10}(\frac{w}{0.96}) & \\ \text{=-1.93+13.98-40log}_{10}(\frac{w}{0.96}) = 12.05 - 40log_{10}(\frac{w}{0.96}) & \\ \text{we have 2 here as there are 1 and -1 frequencies.} & \\ \text{12.05-40log}_{10}(\frac{w}{0.96}) - 20log_10(\frac{w}{1.5}), & \text{if } 1.5 \leq f < 3. \\ \text{12.05-40log}_{10}(\frac{w}{0.96}) - 20log_10(\frac{w}{1.5}) + 20log_10(\frac{w}{3}), & \text{otherwise.} \end{cases} \tag{1}$$

From the function definition we can find approximate axis intersection and max value. They are:$A = 0 \Rightarrow f = 0.40 or f = 1.39$, $f = 0 \Rightarrow A = -1.93$, peak value (at $x = 0.96$) is 19.48 - $f = 0.96$ is also the break frequency value.
Real values from the matlab bode plot are $A = 0 \Rightarrow f \approx 0.2 or f \approx 1.8$, $f = 0 \Rightarrow A \approx -1.93$, peak value (at $x \approx 0.96$) is ≈ 14.5.

Task 3

From lab we have the formula for Total Transfer Function of the same system:

$$T(s) = \frac{W(s)}{1+W(s)}G + \frac{M(s)}{1+W(s)}F$$

In this case $W(s) = \dfrac{s^2+4s+1}{2s^2+5s}$, $M(s) = \dfrac{3}{s-3}$

Hence, $\dfrac{W(s)}{1+W(s)} = \dfrac{(s^2+4s+1)/(2s^2+5s)}{(3s^2+9s+1)/(2s^2+5s)} = \dfrac{s^2+4s+1}{3s^2+9s+1}$

$$\frac{M(s)}{1+W(s)} = \frac{(3)(s-3)}{(3s^2+9s+1)/(2s^2+5s)} = \frac{3(2s^2+5s)}{3(s^3-2s^2-2s-1)} = \frac{2s^2+5s}{s^3-2s^2-2s-1}$$

Therefore, our total transfer function looks like:

$$T(s) = \frac{s^2+4s+1}{3s^2+9s+1}G + \frac{2s^2+5s}{s^3-2s^2-2s-1}F$$

Task 4

System:

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \; B = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \; C = \begin{pmatrix} 3 & 0 \end{pmatrix} \; D = (0)$$

Solution:

$$\textit{Transfer function} = C(sI-A)^{-1}B + D$$

$$(sI-A) = \begin{pmatrix} s-1 & 1 \\ -2 & s-1 \end{pmatrix} \Rightarrow (sI-A)^{-1} = \frac{1}{(s-1)^2+2}\begin{pmatrix} s-1 & 2 \\ -1 & s-1 \end{pmatrix}$$

$$C(sI-A)^{-1}B = \frac{1}{(s-1)^2+2}\begin{pmatrix} 3 & 0 \end{pmatrix}\begin{pmatrix} s-1 & 2 \\ -1 & s-1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} =$$

$$= \frac{1}{(s-1)^2+2}\begin{pmatrix} 3s-3 & 6 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{6s}{(s-1)^2+2}$$

$$\textit{Transfer function} = C(sI-A)^{-1}B + D = \frac{6s}{(s-1)^2+2}$$

Task 5

System:

$$A = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \; B = \begin{pmatrix} 2 & 3 \\ 2 & 0 \end{pmatrix} \; C = \begin{pmatrix} -1 & 4 \end{pmatrix} \; D = \begin{pmatrix} 2 & 1 \end{pmatrix}$$

Solution:

