
Introduction to Artificial Intelligence.

Assignment One.

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Assumptions:

- Start point is always (0,0)
- The field edges are (0,0), (0, 9), (9, 0), (9,9) - inclusively
- Diagonal throws are valid
- Only one throw per search run is valid
- We were not asked to create successful algorithms in terms of correct answer or time. This assumption is important for random search because according to our TA random walk does not analyze adjacent cells, hence, it can step on an ork
- For random search if the player meets org - player dies
- For random search if he/she tries to go out of map bound - player dies
- Random walk makes maximum 100 steps during a search
- All algorithms search for the 1st valid path. This is done because to find all passes on 10 on 10 map is extremely time consuming. In fact this is what happens on the empty map. Time necessary for such test is available further. This was also allowed by Prof. Brown.
- The code was checked on [SWI prolog](#) and [with its system on local machine](#).
- No input files are read. The map conditions should be written in executed file.
- [Link to github with code.](#)

Description of the 3rd algorithm:

My third algorithm is an improved backtracking. The idea is that I assume that we know the point of start, which is (0,0). Hence, we can investigate the map only in 2 directions: up and right. This reduces the number of step twice, comparing with backtracking, which should check all possible directions: up, rights, down, left. This is kind of heuristic. However, with such moving scheme there is a tricky case - Figure 1. In case of figure 1-a any of 3 algorithms will perform successfully only one step - up-right diagonal throw. But for case 1-b and 1-c we have opportunities to move up or right but for such moving scheme some part of the field will stay unvisited, hence, if touchdown will be there - it will not find it. *In other languages we could use a global list of visited cells and than solve it like this: add the second part for our algorithm. If during up-right search the touchdown was not found, hence, we start moving down-left and if the cell was not visited during the first period(up and right exploration), it is not in the global list,hence, we start backtracking from it. To understand, why it is important look at picture 2.* Unfortunately, prolog design does not allow us to have global variables, therefore, in case the first up and right diagonal walk has found no paths, we start backtracking for the initial position. The reasons for this solution to be still improving will be discussed in Part One.

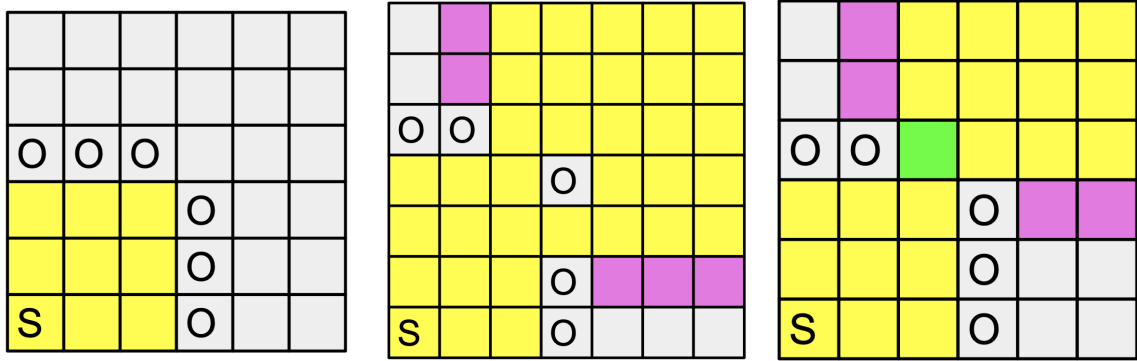


Figure 1: S- start, O -Ork.

Yellow cells - the algorithm will visit them.

Pink cells - the algorithm will visit them only if touchdown is there.

Green cell - the only possibility to go out of orks' angle, this point limits the areas that will be visited.

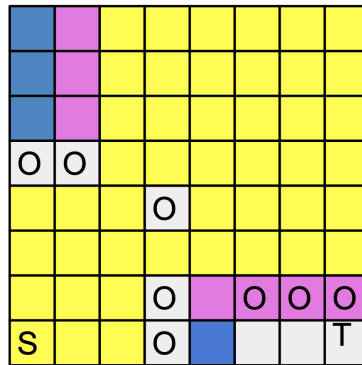


Figure 2: Blue cells - will be additionally visited during left and down iteration. Hence, from such cells we should start backtracking, otherwise, we will not be able to find touchdown in some cases. The example is in the picture. There is no use start backtracking from every cell because backtracking is very time consuming, only from unvisited before cells. If from backtracking we return to the cells from the first(right and up) iteration, we stop this backtracking branch.

Part One First algorithm is on average works much faster than backtracking because it always scans more than half of the map in a very short period of time. The chances that the touchdown point will be found during the up and right search is equal to $\frac{\text{yellow cells} + \text{pink cells} + \text{orks on boundary}}{\text{all cells in the map}}$.

The worst probability to find touchdown during up and right search is 36%. This happens when orks' angle appear is one line along the whole axe - figure 3. This angle has low probability to appear $= 2 * 0.25^9 \approx 7.6 * 10^{-6}$ because each cell has 4 options - ork, nothing, human, touchdown, therefore, we can assume that all of them have equal probability to appear in this cell, therefore, probability for ork is 0.25, and there should be 9 such cells simultaneously on the fixed positions, 2 become from one horizontal and one vertical row of 9 orks. Hence, on average, our chances to find touchdown are much higher and the first iteration is so cheap to perform (comparing with backtrackings hard-to-solve cases), that it definitely worth it. The worst case for all three algorithms is when we have absolutely empty map because they should visit each cell. For the third algorithm it is even worse as it will visit all cells more times - during up and right search and during backtracking search. I have tested all three algorithms on absolutely empty 5 on 5, 6 on 6 maps (7 on 7 empty map was too long to execute - after several hours I have interrupted the execution - in order to save time as empty 10 on 10 map is extremely long to execute) - the difference between 3rd approach and backtracking seems to be neglectable- figure 4,5. Moreover, is the touchdown point will be found during up and right search and there are no humans on the map - the result from 3rd algorithm will be the shortest possible as it makes more rational steps than backtracking. Even with humans it will most probably be better than for the backtracking,

The first and second tests were intended to check the time improvement of the 3rd method over backtracking.

```
%Test one
o(0,3).
o(1,3).
o(3,0).
o(3,1).
o(3,2).
t(4,4).
```

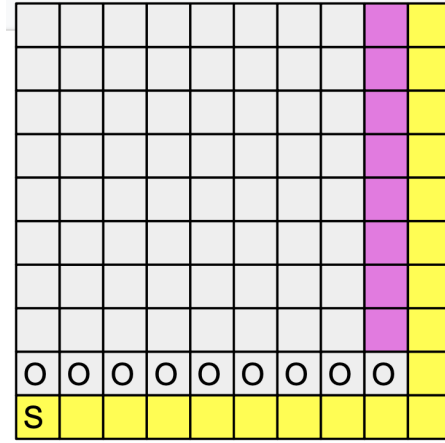


Figure 3: Worst case for 3rd algorithm heuristic.

```
?- main.
Backtracking No solution
% 85,100,281 inferences, 7.522 CPU in 7.526 seconds (100% CPU, 11314196 Lips)
Random search No solution
% 119 inferences, 0.001 CPU in 0.001 seconds (83% CPU, 224105 Lips)
Improved backtracking No solution
% 85,482,854 inferences, 7.530 CPU in 7.533 seconds (100% CPU, 11352801 Lips)
```

Figure 4: Empty 5 on 5 map. Difference between backtracking and third method is 7ms, when execution takes more than 7.5 seconds.

```
Backtracking No solution
% 20,673,344,525 inferences, 1993.505 CPU in 1999.864 seconds (100% CPU, 10370350 Lips)
Random search No solution
% 133 inferences, 0.000 CPU in 0.000 seconds (87% CPU, 306452 Lips)
Improved backtracking No solution
% 20,726,814,104 inferences, 2008.347 CPU in 2014.941 seconds (100% CPU, 10320333 Lips)
true.
```

Figure 5: Empty 6 on 6 map. Difference between backtracking and third method is about 7s, when execution takes more than 2000 seconds.

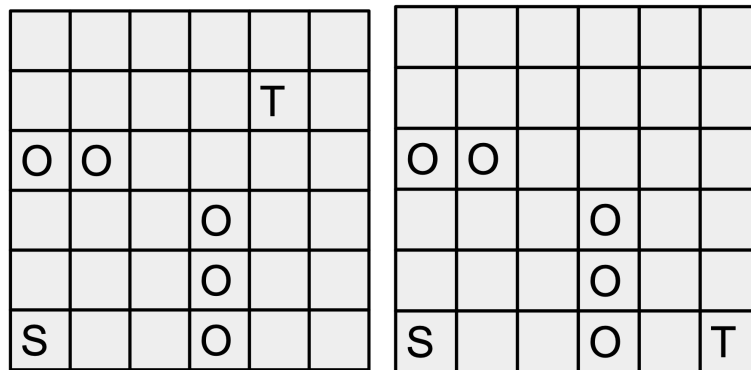


Figure 6: In the first test the touchdown should be found during up and right search, in the second tests it will be found only during backtracking stage.

