Program LLWing

Methodology

The program LLwing solves the wing's lifting line problem using horseshoe vortices and Weissinger's method (see a description in [1,2]). Planar wings in longitudinal flight are considered, but it is possible to simulate wing dihedral and yaw with simple code modifications (the program data structure and organization are ready for that).

The computational code consists of a main Matlab® script named **LLwing.m** in which the user defines the input data (wing geometry, airfoil data –optional– and simulation angles of attack). This script calls several functions to perform the wing discretization (**geo.m**), the assembly of the influence coefficients matrix (**infcoeff.m**) and the solution of the wing's circulations (**getcirc.m**) for the desired angles of attack. The aerodynamic forces and moments are obtained for each wing's bounded vortex using the Kutta-Joukowsky theorem and projecting these contributions on the respective body and wind axes (**KuttaJoukowsky.m**). If the user defines the airfoil data on input, the program accounts for the contributions of their angles of zero lift, drag and pitching moment to the wing coefficients. Otherwise, a symmetrical section is assumed.

Input data

The wing geometry is defined by dimensionless parameters such as the aspect and taper ratio. Sweep angle should be given about the quarter chord line and geometric twist is defined by the tip twist (a linear variation between root and tip chords is assumed). The latter is negative for washout, i.e. tip nose down. The lines in the script file defining these data are shown below.

```
AR = 5.0 ;   % aspect ratio
TR = 1.0 ;   % taper ratio
DE25 = 0.0 ; % sweep angle at c/4 (deg)

ETIP = 0.0; % tip twist (deg, negative for washout)
```

As mentioned, the program can take into account the airfoils' angle of zero lift, free pithing moment and drag. These values must be given for both root and tip sections, and a linear variation is assumed between them. The same applies when the airfoil does not change along the span, always root and tip values must be defined. The sections' drag model is quadratic, i.e. $Cd = Cd_0 + k_1*CL + k_2*CL^2$. Thus, the user must define the values of Cd_0, k_1 and k_2 for the root and tip sections (set k_1=0 for a parabolic polar). Similarly, a linear variation is assumed along the span. The airfoils' drag contribution are calculated in the program as in M3_2 p. 28.

```
A0p = [ a0root a0tip ]; % root and tip section zero-lift angles (deg)
CM0p = [ Cm0root Cm0tip]; % root and tip section free moments
CDP = [ Cd0root k1_root k2_root;    % CD root section = CD0+k1*CL+k2*CL²
        Cd0tip   k1_tip k2_tip] ;   % CD tip section
```

In order to simulate symmetrical deflection of control surfaces, the user must define the flap/aileron initial and final span positions, the flap's chord ratio and deflection angle. The incremental angle of zero lift and free pitching moment of the airfoils due to flap are calculated using the thin-airfoil theory (see M2_2 pp. 29-30) and added to the corresponding wing stations. It is also possible to introduce a flap correction factor (<1) to increase the accuracy of the theoretical estimates. In the example below, a plain flap with chord ratio 0.2 spanning from the wing root to 30% of the half-span is rotated down 10 deg. The correction factor is obtained from M2_2 p. 36.

```
YF_pos = [ 0.0 0.3]; % 2y/b initial and final position of the
flap/aileron in the half-wing (0 is the wing`s root, 1 is the tip)
CF_ratio = 0.2 ;  % flap_chord/chord ratio
DE_flap = 10.0; % flap deflection (deg, positive:down)
FlapCorr = 0.8 ; % flap effectiveness (<=1)
```

Finally, the user must define the number of horseshoe vortices along the span (do not forget to do convergence analysis for that!) and a set of angles of attack of analysis (5 and 10 deg is selected below). There is no limitation in the number of angles of attack, but take into account that two angles of attack are enough for obtaining wing derivatives and the basic and additional lift (the model is linear). However, additional input angles of attack are required to reproduce the wing drag curve when the profile drag is taken into account.

```
N = 50 ; % number of panels along the span

ALPHA = [ 5.0 10.0 ] ; % angles of attack for analysis (deg)
```

Output results

For each angle of attack of analysis, the function **KuttaJoukowsky.m** calculates the lift distribution along de span and the integrated aerodynamic coefficients in body and wing axes. The local lift distributions (Cl(y)) are stored in the array *cl_local(1:N,1:nalfa)*, being *nalfa* the number of angles of attack of analysis. The columns correspond to each angle of attack (in the order defined in vector ALPHA) and the rows to the wing stations, going from the left tip to the right tip. The (x,y,z) position of each wing station is given in the array *c4nods*(1:3,1:N). The local lift vector can be assumed to be located at the midpoint of the bounded vortex (circulation is constant along each strip).

The integrated wing's force coefficients are given in the array *force_coeff(1:11,nalfa)*. Again, the columns corresponds to each angle of attack of analysis. The rows are organized as follows:

1 to 3 : CFX, CFY and CFZ (body axes)
4 to 6 : CMX, CMY and CMZ (CMY includes the airfoils' Cm0 if defined by the user)
7 to 9 : CL, CS and induced drag CDi (wind axes)

10 : profile drag contribution CDp (if airfoil data is defined by the user)
11 : total wing drag CD (induced + profile)

Other geometrical data useful for post-process can be obtained from the function **geo.m**, for example: *chord*(1:N) stores the chord (length) of each wing station, *s_pan*(1:N) the surface area of each wing panel, *mac* gives the mean aerodynamic chord and *S* the total surface of the wing. Note that **all lengths and derivate dimensions in LLWing are dimensionless with the wing span**, so it is necessary to multiply the output values by the wing span. Example: *output_mac * span* (in meters) = *mac* (in m); *output_S* * $span^2$ = *S* (in m^2).

References

1. Ortega E. *Horseshoe vortex method*. Class notes, 220024 Aerodynamics, ESEIAAT-UPC, 2020.
2. Schlichting, Hermann T., and Erich A. Truckenbrodt. *Aerodynamics of the Airplane*. McGraw-Hill Companies, 1979.