$Transfer\ function = C(sI - A)^{-1}B + D$

$$(sI - A) = \begin{pmatrix} s-1 & 2 \\ -2 & s-1 \end{pmatrix} \Rightarrow (sI - A)^{-1} = \frac{1}{s^2+3} \begin{pmatrix} s-1 & -2 \\ 2 & s-1 \end{pmatrix}$$

$$C(sI - A)^{-1}B = \frac{1}{s^2+3} \begin{pmatrix} -1 & 4 \end{pmatrix} \begin{pmatrix} s-1 & -2 \\ 2 & s-1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 2 & 0 \end{pmatrix} =$$

$$= \frac{1}{s^2+3} \begin{pmatrix} 7-s & 4s-2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 2 & 0 \end{pmatrix} = \frac{1}{s^2+3} \begin{pmatrix} 10+6s & 21-3s \end{pmatrix}$$

$$Transfer\ function = C(sI - A)^{-1}B + D = \frac{1}{s^2+3} \begin{pmatrix} 12+6s, & 22-3s \end{pmatrix}$$

Task 6

$$X = 0 \Rightarrow \Phi_f(x) = \frac{W_5W_6W_7}{1 - W_4W_6} * \frac{W_3}{1 - \frac{W_3W_5W_6W_8}{(1-W_4W_6)}}$$

$$F = 0 \Rightarrow \Phi(x) = \frac{W_5W_6W_7}{1 - W_4W_6} * \frac{W_3}{1 - \frac{W_3W_5W_6W_8}{(1-W_4W_6)}} * \frac{W_2}{1 - \frac{W_1W_2}{W_3}}$$