Backtracking 60[[, (0,0), (0,1), (0,2), (1,2), (2,2), (2,3), (2,4), (2,5), (2,6), (2,7), (2,8), (2,9), (3,9), (4,9), (5,9), (6,9), (7,9), (8,9), (9,9), (9,8), (9,7), (9,6), (9,5), (9,4), (9,3), (9,2), (9,1), (9,0), (8,0), (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (8,8), (7,8), (7,7), (7,6), (7,5), (7,4), (7,3), (7,2), (7,1), (7,0), (6,0), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6), (6,7), (6,8), (5,8), (5,7), (5,6), (5,5), (5,4), (4,4)]_4006]	
74,810 inferences, 0.014 CPU in 0.014 seconds (100% CPU, 5353606 Lips)	
Random search No solution	
354 inferences, 0.001 CPU in 0.001 seconds (98% CPU, 339680 Lips)	
Improved backtracking 10[[, (0,0), (0,1), (0,2), (1,2), (2,2), (2,3), (2,4), (2,5), (3,5), (4,5), (4,4)]_3392]	
545,923 inferences, 0.064 CPU in 0.065 seconds (100% CPU, 8463987 Lips)	
true	1
Test 1	
Backtracking 47[[, (0,0), (0,1), (0,2), (1,2), (2,2), (2,3), (2,4), (2,5), (2,6), (2,7), (2,8), (2,9), (3,9), (4,9), (5,9), (6,9), (7,9), (8,9), (9,9), (9,8), (9,7), (9,6), (9,5), (9,4), (9,3), (9,2), (9,1), (9,0), (8,0), (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (8,8), (7,8), (7,7), (7,6), (7,5), (7,4), (7,3), (7,2), (7,1), (7,0), (6,0), (5,0)]_3850]	
57,406 inferences, 0.011 CPU in 0.011 seconds (100% CPU, 5332545 Lips)	
Random search No solution	
353 inferences, 0.001 CPU in 0.001 seconds (98% CPU, 338249 Lips)	
Improved backtracking in backtracking47[[, (0,0), (0,0), (0,1), (0,2), (1,2), (2,2), (2,3), (2,4), (2,5), (2,6), (2,7), (2,8), (2,9), (3,9), (4,9), (5,9), (6,9), (7,9), (8,9), (9,9), (9,8), (9,7), (9,6), (9,5), (9,4), (9,3), (9,2), (9,1), (9,0), (8,0), (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (8,8), (7,8), (7,7), (7,6), (7,5), (7,4), (7,3), (7,2), (7,1), (7,0), (6,0), (5,0)]_3848]	
24,669,457 inferences, 2.865 CPU in 2.865 seconds (100% CPU, 8609383 Lips)	
true	1
Test 2	

Figure 7: Results for test 1(up) and test 2 (down).In test one algorithm 3 reaches the touchdown during the up and right stage, hence it provides much more rational path. However, as algorithm is also kind of 2-directional backtracking it starts recursion which is rather time consuming. During the 2nd test, when the 3rd algorithm will go to backtracking stage this is even more obvious. Note that the path output looks like $[[, actual\ path \mid _something]$, the empty list in the beginning and everything after \mid can be neglected, it is shown just because I have used difference list for my inner implementation.

Backtracking 32[[, (0,0), (0,1), (4,5), (4,6), (4,7), (4,8), (4,9), (5,9), (6,9), (7,9), (8,9), (9,9), (9,8), (9,7), (9,6), (9,5), (9,4), (9,3), (9,2), (9,1), (9,0), (8,0), (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (8,8), (7,8), (7,7), (7,6), (6,6)]_3680]	
22,702 inferences, 0.006 CPU in 0.006 seconds (100% CPU, 4091345 Lips)	
Random search No solution	
395 inferences, 0.001 CPU in 0.001 seconds (99% CPU, 374538 Lips)	
Improved backtracking 6[[, (0,0), (0,1), (4,5), (4,6), (4,7), (5,7), (6,7), (6,6)]_24132]	
8,943 inferences, 0.002 CPU in 0.002 seconds (100% CPU, 5285586 Lips)	
true	1
Backtracking 17[[, (0,0), (0,1), (0,2), (1,2), (2,2), (2,3), (2,4), (2,5), (2,6), (2,7), (2,8), (2,9), (3,9), (4,9), (5,9), (6,9), (7,9), (8,9)]_36142]	
20,268 inferences, 0.004 CPU in 0.004 seconds (100% CPU, 5298814 Lips)	
Random search No solution	
409 inferences, 0.001 CPU in 0.001 seconds (100% CPU, 391442 Lips)	
Improved backtracking 17[[, (0,0), (0,1), (0,2), (1,2), (2,2), (2,3), (2,4), (2,5), (2,6), (2,7), (2,8), (2,9), (3,9), (4,9), (5,9), (6,9), (7,9), (8,9)]_3476]	
19,766 inferences, 0.004 CPU in 0.004 seconds (100% CPU, 5114232 Lips)	
true	1

Test 3

Test 4

Figure 8: Tests 3 and 4 have same orks positions as for tests 1 and 2, not other touchdown positions and also in test 3 a human was added. Results for test 3(up) and test 4(down). Time difference is noticeable, it is 3 times less for the 3rd algorithm. For the 4th test, when 3rd algorithm when on the backtracking stage the time difference is not noticeable. Both tests were run on 10 on 10 map.

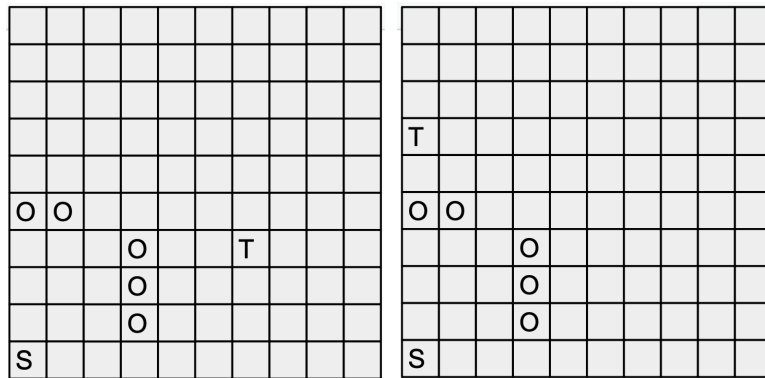


Figure 9: Maps for Test 6 and Test 7.

```

Backtracking 43[[], (0,0), (0,1), (0,2), (0,3), (1,3), (2,3), (2,4), (2,5), (2,6), (2,7), (2,8), (2,9), (3,9), (4,9), (5,9), (6,9), (7,9),
(8,9), (9,9), (9,8), (9,7), (9,6), (9,5), (9,4), (9,3), (9,2), (9,1), (9,0), (8,0), (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (8,8),
(7,8), (7,7), (7,6), (7,5), (7,4), (7,3), (6,3)]_3854]
51,871 inferences, 0.010 CPU in 0.010 seconds (100% CPU, 5192383 Lips)
Random search 1 No solution
403 inferences, 0.001 CPU in 0.001 seconds (99% CPU, 389662 Lips)
Improved backtracking 11[[], (0,0), (0,1), (0,2), (0,3), (1,3), (2,3), (2,4), (3,4), (4,4), (5,4), (6,4), (6,3)]_36074]
1,757,265 inferences, 0.205 CPU in 0.205 seconds (100% CPU, 8579730 Lips)

```

Test6

```

Backtracking 10[[[], (0,0), (0,1), (0,2), (0,3), (1,3), (2,3), (2,4), (2,5), (2,6), (1,6), (0,6)]_8174]
% 69,477,641 inferences, 6.869 CPU in 6.874 seconds (100% CPU, 10114093 Lips)
Random search 1 No solution
% 78 inferences, 0.001 CPU in 0.001 seconds (85% CPU, 133080 Lips)
Improved backtracking in backtracking10[[[], (0,0), (0,0), (0,1), (0,2), (0,3), (1,3), (2,3), (2,4), (2,5), (2,6), (1,6), (0,6)]_10338]
% 78,012,488 inferences, 6.931 CPU in 6.935 seconds (100% CPU, 10101688 Lips)
true.
% 27,610,669,391 inferences, 3128.416 CPU in 3139.489 seconds (100% CPU, 8825767 Lips)
% Execution Aborted

```

Test7-1

Test7-2

Figure 10: Test 6 was run on 10 on 10 map. Test 7 was run on 7 on 7 map. Because of time reasons. Test 7 on 10 on 10 map has not terminated in an hour for backtracking on my machine. This is also reasoning for not looking for the best path.

```

%Test two
o(0,3).
o(1,3).
o(3,0).
o(3,1).
o(3,2).
t(5,0).
%Test three
o(0,3).
o(1,3).
o(3,0).
o(3,1).
o(3,2).
h(4,5).
t(6,6).
%Test four
o(0,3).
o(1,3).
o(3,0).
o(3,1).
o(3,2).
t(8,9).
%test five
h(3,3).

```



```

t(4,4).
o(0,-1).
o(0,3).
o(1,3).
o(2,3).
o(3,2).
o(3,1).
o(3,0).
%test six
o(0,4).
o(1,4).
o(3,3).
o(3,1).
o(3,2).
t(6,3).
%test seven
o(0,4).
o(1,4).
o(3,3).
o(3,1).
o(3,2).
t(6,3).
%Test eight
t(0,2).
%Test nine
t(5,5).

