Control Theory.
Homework Five.
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Task 1

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Task 2

My variant: $M = 5.3$, $m = 3.2$, $l = 1.15$.

From the previous assignment we have the following matrixes for the linearized system:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mg}{M} & 0 & 0 \\ 0 & \frac{(m+M)g}{Ml} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 5.92 & 0 & 0 \\ 0 & 13.67 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ \frac{1}{Ml} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.19 \\ 0.16 \end{bmatrix}$$

As in task explanation there was stated that $y = [x\theta]^T$, hence, D=0, $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

A

Rank of A matrix is 4, hence, observability matrix $\Omega = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}$ $\Omega = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 5.9200 & 0 & 0 \\ 0 & 13.6700 & 0 & 0 \\ 0 & 0 & 0 & 5.9200 \\ 0 & 0 & 0 & 13.6700 \end{bmatrix}$

$rank(\Omega) = 4$, which is equal to the size of A, hence, it is possible to design state observer of the linearized system.

B

For open-loop system, state observer has form $\hat{z}' = A\hat{z} + Bu$, hence, $\epsilon' = A'\epsilon$, therefore, for a system to be observable, A should be stable.

$$eig(A) = det\left(\begin{bmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ 0 & 5.92 & -\lambda & 0 \\ 0 & 13.67 & 0 & -\lambda \end{bmatrix}\right) = -\lambda\left(\begin{bmatrix} -\lambda & 0 & 1 \\ 5.92 & -\lambda & 0 \\ 13.67 & 0 & -\lambda \end{bmatrix}\right) = \lambda^2(\lambda^2 - 13.67)$$

Eigenvalues of matrix A are $0, 0, \approx \pm 3.6973$ and because on eigenvalues is positive $\Rightarrow$ the system is unstable $\Rightarrow$ error dynamics is NOT stable.

Solutions for C-J are available in the colab notebook via the link Link