$Total\ Transfer\ Function = \Phi(s)G + \Phi_f(s)F$

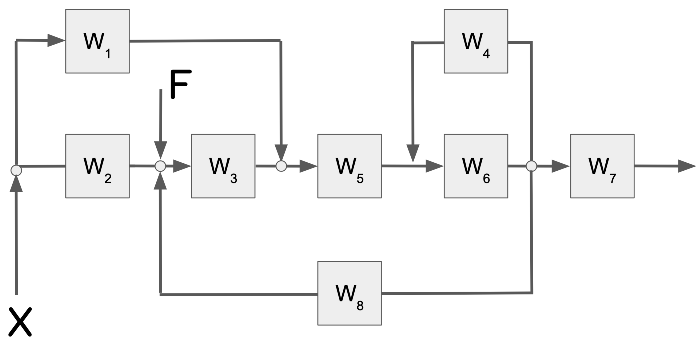


Figure 6: Initial scheme

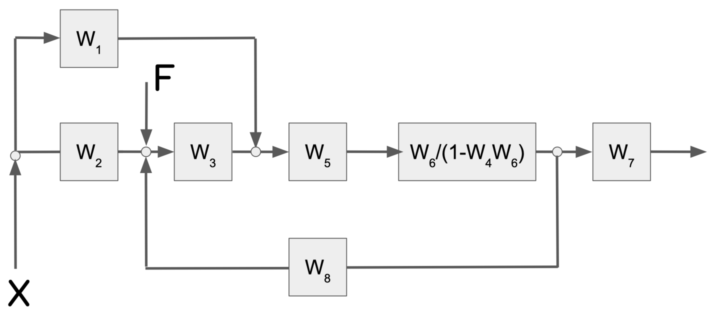


Figure 7: Simplified loop with W_4 and W_6

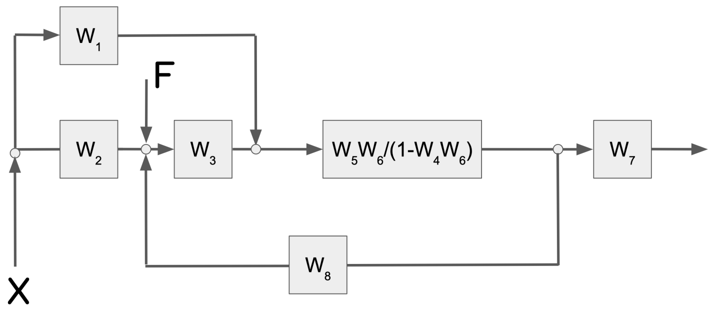


Figure 8: Combined W_5 with former loop

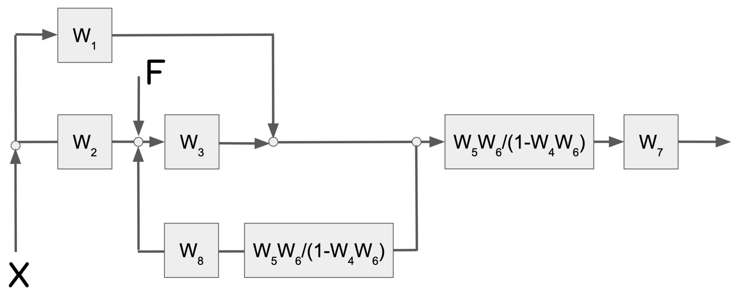


Figure 9: Moving take-off point before a block

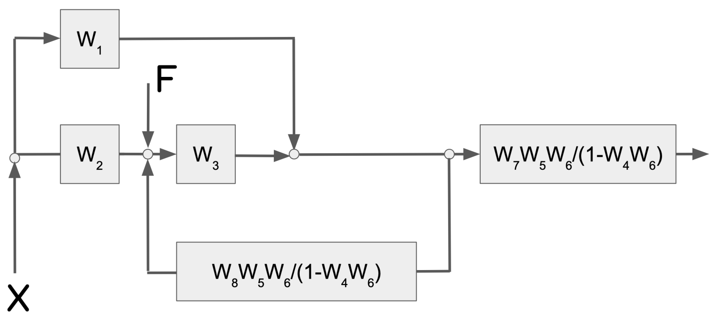


Figure 10: Combined W_8 and W_7 with their branches

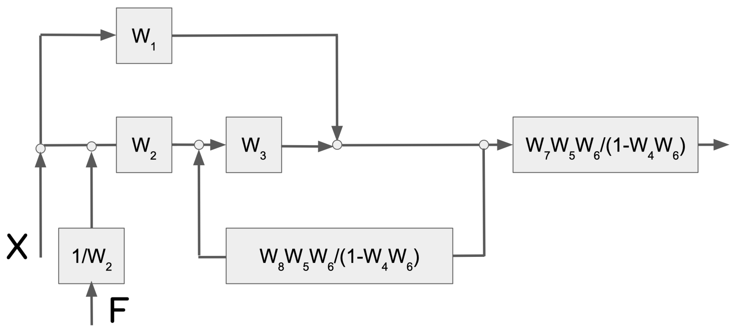


Figure 11: Moving summing point with F before W_2 block

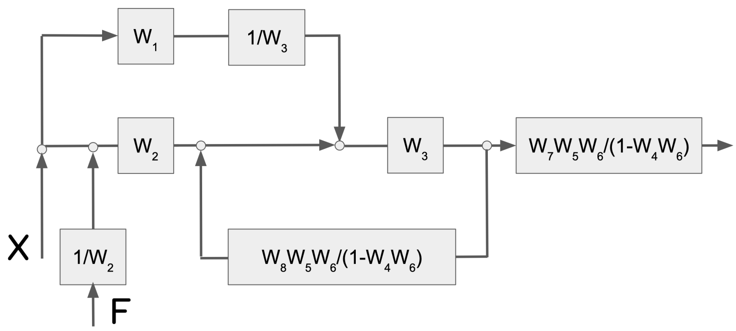