```

One can notice that random search has not succeeded ever. I have run random walk on the empty 10 on 10 map and analyzed the paths it has chosen in order to try to find the type of maps that can be solved with it. In the output A is value of current randomly generated number. For A - 0 is up throw, 1 is right throw, 2 is down throw, 3 is left throw, 4 is up-left throw, 5 is up-right throw, 6 is down-right throw, 7 is down-left throw, 8 is up step, 9 is right step, 10 is down step, 11 is left step.

Paths from random walk on the empty map:

1. X=0 Y=0 A=5 No solution
2. X=0 Y=0 A=6 No solution
3. X=0 Y=0 A=5 No solution
4. X=0 Y=0 A=6 No solution

5. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=9$ $X=2$ $Y=0$ $A=1$ No solution
6. $X=0$ $Y=0$ $A=4$ No solution
7. $X=0$ $Y=0$ $A=0$ No solution
8. $X=0$ $Y=0$ $A=3$ No solution
9. $X=0$ $Y=0$ $A=0$ No solution
10. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=6$ No solution
11. $X=0$ $Y=0$ $A=0$ No solution
12. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=0$ No solution
13. $X=0$ $Y=0$ $A=0$ No solution
14. $X=0$ $Y=0$ $A=1$ No solution
15. $X=0$ $Y=0$ $A=0$ No solution
16. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=8$ $X=1$ $Y=1$ $A=7$ No solution
17. $X=0$ $Y=0$ $A=1$ No solution
18. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=10$ $X=1$ $Y=-1$ No solution
19. $X=0$ $Y=0$ $A=3$ No solution
20. $X=0$ $Y=0$ $A=3$ No solution
21. $X=0$ $Y=0$ $A=1$ No solution
22. $X=0$ $Y=0$ $A=0$ No solution
23. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=3$ No solution
24. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=11$ $X=2$ $Y=1$ $A=8$ $X=2$ $Y=2$ $A=6$ No solution
25. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=7$ No solution
26. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=3$ No solution
27. $X=0$ $Y=0$ $A=10$ $X=0$ $Y=-1$ No solution
28. $X=0$ $Y=0$ $A=6$ No solution
29. $X=0$ $Y=0$ $A=5$ No solution
30. $X=0$ $Y=0$ $A=10$ $X=0$ $Y=-1$ No solution
31. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=1$ No solution
32. $X=0$ $Y=0$ $A=3$ No solution
33. $X=0$ $Y=0$ $A=1$ No solution
34. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=0$ No solution
35. $X=0$ $Y=0$ $A=6$ No solution
36. $X=0$ $Y=0$ $A=5$ No solution
37. $X=0$ $Y=0$ $A=6$ No solution
38. $X=0$ $Y=0$ $A=7$ No solution

39. $X=0$ $Y=0$ $A=6$ No solution
40. $X=0$ $Y=0$ $A=2$ No solution
41. $X=0$ $Y=0$ $A=4$ No solution
42. $X=0$ $Y=0$ $A=2$ No solution
43. $X=0$ $Y=0$ $A=2$ No solution
44. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=11$ $X=2$ $Y=1$ $A=5$ No solution
45. $X=0$ $Y=0$ $A=0$ No solution
46. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=7$ No solution
47. $X=0$ $Y=0$ $A=5$ No solution
48. $X=0$ $Y=0$ $A=7$ No solution
49. $X=0$ $Y=0$ $A=2$ No solution
50. $X=0$ $Y=0$ $A=3$ No solution
51. $X=0$ $Y=0$ $A=10$ $X=0$ $Y=-1$ No solution
52. $X=0$ $Y=0$ $A=3$ No solution
53. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=3$ No solution
54. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=2$ No solution
55. $X=0$ $Y=0$ $A=2$ No solution
56. $X=0$ $Y=0$ $A=3$ No solution
57. $X=0$ $Y=0$ $A=3$ No solution
58. $X=0$ $Y=0$ $A=4$ No solution
59. $X=0$ $Y=0$ $A=5$ No solution
60. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=2$ No solution
61. $X=0$ $Y=0$ $A=1$ No solution
62. $X=0$ $Y=0$ $A=2$ No solution
63. $X=0$ $Y=0$ $A=4$ No solution
64. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=2$ No solution
65. $X=0$ $Y=0$ $A=5$ No solution
66. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=2$ No solution
67. $X=0$ $Y=0$ $A=6$ No solution
68. $X=0$ $Y=0$ $A=2$ No solution
69. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=0$ No solution
70. $X=0$ $Y=0$ $A=0$ No solution
71. $X=0$ $Y=0$ $A=4$ No solution
72. $X=0$ $Y=0$ $A=0$ No solution

73. $X=0$ $Y=0$ $A=4$ No solution
74. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=3$ No solution
75. $X=0$ $Y=0$ $A=2$ No solution
76. $X=0$ $Y=0$ $A=0$ No solution
77. $X=0$ $Y=0$ $A=0$ No solution
78. $X=0$ $Y=0$ $A=10$ $X=0$ $Y=-1$ No solution
79. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=6$ No solution
80. $X=0$ $Y=0$ $A=2$ No solution
81. $X=0$ $Y=0$ $A=1$ No solution
82. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=0$ No solution
83. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=1$ No solution
84. $X=0$ $Y=0$ $A=7$ No solution
85. $X=0$ $Y=0$ $A=5$ No solution
86. $X=0$ $Y=0$ $A=4$ No solution
87. $X=0$ $Y=0$ $A=2$ No solution
88. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=6$ No solution
89. $X=0$ $Y=0$ $A=3$ No solution
90. $X=0$ $Y=0$ $A=1$ No solution
91. $X=0$ $Y=0$ $A=3$ No solution
92. $X=0$ $Y=0$ $A=4$ No solution
93. $X=0$ $Y=0$ $A=6$ No solution
94. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=2$ No solution
95. $X=0$ $Y=0$ $A=6$ No solution
96. $X=0$ $Y=0$ $A=3$ No solution
97. $X=0$ $Y=0$ $A=5$ No solution
98. $X=0$ $Y=0$ $A=5$ No solution
99. $X=0$ $Y=0$ $A=4$ No solution
100. $X=0$ $Y=0$ $A=5$ No solution

21 of the paths had length 2, 3 had length 3 and one had length 4. At total here were 141 choosing of random number, here is statistics about it:

Backtracking 3[[], (0,0), (3,3), (3,4), (4,4)]_34152]
3,278 inferences, 0.001 CPU in 0.001 seconds (99% CPU, 3535040 Lips)
Random search 1 No solution
341 inferences, 0.001 CPU in 0.001 seconds (99% CPU, 306406 Lips)
Improved backtracking 3[[], (0,0), (3,3), (3,4), (4,4)]_42880]
3,271 inferences, 0.001 CPU in 0.001 seconds (101% CPU, 3759373 Lips)

Figure 11: Results of test 5 for all 3 algorithms. Proves that random search is extremely inefficient.

Number	Direction	Times	Percentage
0	Up Throw	15	10.6
1	Right Throw	21	14.9
2	Down Throw	15	10.6
3	Left Throw	15	10.6
4	Up-Right Throw	9	6.4
5	Up-Left Throw	12	8.5
6	Down-Right Throw	13	9.2
7	Down-Left Throw	6	4.3
8	Up Move	12	8.5
9	Right Move	12	8.5
10	Down Move	5	3.5
11	Left Move	6	4.3

From here we see that the choose of the move is not purely uniformly distributed (because random is not pure random), however, each choice has competitive chances. The most popular values are throws, hence, maps with people catching ball from right or up sides may be successful for the random search. So lets try. The result of 400 runs of the test 5 are written down. The average execution time is less 0.001s. However, *only 3% of runs succeeded to find the touchdown and only 1.5 of them found the most optimal path.* Hence, we conclude that random search is useless. It may be crucially improved if it would analyze its surroundings but in the assumptions it is written that we were asked not to do it. *Scroll for page 25 to see the results.*

Note that for some outputs X or Y are equal to -1, this happened because print was put before checking if the cell is in bound.

1. X=0 Y=0 A=3 No solution
2. X=0 Y=0 A=10 No solution
3. X=0 Y=0 A=7 No solution
4. X=0 Y=0 A=6 No solution
5. X=0 Y=0 A=10 No solution

6. $X=0$ $Y=0$ $A=6$ No solution
7. $X=0$ $Y=0$ $A=4$ No solution
8. $X=0$ $Y=0$ $A=6$ No solution
9. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=9$ $X=2$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=10$ No solution
10. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=5$ No solution
11. $X=0$ $Y=0$ $A=10$ No solution
12. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=10$ No solution
13. $X=0$ $Y=0$ $A=10$ No solution
14. $X=0$ $Y=0$ $A=10$ No solution
15. $X=0$ $Y=0$ $A=10$ No solution
16. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=8$ $X=0$ $Y=2$ $A=11$ $X=3$ $Y=2$ No solution
17. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=9$ $X=1$ $Y=1$ $A=2$ No solution
18. $X=0$ $Y=0$ $A=2$ No solution
19. $X=0$ $Y=0$ $A=0$ No solution
20. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=0$ No solution
21. $X=0$ $Y=0$ $A=7$ No solution
22. $X=0$ $Y=0$ $A=0$ No solution
23. $X=0$ $Y=0$ $A=7$ No solution
24. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=10$ $X=3$ $Y=2$ No solution
25. $X=0$ $Y=0$ $A=0$ No solution
26. $X=0$ $Y=0$ $A=1$ No solution
27. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=9$ $X=4$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=10$ $X=4$ $Y=2$ $A=9$ $X=5$ $Y=2$ $A=8$ $X=5$ $Y=3$ $A=10$ $X=5$ $Y=2$ $A=8$ $X=5$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=8$ $X=4$ $Y=4$ $\text{cost}=10$ $\text{Path}=[[], (0,0), (3,3), (4,3), (4,3), (4,2), (5,2), (5,3), (5,2), (5,3), (4,3), (4,4)]$
28. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=11$ $X=2$ $Y=1$ $A=8$ $X=2$ $Y=2$ $A=0$ No solution
29. $X=0$ $Y=0$ $A=4$ No solution
30. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=10$ No solution
31. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=10$ $X=3$ $Y=2$ No solution
32. $X=0$ $Y=0$ $A=10$ No solution
33. $X=0$ $Y=0$ $A=3$ No solution
34. $X=0$ $Y=0$ $A=4$ No solution
35. $X=0$ $Y=0$ $A=3$ No solution
36. $X=0$ $Y=0$ $A=1$ No solution
37. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=9$ $X=2$ $Y=0$ $A=2$ No solution
38. $X=0$ $Y=0$ $A=10$ No solution
39. $X=0$ $Y=0$ $A=2$ No solution

40. $X=0$ $Y=0$ $A=2$ No solution
41. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=10$ No solution
42. $X=0$ $Y=0$ $A=2$ No solution
43. $X=0$ $Y=0$ $A=7$ No solution
44. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=0$ No solution
45. $X=0$ $Y=0$ $A=3$ No solution
46. $X=0$ $Y=0$ $A=0$ No solution
47. $X=0$ $Y=0$ $A=7$ No solution
48. $X=0$ $Y=0$ $A=0$ No solution
49. $X=0$ $Y=0$ $A=7$ No solution
50. $X=0$ $Y=0$ $A=3$ No solution
51. $X=0$ $Y=0$ $A=0$ No solution
52. $X=0$ $Y=0$ $A=7$ No solution
53. $X=0$ $Y=0$ $A=3$ No solution
54. $X=0$ $Y=0$ $A=1$ No solution
55. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=6$ No solution
56. $X=0$ $Y=0$ $A=10$ No solution
57. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=4$ No solution
58. $X=0$ $Y=0$ $A=10$ No solution
59. $X=0$ $Y=0$ $A=2$ No solution
60. $X=0$ $Y=0$ $A=4$ No solution
61. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=10$ $X=4$ $Y=2$ $A=8$ $X=4$ $Y=3$ $A=8$ $X=4$ $Y=4$ cost =5 Path=[[], (0,0), (3,3), (4,3), (4,2), (4,3), (4,4)]
62. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=5$ No solution
63. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=10$ $X=0$ $Y=0$ $A=3$ No solution
64. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=4$ No solution
65. $X=0$ $Y=0$ $A=6$ No solution
66. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=10$ $X=0$ $Y=0$ $A=7$ No solution
67. $X=0$ $Y=0$ $A=10$ No solution
68. $X=0$ $Y=0$ $A=1$ No solution
69. $X=0$ $Y=0$ $A=2$ No solution
70. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=9$ $X=4$ $Y=3$ $A=9$ $X=5$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=9$ $X=5$ $Y=3$ $A=8$ $X=5$ $Y=4$ $A=9$ $X=6$ $Y=4$ $A=11$ $X=5$ $Y=4$ $A=8$ $X=5$ $Y=5$ $A=9$ $X=6$ $Y=5$ $A=10$ $X=6$ $Y=4$ $A=10$ $X=6$ $Y=3$ $A=10$ $X=6$ $Y=2$ $A=8$ $X=6$ $Y=3$ $A=10$ $X=6$ $Y=2$ $A=11$ $X=3$ $Y=2$ No solution
71. $X=0$ $Y=0$ $A=2$ No solution
72. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=10$ $X=4$ $Y=2$ $A=10$ $X=4$ $Y=1$ $A=9$ $X=5$ $Y=1$ $A=11$ $X=2$ $Y=1$ $A=10$ $X=2$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=8$ $X=1$ $Y=1$ $A=9$ $X=2$ $Y=1$ $A=11$ $X=2$ $Y=1$ $A=11$ $X=2$ $Y=1$ $A=8$ $X=2$ $Y=2$ $A=8$ $X=2$ $Y=3$ No solution

73. $X=0$ $Y=0$ $A=4$ No solution
74. $X=0$ $Y=0$ $A=4$ No solution
75. $X=0$ $Y=0$ $A=7$ No solution
76. $X=0$ $Y=0$ $A=0$ No solution
77. $X=0$ $Y=0$ $A=6$ No solution
78. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=11$ $X=2$ $Y=1$ $A=9$ $X=3$ $Y=1$ No solution
79. $X=0$ $Y=0$ $A=7$ No solution
80. $X=0$ $Y=0$ $A=4$ No solution
81. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=7$ No solution
82. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=1$ No solution
83. $X=0$ $Y=0$ $A=6$ No solution
84. $X=0$ $Y=0$ $A=1$ No solution
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86. $X=0$ $Y=0$ $A=10$ No solution
87. $X=0$ $Y=0$ $A=10$ No solution
88. $X=0$ $Y=0$ $A=10$ No solution
89. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=11$ $X=2$ $Y=1$ $A=5$ No solution
90. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=1$ No solution
91. $X=0$ $Y=0$ $A=6$ No solution
92. $X=0$ $Y=0$ $A=7$ No solution
93. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=9$ $X=4$ $Y=3$ $A=9$ $X=5$ $Y=3$ $A=9$ $X=6$ $Y=3$ $A=10$ $X=6$ $Y=2$ $A=11$ $X=3$ $Y=2$ No solution
94. $X=0$ $Y=0$ $A=2$ No solution
95. $X=0$ $Y=0$ $A=1$ No solution
96. $X=0$ $Y=0$ $A=6$ No solution
97. $X=0$ $Y=0$ $A=0$ No solution
98. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=8$ $X=3$ $Y=4$ $A=8$ $X=3$ $Y=5$ $A=11$ $X=6$ $Y=5$ $A=8$ $X=6$ $Y=6$ $A=11$ $X=7$ $Y=6$ $A=9$ $X=8$ $Y=6$ $A=10$ $X=8$ $Y=5$ $A=9$ $X=9$ $Y=5$ $A=11$ $X=6$ $Y=5$ $A=10$ $X=6$ $Y=4$ $A=11$ $X=5$ $Y=4$ $A=8$ $X=5$ $Y=5$ $A=8$ $X=5$ $Y=6$ $A=9$ $X=6$ $Y=6$ $A=9$ $X=7$ $Y=6$ $A=8$ $X=7$ $Y=7$ $A=10$ $X=7$ $Y=6$ $A=8$ $X=7$ $Y=7$ $A=8$ $X=7$ $Y=8$ $A=11$ $X=9$ $Y=8$ $A=10$ $X=9$ $Y=7$ $A=8$ $X=9$ $Y=8$ $A=9$ No solution
99. $X=0$ $Y=0$ $A=1$ No solution
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101. $X=0$ $Y=0$ $A=2$ No solution
102. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=3$ No solution
103. $X=0$ $Y=0$ $A=7$ No solution
104. $X=0$ $Y=0$ $A=6$ No solution
105. $X=0$ $Y=0$ $A=6$ No solution

106. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=8$ $X=3$ $Y=4$ $A=11$ $X=5$ $Y=4$ $A=8$ $X=5$ $Y=5$ $A=9$ $X=6$ $Y=5$ $A=10$ $X=6$ $Y=4$ $A=9$ $X=7$ $Y=4$ $A=9$ $X=8$ $Y=4$ $A=9$ $X=9$ $Y=4$ $A=8$ $X=9$ $Y=5$ $A=11$ $X=6$ $Y=5$ $A=11$ $X=6$ $Y=5$ $A=10$ $X=6$ $Y=4$ $A=10$ $X=6$ $Y=3$ $A=8$ $X=6$ $Y=4$ $A=9$ $X=7$ $Y=4$ $A=10$ $X=7$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=9$ $X=5$ $Y=3$ $A=10$ $X=5$ $Y=2$ $A=10$ $X=5$ $Y=1$ $A=8$ $X=5$ $Y=2$ $A=10$ $X=5$ $Y=1$ $A=10$ $X=5$ $Y=0$ $A=9$ $X=6$ $Y=0$ $A=9$ $X=7$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=9$ $X=2$ $Y=0$ $A=8$ $X=2$ $Y=1$ $A=8$ $X=2$ $Y=2$ $A=10$ $X=2$ $Y=1$ $A=9$ $X=3$ $Y=1$ No solution
107. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=5$ No solution
108. $X=0$ $Y=0$ $A=2$ No solution
109. $X=0$ $Y=0$ $A=10$ No solution
110. $X=0$ $Y=0$ $A=0$ No solution
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114. $X=0$ $Y=0$ $A=3$ No solution
115. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=10$ No solution
116. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=1$ No solution
117. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=1$ No solution
118. $X=0$ $Y=0$ $A=10$ No solution
119. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=9$ $X=1$ $Y=1$ $A=2$ No solution
120. $X=0$ $Y=0$ $A=2$ No solution
121. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=0$ No solution
122. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=4$ No solution
123. $X=0$ $Y=0$ $A=10$ No solution
124. $X=0$ $Y=0$ $A=0$ No solution
125. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=7$ No solution
126. $X=0$ $Y=0$ $A=3$ No solution
127. $X=0$ $Y=0$ $A=1$ No solution
128. $X=0$ $Y=0$ $A=3$ No solution
129. $X=0$ $Y=0$ $A=2$ No solution
130. $X=0$ $Y=0$ $A=10$ No solution
131. $X=0$ $Y=0$ $A=0$ No solution
132. $X=0$ $Y=0$ $A=6$ No solution
133. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=7$ No solution
134. $X=0$ $Y=0$ $A=10$ No solution
135. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=0$ No solution
136. $X=0$ $Y=0$ $A=7$ No solution
137. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=6$ No solution
138. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=0$ No solution

139. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=1$ No solution
140. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=1$ No solution
141. $X=0$ $Y=0$ $A=7$ No solution
142. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=1$ No solution
143. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=0$ No solution
144. $X=0$ $Y=0$ $A=10$ No solution
145. $X=0$ $Y=0$ $A=4$ No solution
146. $X=0$ $Y=0$ $A=1$ No solution
147. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=1$ No solution
148. $X=0$ $Y=0$ $A=7$ No solution
149. $X=0$ $Y=0$ $A=0$ No solution
150. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=7$ No solution
151. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=10$ $X=3$ $Y=2$ No solution
152. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=9$ $X=4$ $Y=3$ $A=9$ $X=5$ $Y=3$ $A=9$ $X=6$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=9$ $X=5$ $Y=3$ $A=9$ $X=6$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=9$ $X=5$ $Y=3$ $A=10$ $X=5$ $Y=2$ $A=8$ $X=5$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=8$ $X=4$ $Y=4$ cost =13 Path=[[], (0,0), (3,3), (4,3), (5,3), (6,3), (4,3), (5,3), (6,3), (4,3), (5,3), (5,2), (5,3), (4,3), (4,4)]
153. $X=0$ $Y=0$ $A=10$ No solution
154. $X=0$ $Y=0$ $A=7$ No solution
155. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=9$ $X=4$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=9$ $X=5$ $Y=3$ $A=9$ $X=6$ $Y=3$ $A=8$ $X=6$ $Y=4$ $A=9$ $X=7$ $Y=4$ $A=8$ $X=7$ $Y=5$ $A=9$ $X=8$ $Y=5$ $A=8$ $X=8$ $Y=6$ $A=10$ $X=8$ $Y=5$ $A=8$ $X=8$ $Y=6$ $A=9$ $X=9$ $Y=6$ $A=10$ $X=9$ $Y=5$ $A=9$ No solution
156. $X=0$ $Y=0$ $A=3$ No solution
157. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=10$ $X=3$ $Y=2$ No solution
158. $X=0$ $Y=0$ $A=10$ No solution
159. $X=0$ $Y=0$ $A=3$ No solution
160. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=5$ No solution
161. $X=0$ $Y=0$ $A=2$ No solution
162. $X=0$ $Y=0$ $A=0$ No solution
163. $X=0$ $Y=0$ $A=10$ No solution
164. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=4$ No solution
165. $X=0$ $Y=0$ $A=6$ No solution
166. $X=0$ $Y=0$ $A=10$ No solution
167. $X=0$ $Y=0$ $A=7$ No solution
168. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=10$ No solution
169. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=8$ $X=3$ $Y=4$ $A=8$ $X=3$ $Y=5$ $A=10$ $X=3$ $Y=4$ $A=8$ $X=3$ $Y=5$ $A=10$ $X=3$ $Y=4$ $A=11$ $X=5$ $Y=4$ $A=9$ $X=6$ $Y=4$ $A=11$ $X=5$ $Y=4$ $A=9$ $X=6$ $Y=4$ $A=8$ $X=6$ $Y=5$ $A=8$ $X=6$ $Y=6$ $A=9$ $X=7$ $Y=6$ $A=11$ $X=7$ $Y=6$ $A=9$ $X=8$ $Y=6$ $A=8$ $X=8$ $Y=7$ $A=8$ $X=8$ $Y=8$ $A=9$ $X=9$ $Y=8$ $A=8$ $X=9$ $Y=9$ $A=9$ No solution
170. $X=0$ $Y=0$ $A=3$ No solution

171. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=8$ $X=1$ $Y=1$ $A=1$ No solution
172. $X=0$ $Y=0$ $A=4$ No solution
173. $X=0$ $Y=0$ $A=0$ No solution
174. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=1$ No solution
175. $X=0$ $Y=0$ $A=1$ No solution
176. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=4$ No solution
177. $X=0$ $Y=0$ $A=4$ No solution
178. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=10$ $X=3$ $Y=2$ No solution
179. $X=0$ $Y=0$ $A=2$ No solution
180. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=7$ No solution
181. $X=0$ $Y=0$ $A=0$ No solution
182. $X=0$ $Y=0$ $A=4$ No solution
183. $X=0$ $Y=0$ $A=7$ No solution
184. $X=0$ $Y=0$ $A=1$ No solution
185. $X=0$ $Y=0$ $A=6$ No solution
186. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=10$ $X=0$ $Y=0$ $A=7$ No solution
187. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=3$ No solution
188. $X=0$ $Y=0$ $A=6$ No solution
189. $X=0$ $Y=0$ $A=1$ No solution
190. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=10$ $X=4$ $Y=2$ $A=9$ $X=5$ $Y=2$ $A=10$ $X=5$ $Y=1$ $A=10$ $X=5$ $Y=0$ $A=10$ No solution
191. $X=0$ $Y=0$ $A=2$ No solution
192. $X=0$ $Y=0$ $A=6$ No solution
193. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=0$ No solution
194. $X=0$ $Y=0$ $A=0$ No solution
195. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=5$ No solution
196. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=8$ $X=3$ $Y=4$ $A=11$ $X=5$ $Y=4$ $A=11$ $X=5$ $Y=4$ $A=10$ $X=5$ $Y=3$ $A=10$ $X=5$ $Y=2$ $A=11$ $X=3$ $Y=2$ No solution
197. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=4$ No solution
198. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=0$ No solution
199. $X=0$ $Y=0$ $A=3$ No solution
200. $X=0$ $Y=0$ $A=7$ No solution
201. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=2$ No solution
202. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=2$ No solution
203. $X=0$ $Y=0$ $A=3$ No solution
204. $X=0$ $Y=0$ $A=1$ No solution

205. $X=0$ $Y=0$ $A=2$ No solution
206. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=9$ $X=4$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=8$ $X=4$ $Y=4$ cost =4 Path=[[], (0,0), (3,3), (4,3), (4,3), (4,4)] $X=0$ $Y=0$ $A=0$ No solution
207. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=10$ No solution
208. $X=0$ $Y=0$ $A=1$ No solution
209. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=8$ $X=4$ $Y=4$ cost =3 Path=[[], (0,0), (3,3), (4,3), (4,4)]
210. $X=0$ $Y=0$ $A=10$ No solution
211. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=1$ No solution
212. $X=0$ $Y=0$ $A=1$ No solution
213. $X=0$ $Y=0$ $A=3$ No solution
214. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=9$ $X=1$ $Y=1$ $A=6$ No solution
215. $X=0$ $Y=0$ $A=3$ No solution
216. $X=0$ $Y=0$ $A=4$ No solution
217. $X=0$ $Y=0$ $A=6$ No solution
218. $X=0$ $Y=0$ $A=2$ No solution
219. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=5$ No solution
220. $X=0$ $Y=0$ $A=7$ No solution
221. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=10$ $X=3$ $Y=2$ No solution
222. $X=0$ $Y=0$ $A=6$ No solution
223. $X=0$ $Y=0$ $A=0$ No solution
224. $X=0$ $Y=0$ $A=10$ No solution
225. $X=0$ $Y=0$ $A=2$ No solution
226. $X=0$ $Y=0$ $A=2$ No solution
227. $X=0$ $Y=0$ $A=0$ No solution
228. $X=0$ $Y=0$ $A=6$ No solution
229. $X=0$ $Y=0$ $A=10$ No solution
230. $X=0$ $Y=0$ $A=0$ No solution
231. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=9$ $X=1$ $Y=1$ $A=11$ $X=2$ $Y=1$ $A=3$ No solution
232. $X=0$ $Y=0$ $A=4$ No solution
233. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=3$ No solution
234. $X=0$ $Y=0$ $A=7$ No solution
235. $X=0$ $Y=0$ $A=7$ No solution
236. $X=0$ $Y=0$ $A=1$ No solution
237. $X=0$ $Y=0$ $A=0$ No solution
238. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=7$ No solution