Figure 12: Moving summing point from bottom branch before W_3 block

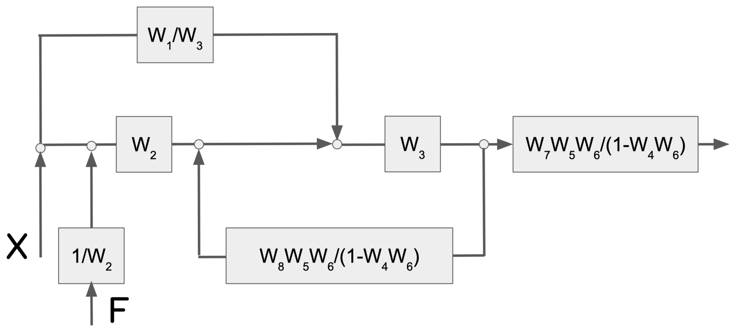


Figure 13: Combined W_1 with its branch

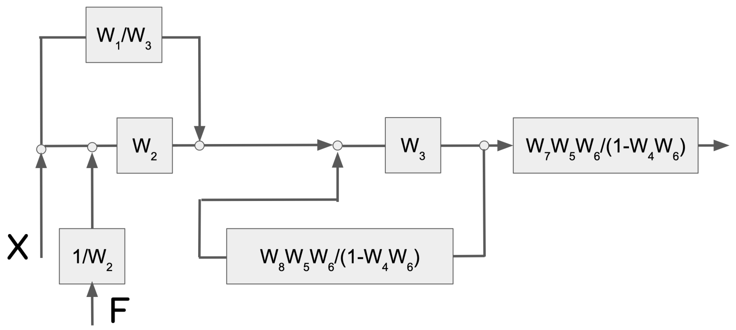


Figure 14: Associative Law - changed the order of summing points without a block between them

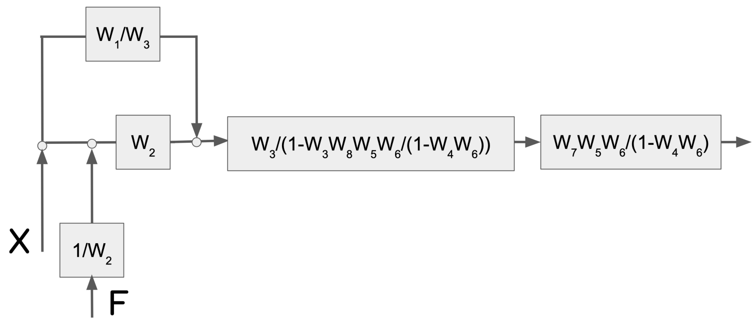


Figure 15: Simplified loop with W_3

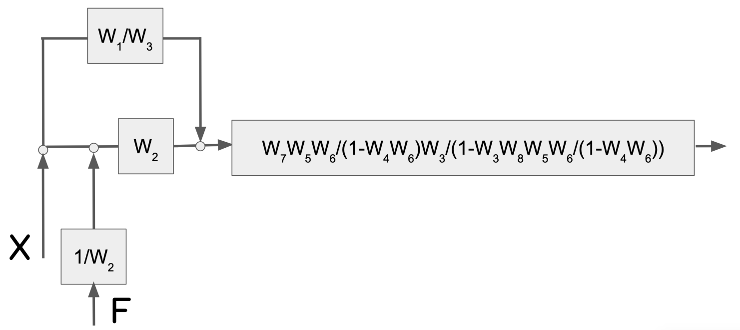


Figure 16: Combined two last blocks before output

$$\frac{W_5W_6W_7}{1-W_4W_6} * \frac{W_3}{1-\frac{W_3W_5W_6W_8}{(1-W_4W_6)}}$$

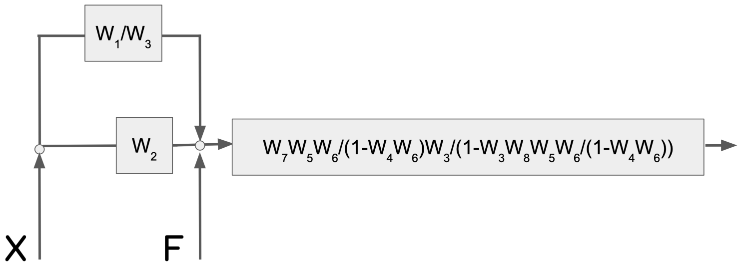


Figure 17: Move summing point after a block.
As $W_2 * \dfrac{1}{W_2} = 1$ it becomes just F