239. $X=0$ $Y=0$ $A=7$ No solution
240. $X=0$ $Y=0$ $A=3$ No solution
241. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=9$ $X=5$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=8$ $X=4$ $Y=4$ cost =5 Path=[[], (0,0), (3,3), (4,3), (5,3), (4,3), (4,4)]
242. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=0$ No solution
243. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=9$ $X=4$ $Y=3$ $A=8$ $X=4$ $Y=4$ cost =3 Path=[[], (0,0), (3,3), (4,3), (4,4)]
244. $X=0$ $Y=0$ $A=1$ No solution
245. $X=0$ $Y=0$ $A=10$ No solution
246. $X=0$ $Y=0$ $A=4$ No solution
247. $X=0$ $Y=0$ $A=7$ No solution
248. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=7$ No solution
249. $X=0$ $Y=0$ $A=1$ No solution
250. $X=0$ $Y=0$ $A=1$ No solution
251. $X=0$ $Y=0$ $A=4$ No solution
252. $X=0$ $Y=0$ $A=7$ No solution
253. $X=0$ $Y=0$ $A=3$ No solution
254. $X=0$ $Y=0$ $A=4$ No solution
255. $X=0$ $Y=0$ $A=10$ No solution
256. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=0$ No solution
257. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=10$ $X=3$ $Y=2$ No solution
258. $X=0$ $Y=0$ $A=2$ No solution
259. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=10$ $X=4$ $Y=2$ $A=11$ $X=3$ $Y=2$ No solution
260. $X=0$ $Y=0$ $A=1$ No solution
261. $X=0$ $Y=0$ $A=0$ No solution
262. $X=0$ $Y=0$ $A=4$ No solution
263. $X=0$ $Y=0$ $A=6$ No solution
264. $X=0$ $Y=0$ $A=0$ No solution
265. $X=0$ $Y=0$ $A=10$ No solution
266. $X=0$ $Y=0$ $A=1$ No solution
267. $X=0$ $Y=0$ $A=7$ No solution
268. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=1$ No solution
269. $X=0$ $Y=0$ $A=0$ No solution
270. $X=0$ $Y=0$ $A=6$ No solution
271. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=9$ $X=2$ $Y=0$ $A=5$ No solution
272. $X=0$ $Y=0$ $A=4$ No solution

273. $X=0$ $Y=0$ $A=2$ No solution
 274. $X=0$ $Y=0$ $A=3$ No solution
 275. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=3$ No solution
 276. $X=0$ $Y=0$ $A=1$ No solution
 277. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=10$ $X=3$ $Y=2$ No solution
 278. $X=0$ $Y=0$ $A=0$ No solution
 279. $X=0$ $Y=0$ $A=3$ No solution
 280. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=2$ No solution
 281. $X=0$ $Y=0$ $A=1$ No solution
 282. $X=0$ $Y=0$ $A=6$ No solution
 283. $X=0$ $Y=0$ $A=1$ No solution
 284. $X=0$ $Y=0$ $A=3$ No solution
 285. $X=0$ $Y=0$ $A=0$ No solution
 286. $X=0$ $Y=0$ $A=3$ No solution
 287. $X=0$ $Y=0$ $A=1$ No solution
 288. $X=0$ $Y=0$ $A=10$ No solution
 289. $X=0$ $Y=0$ $A=4$ No solution
 290. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=8$ $X=4$ $Y=4$ cost =3 Path=[[], (0,0), (3,3), (4,3), (4,4)]
 291. $X=0$ $Y=0$ $A=3$ No solution
 292. $X=0$ $Y=0$ $A=0$ No solution
 293. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=6$ No solution
 294. $X=0$ $Y=0$ $A=7$ No solution
 295. $X=0$ $Y=0$ $A=3$ No solution
 296. $X=0$ $Y=0$ $A=4$ No solution
 297. $X=0$ $Y=0$ $A=1$ No solution
 298. $X=0$ $Y=0$ $A=6$ No solution
 299. $X=0$ $Y=0$ $A=0$ No solution
 300. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=6$ No solution
 301. $X=0$ $Y=0$ $A=3$ No solution
 302. $X=0$ $Y=0$ $A=4$ No solution
 303. $X=0$ $Y=0$ $A=1$ No solution
 304. $X=0$ $Y=0$ $A=6$ No solution
 305. $X=0$ $Y=0$ $A=1$ No solution
 306. $X=0$ $Y=0$ $A=1$ No solution

307. $X=0$ $Y=0$ $A=10$ No solution
308. $X=0$ $Y=0$ $A=2$ No solution
309. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=9$ $X=4$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=8$ $X=4$ $Y=4$ $\text{cost} = 4$ $\text{Path}=[[], (0,0), (3,3), (4,3), (4,3), (4,4)]$
310. $X=0$ $Y=0$ $A=0$ No solution
311. $X=0$ $Y=0$ $A=4$ No solution
312. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=0$ No solution
313. $X=0$ $Y=0$ $A=2$ No solution
314. $X=0$ $Y=0$ $A=3$ No solution
315. $X=0$ $Y=0$ $A=6$ No solution
316. $X=0$ $Y=0$ $A=6$ No solution
317. $X=0$ $Y=0$ $A=4$ No solution
318. $X=0$ $Y=0$ $A=3$ No solution
319. $X=0$ $Y=0$ $A=3$ No solution
320. $X=0$ $Y=0$ $A=7$ No solution
321. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=1$ No solution
322. $X=0$ $Y=0$ $A=0$ No solution
323. $X=0$ $Y=0$ $A=0$ No solution
324. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=0$ No solution
325. $X=0$ $Y=0$ $A=7$ No solution
326. $X=0$ $Y=0$ $A=2$ No solution
327. $X=0$ $Y=0$ $A=10$ No solution
328. $X=0$ $Y=0$ $A=10$ No solution
329. $X=0$ $Y=0$ $A=10$ No solution
330. $X=0$ $Y=0$ $A=10$ No solution
331. $X=0$ $Y=0$ $A=1$ No solution
332. $X=0$ $Y=0$ $A=0$ No solution
333. $X=0$ $Y=0$ $A=6$ No solution
334. $X=0$ $Y=0$ $A=7$ No solution
335. $X=0$ $Y=0$ $A=3$ No solution
336. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=4$ No solution
337. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=5$ No solution
338. $X=0$ $Y=0$ $A=0$ No solution
339. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=8$ $X=0$ $Y=2$ $A=10$ $X=0$ $Y=1$ $A=8$ $X=0$ $Y=2$ $A=4$ No solution
340. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=10$ $X=4$ $Y=2$ $A=10$ $X=4$ $Y=1$ $A=9$ $X=5$ $Y=1$ $A=10$ $X=5$ $Y=0$ $A=9$ $X=6$ $Y=0$ $A=10$ No solution

341. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=10$ $X=0$ $Y=0$ $A=7$ No solution
342. $X=0$ $Y=0$ $A=7$ No solution
343. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=0$ No solution
344. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=11$ $X=2$ $Y=1$ $A=11$ $X=2$ $Y=1$ $A=3$ No solution
345. $X=0$ $Y=0$ $A=3$ No solution
346. $X=0$ $Y=0$ $A=7$ No solution
347. $X=0$ $Y=0$ $A=3$ No solution
348. $X=0$ $Y=0$ $A=9$ $X=1$ $Y=0$ $A=2$ No solution
349. $X=0$ $Y=0$ $A=6$ No solution
350. $X=0$ $Y=0$ $A=6$ No solution
351. $X=0$ $Y=0$ $A=4$ No solution
352. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=0$ No solution
353. $X=0$ $Y=0$ $A=2$ No solution
354. $X=0$ $Y=0$ $A=11$ $X=1$ $Y=0$ $A=9$ $X=2$ $Y=0$ $A=10$ No solution
355. $X=0$ $Y=0$ $A=7$ No solution
356. $X=0$ $Y=0$ $A=3$ No solution
357. $X=0$ $Y=0$ $A=2$ No solution
358. $X=0$ $Y=0$ $A=3$ No solution
359. $X=0$ $Y=0$ $A=10$ No solution
360. $X=0$ $Y=0$ $A=4$ No solution
361. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=8$ $X=3$ $Y=4$ $A=11$ $X=5$ $Y=4$ $A=11$ $X=5$ $Y=4$ $A=10$ $X=5$ $Y=3$ $A=8$ $X=5$ $Y=4$ $A=9$ $X=6$ $Y=4$ $A=8$ $X=6$ $Y=5$ $A=9$ $X=7$ $Y=5$ $A=11$ $X=6$ $Y=5$ $A=9$ $X=7$ $Y=5$ $A=8$ $X=7$ $Y=6$ $A=9$ $X=8$ $Y=6$ $A=10$ $X=8$ $Y=5$ $A=8$ $X=8$ $Y=6$ $A=11$ $X=7$ $Y=6$ $A=10$ $X=7$ $Y=5$ $A=10$ $X=7$ $Y=4$ $A=10$ $X=7$ $Y=3$ $A=11$ $X=4$ $Y=3$ $A=8$ $X=4$ $Y=4$ $\text{cost} = 30$ $\text{Path} = [[], (0,0), (3,3), (3,4), (5,4), (5,4), (5,3), (5,4), (6,4), (6,5), (7,5), (6,5), (7,5), (7,6), (8,6), (8,5), (8,6), (7,6), (7,5), (7,4), (7,3), (4,3), (4,4)]$
362. $X=0$ $Y=0$ $A=6$ No solution
363. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=3$ No solution
364. $X=0$ $Y=0$ $A=3$ No solution
365. $X=0$ $Y=0$ $A=10$ No solution
366. $X=0$ $Y=0$ $A=1$ No solution
367. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=4$ No solution
368. $X=0$ $Y=0$ $A=8$ $X=0$ $Y=1$ $A=1$ No solution
369. $X=0$ $Y=0$ $A=7$ No solution
370. $X=0$ $Y=0$ $A=5$ $X=3$ $Y=3$ $A=8$ $X=3$ $Y=4$ $A=11$ $X=5$ $Y=4$ $A=9$ $X=6$ $Y=4$ $A=8$ $X=6$ $Y=5$ $A=9$ $X=7$ $Y=5$ $A=10$ $X=7$ $Y=4$ $A=9$ $X=8$ $Y=4$ $A=11$ $X=5$ $Y=4$ $A=9$ $X=6$ $Y=4$ $A=8$ $X=6$ $Y=5$ $A=8$ $X=6$ $Y=6$ $A=11$ $X=7$ $Y=6$ $A=8$ $X=7$ $Y=7$ $A=10$ $X=7$ $Y=6$ $A=10$ $X=7$ $Y=5$ $A=10$ $X=7$ $Y=4$ $A=8$ $X=7$ $Y=5$ $A=11$ $X=6$ $Y=5$ $A=10$ $X=6$ $Y=4$ $A=8$ $X=6$ $Y=5$ $A=9$ $X=7$ $Y=5$ $A=9$ $X=8$ $Y=5$ $A=8$ $X=8$ $Y=6$ $A=8$ $X=8$ $Y=7$ $A=9$ $X=9$ $Y=7$ $A=11$ $X=8$ $Y=7$ $A=8$ $X=8$ $Y=8$ $A=11$ $X=9$ $Y=8$ $A=10$ $X=9$ $Y=7$ $A=10$ $X=9$ $Y=6$ $A=9$ No solution
371. $X=0$ $Y=0$ $A=4$ No solution

372. X=0 Y=0 A=8 X=0 Y=1 A=11 X=2 Y=1 A=1 No solution
373. X=0 Y=0 A=11 X=1 Y=0 A=6 No solution
374. X=0 Y=0 A=11 X=1 Y=0 A=7 No solution
375. X=0 Y=0 A=8 X=0 Y=1 A=0 No solution
376. X=0 Y=0 A=5 X=3 Y=3 A=9 X=4 Y=3 A=10 X=4 Y=2 A=9 X=5 Y=2 A=8 X=5 Y=3 A=9 X=6 Y=3 A=8 X=6 Y=4 A=10 X=6 Y=3 A=10 X=6 Y=2 A=11 X=3 Y=2 No solution
377. X=0 Y=0 A=3 No solution
378. X=0 Y=0 A=5 X=3 Y=3 A=11 X=4 Y=3 A=8 X=4 Y=4 cost =3 Path=[[], (0,0), (3,3), (4,3), (4,4)]
379. X=0 Y=0 A=6 No solution
380. X=0 Y=0 A=4 No solution
381. X=0 Y=0 A=2 No solution
382. X=0 Y=0 A=0 No solution
383. X=0 Y=0 A=5 X=3 Y=3 A=11 X=4 Y=3 A=11 X=4 Y=3 A=10 X=4 Y=2 A=11 X=3 Y=2 No solution
384. X=0 Y=0 A=10 No solution
385. X=0 Y=0 A=8 X=0 Y=1 A=11 X=2 Y=1 A=3 No solution
386. X=0 Y=0 A=7 No solution
387. X=0 Y=0 A=5 X=3 Y=3 A=11 X=4 Y=3 A=10 X=4 Y=2 A=8 X=4 Y=3 A=10 X=4 Y=2 A=10 X=4 Y=1 A=8 X=4 Y=2 A=8 X=4 Y=3 A=9 X=5 Y=3 A=8 X=5 Y=4 A=11 X=5 Y=4 A=8 X=5 Y=5 A=11 X=6 Y=5 A=11 X=6 Y=5 A=11 X=6 Y=5 A=8 X=6 Y=6 A=9 X=7 Y=6 A=8 X=7 Y=7 A=11 X=8 Y=7 A=8 X=8 Y=8 A=11 X=9 Y=8 A=11 X=9 Y=8 A=9 No solution
388. X=0 Y=0 A=2 No solution
389. X=0 Y=0 A=2 No solution
390. X=0 Y=0 A=9 X=1 Y=0 A=10 No solution
391. X=0 Y=0 A=1 No solution
392. X=0 Y=0 A=9 X=1 Y=0 A=5 No solution
393. X=0 Y=0 A=1 No solution
394. X=0 Y=0 A=11 X=1 Y=0 A=5 No solution
395. X=0 Y=0 A=1 No solution
396. X=0 Y=0 A=11 X=1 Y=0 A=6 No solution
397. X=0 Y=0 A=2 No solution
398. X=0 Y=0 A=9 X=1 Y=0 A=7 No solution
399. X=0 Y=0 A=3 No solution
400. X=0 Y=0 A=5 X=3 Y=3 A=8 X=3 Y=4 A=9 X=4 Y=4 cost =3 Path=[[], (0,0), (3,3), (3,4), (4,4)]

Conclusion: Third algorithm improves backtracking from optimal path search perspective if the point is found during up and right stage. For the worst case the time require3d for the 1st iteration is neglactable comparing with backtracking stage especially for the hard-to-solve-maps (see Part Three). Random search is useless in this variation.

Part Two No significant changes will appear to random search because it does not use info about adjacent cells. For the 3rd algorithm and backtracking it is a nice chance to make a right move, the increment of the seen area can reduce time number of steps for crucially. See examples and explanations under them.

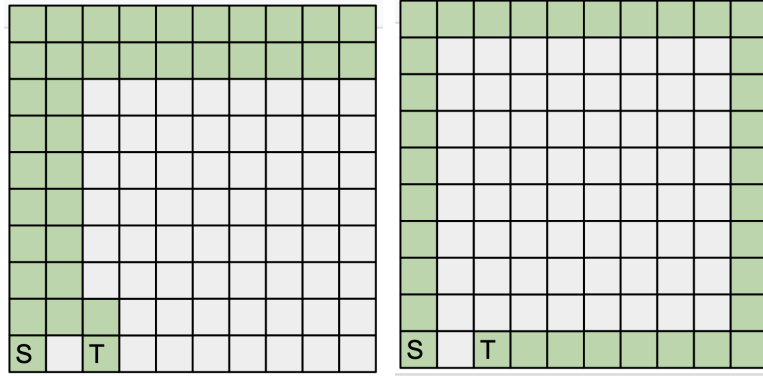


Figure 12: Green cells - the ones visited before the touchdown in case of radius of vision =1. In case of $r=2$ it will take 2 steps in both algorithms. In case of $r=1$ findind touchdown will take 37 cells for the 3rd algorithm, right picture, and 35 cells for backtracking, left picture.

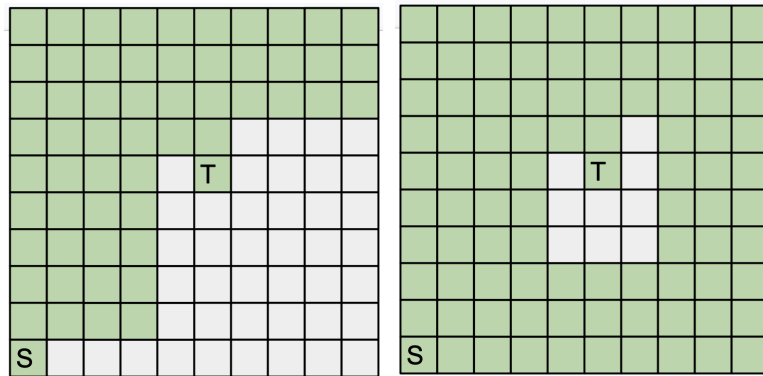


Figure 13: Green cells - the ones visited before the touchdown in case of radius of vision =1. In case of $r=1$ findind touchdown will take 58 cells for the 3rd algorithm, right picture, and 91 cells for backtracking, left picture.