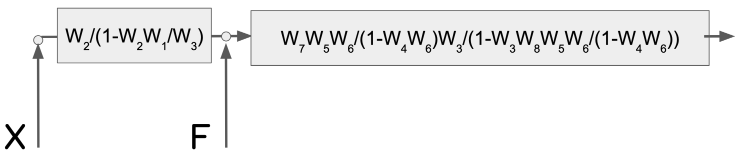


Figure 18: Simplified loop with W_2 and $\dfrac{W_1}{W_3}$

5.3. Rules for reduction of Block Diagram model:

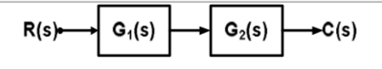
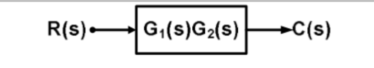
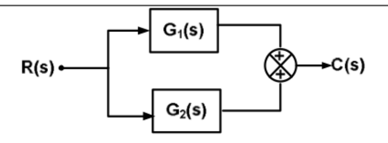
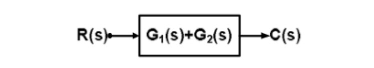
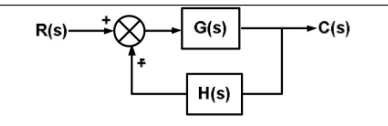
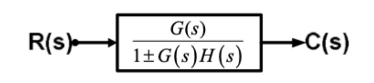
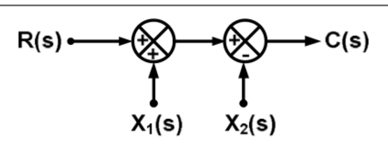
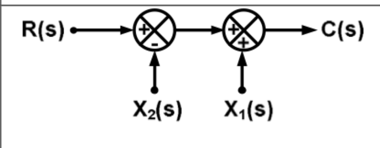
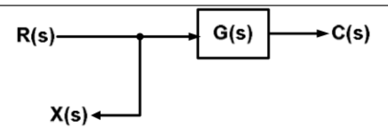
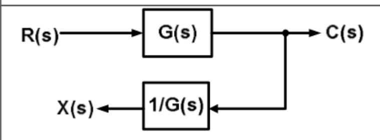
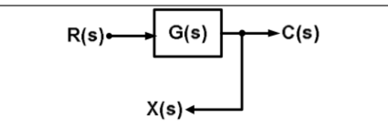
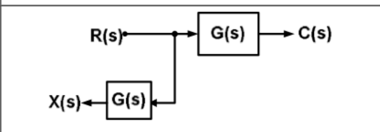
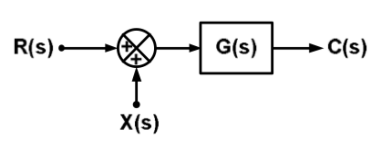
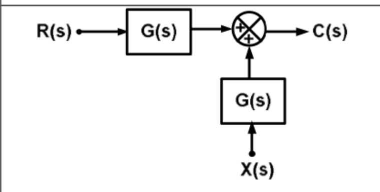
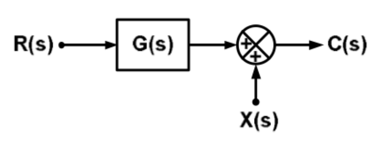
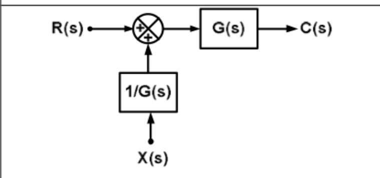
Sl. No.	Rule No.	Configuration	Equivalent	Name
1	Rule 1			Cascade
2	Rule 2			Parallel
3	Rule 3			Loop
4	Rule 4			Associative Law
5	Rule 5			Move take-off point after a block
6	Rule 6			Move take-off point before a block
7	Rule 7			Move summing-point point after a block
8	Rule 8			Move summing-point point before a block

Figure 19: **P.S.** Here is the list of rules used during simplification process.