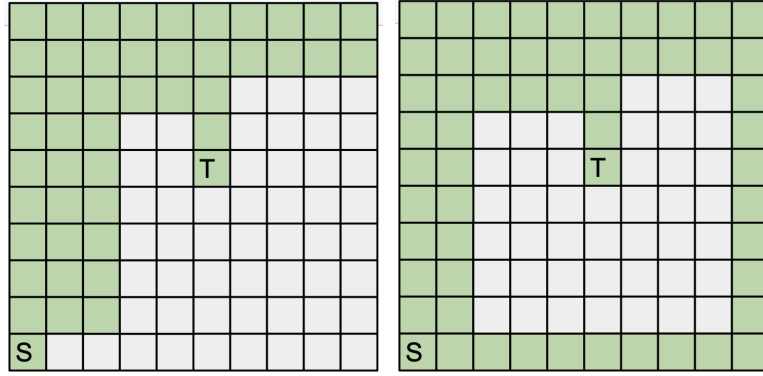


Figure 14: Green cells - the ones visited before the touchdown in case of radius of vision = 2. In case of $r=2$ findind touchdown will take 47 cells for the 3rd algorithm, right picture, and 57 cells for backtracking, left picture. For 3rd algorithm improvement is 11 cells, for backtracking it is 34 cells.

```

Backtracking 82[[],(0,0),(0,1),(0,2),(0,3),(0,4),(0,5),(0,6),(0,7),(0,8),(0,9),(1,9),(2,9),(3,9),(4,9),(5,9),(6,9),(7,9),
(8,9),(9,9),(9,8),(9,7),(9,6),(9,5),(9,4),(9,3),(9,2),(9,1),(9,0),(8,0),(8,1),(8,2),(8,3),(8,4),(8,5),(8,6),(8,7),(8,8),(7
,8),(7,7),(7,6),(7,5),(7,4),(7,3),(7,2),(7,1),(7,0),(6,0),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6),(6,7),(6,8),(5,8),(5,7),(5,6
),(5,5),(5,4),(5,3),(5,2),(5,1),(5,0),(4,0),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(4,7),(4,8),(3,8),(3,7),(3,6),(3,5),(3,4),
(3,3),(3,2),(3,1),(3,0),(2,0)]_50510]
% 66,855 inferences, 0.008 CPU in 0.008 seconds (99% CPU, 8299814 Lips)
Random search 1 No solution
% 194 inferences, 0.001 CPU in 0.001 seconds (84% CPU, 335060 Lips)
Improved backtracking 4[[],(0,0),(0,1),(1,1),(2,1),(2,0)]_51742]
% 44,280,416 inferences, 3.099 CPU in 3.102 seconds (100% CPU, 14290634 Lips)

Backtracking 2[[],(0,0),(1,0),(2,0)]_7310]
% 148 inferences, 0.000 CPU in 0.000 seconds (96% CPU, 1494949 Lips)
Random search 1 No solution
% 243 inferences, 0.001 CPU in 0.001 seconds (88% CPU, 404326 Lips)
Improved backtracking 2[[],(0,0),(1,0),(2,0)]_8494]
% 80 inferences, 0.000 CPU in 0.000 seconds (92% CPU, 2352941 Lips)
true.

Backtracking 52[[],(0,0),(0,1),(0,2),(0,3),(0,4),(0,5),(0,6),(0,7),(0,8),(0,9),(1,9),(2,9),(3,9),(4,9),(5,9),(6,9),(7,9),
(8,9),(9,9),(9,8),(9,7),(9,6),(9,5),(9,4),(9,3),(9,2),(9,1),(9,0),(8,0),(8,1),(8,2),(8,3),(8,4),(8,5),(8,6),(8,7),(8,8),(7
,8),(7,7),(7,6),(7,5),(7,4),(7,3),(7,2),(7,1),(7,0),(6,0),(6,1),(6,2),(6,3),(6,4),(6,5),(5,5)]_25310]
% 38,087 inferences, 0.004 CPU in 0.004 seconds (100% CPU, 8733547 Lips)
Random search 1 No solution
% 52 inferences, 0.001 CPU in 0.001 seconds (87% CPU, 90121 Lips)
Improved backtracking 12[[],(0,0),(0,1),(0,2),(0,3),(0,4),(0,5),(0,6),(1,6),(2,6),(3,6),(4,6),(5,6),(5,5)]_27094]
% 589,440 inferences, 0.050 CPU in 0.050 seconds (100% CPU, 11696631 Lips)

Backtracking 42[[],(0,0),(0,1),(0,2),(0,3),(0,4),(0,5),(0,6),(0,7),(0,8),(0,9),(1,9),(2,9),(3,9),(4,9),(5,9),(6,9),(7,9),
(8,9),(9,9),(9,8),(9,7),(9,6),(9,5),(9,4),(9,3),(9,2),(9,1),(9,0),(8,0),(8,1),(8,2),(8,3),(8,4),(8,5),(8,6),(8,7),(8,8),(7
,8),(7,7),(7,6),(7,5),(6,5),(5,5)]_19310]
% 30,362 inferences, 0.004 CPU in 0.004 seconds (97% CPU, 8277535 Lips)
Random search 1 No solution
% 52 inferences, 0.001 CPU in 0.001 seconds (87% CPU, 89655 Lips)
Improved backtracking 14[[],(0,0),(0,1),(0,2),(0,3),(0,4),(0,5),(0,6),(0,7),(1,7),(2,7),(3,7),(4,7),(5,7),(5,6),(5,5)]_21
322]
% 165,298 inferences, 0.014 CPU in 0.014 seconds (100% CPU, 11766657 Lips)

```

Figure 15: 1st picture- test for figure 12, with $r=1$, test 8. 2nd picture- test for figure 12, with $r=2$, test 8. 3rd picture- test for figure 13, with $r=1$, test 9. 4th picture- test for figure 13, with $r=2$, test 9. For backtracking we have both time and path improvement, for 3rd algorithm we have only significant time improvement.

Part Three

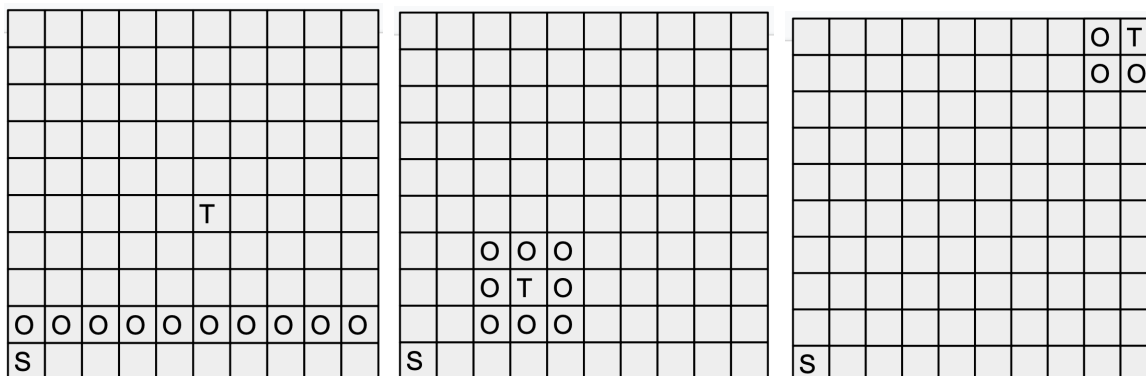


Figure 16: Obvious impossible to solve maps - when the touchdown is not reachable. Isolated player, surrounded touchdown, and partially surrounded on an edge touchdown

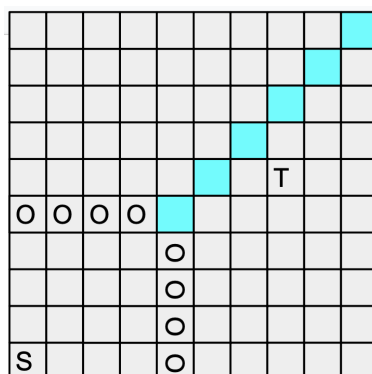


Figure 17: The map is impossible to solve if and only if there is no human staying on the blue line. Otherwise, up-right diagonal throw solves the map.

Impossible to solve maps maps: If a map is a variation (e.g. there is a line of empty cells between touchdown and isolation orks circle) of those or all touchdowns in a map are a combination of those \Rightarrow the map is unsolvable. The other type of impossible to solve maps are when there is no touchdown at all. The common thing between all of them is that the player never gets to the part of the map surrounded by orks, where the touchdown is.

Note: by impossible to solve maps I mean maps, where there is no path for touchdown. If we assume that "no path exists" is also a solution (if it is correct answer from the logical side as well) than backtracking and third algorithm always return a solution, therefore, all maps are solvable

for them.

Hard to solve maps:

For the 3rd algorithm hard-to-solve maps are those, for which we have to go to the second stage with backtracking. Example of such a map was provided in Part 1, figure 2.

For the backtracking, hard to solve maps are ones without touchdowns because we have to check all the cells. The most difficult ones are absolutely empty or only with people. They are the most difficult because each cell generates 10 options of acting (6 throws and 4 steps). Moreover, we get to the same cell during several distinct backtracking branches. For point with coordinates (x,y) , counting from $(0,0)$, we get $C_{(x+1)+(y+1)}^{x+1}$ times - this is a well-known fact from combinatorics, look [here](#) for explanations. Hence, each cell with an ork reduces the number of choices for $10C_{(x+1)+(y+1)}^x$ cases, which is on average -for a cell $(4,4)$ - is 1260 cases.

The other case of difficult to solve maps also strongly connected with the number of paths that should be checked before. We should understand that in prolog the order of function call really matters. During backtracking we call 10 operations, but let's focus on the 4 of them, which are steps. The order you call them is the order in which this branches will be executed. As far as I choose the 1st satisfactory path, the worst case is when it is met during the last branch execution. In my implementation the order is : up, right, down, left. Therefore, empty maps with a touchdown located at the most down sides of the map with half-surrounded touchdown are difficult to solve as during backtracking other branches because of the orks surrounding the backtracking will not be able to see the touchdown and it will find it only during checking the last branches. It is also important that from the right edge of the map till exit surrounding there should be orks, otherwise it will be solved much faster. Example - figure 18. However, this case is much easier to solve than empty map as coming to the appropriate cells ones will finish the execution immediately And many other cells will no be visited at all.

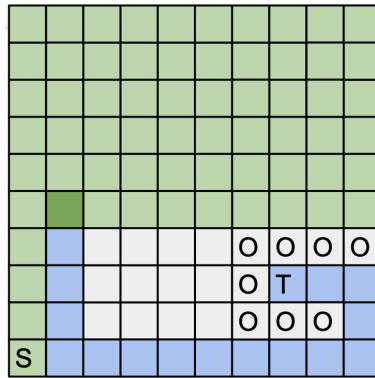


Figure 18: Before the touchdown will be found - backtracking will visit all light green cells, than coming up in the recursion tree from the dark green cell we will step on the correct blue path to the exit.