Programming Assignment 1: Learning Distributed Word Representations

Version: 1.2

Changes by Version:

- (v1.1)
    1. Part 1 Description: indicated that each word is associated with two embedding vectors and two biases
    2. Part 1: Updated `calculate_log_co_occurence` to include the last pair of consecutive words as well
    3. Part 2: Updated question description for 2.1
    4. Part 4: Updated answer requirement for 4.1
    5. (1.3) Fixed symmetric GLoVE gradient
    6. (1.3) Clarified that W_tilde and b_tilde gradients also need to be implemented
    7. (2) Removed extra space leading up to docstring for `compute_loss_derivative`
- (v1.2)
    1. (1.4) Updated the training function `train_GLoVE` to not use inplace update (e.g. `W = W - learning_rate * grad_W` instead), so the initial weight variables are not overwritten between asymmetric and symmetric GLoVE models.
    2. (2) Noted that `compute_loss_derivative` input argument `target_mask` is 3D tensor with shape `[batch_size x context_len x 1]`

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Due Date:Thursday, Feb. 4, at 11:59pm

Based on an assignment by George Dahl

For CSC413/2516 in Winter 2021 with Professor Jimmy Ba and Professor Bo Wang

Submission: You must submit two files through MarkUs:

1. ☐ A PDF file containing your writeup, titled *a1-writeup.pdf*, which will be the PDF export of this notebook (i.e., by printing this notebook webpage as PDF). Your writeup must be typed. There will be sections in the notebook for you to write your responses. Make sure that the relevant outputs (e.g. `print_gradients()` outputs, plots, etc.) are included and clearly visible.

2. ☐ This `a1-code.ipynb` iPython Notebook.

The programming assignments are individual work. See the Course Syllabus for detailed policies.

You should attempt all questions for this assignment. Most of them can be answered at least partially even if you were unable to finish earlier questions. If you think your computational results are incorrect, please say so; that may help you get partial credit.

The teaching assistants for this assignment are Harris Chan and Summer Tao. Send your email with subject "*[CSC413] PA1*" to mailto:csc413-2021-01-tas@cs.toronto.edu or post on Piazza with the tag `pa1`.

Introduction

In this assignment we will learn about word embeddings and make neural networks learn about words. We could try to match statistics about the words, or we could train a network that takes a sequence of words as input and learns to predict the word that comes next.

This assignment will ask you to implement a linear embedding and then the backpropagation computations for a neural language model and then run some experiments to analyze the learned representation. The amount of code you have to write is very short but each line will require you to think very carefully. You will need to derive the updates mathematically, and then implement them using matrix and vector operations in NumPy.

Starter code and data

First, perform the required imports for your code:

```
import collections
import pickle
import numpy as np
import os
from tqdm import tqdm
import pylab
from six.moves.urllib.request import urlretrieve
import tarfile
import sys

TINY = 1e-30
EPS = 1e-4
nax = np.newaxis
```

If you're using colaboratory, this following script creates a folder - here we used 'CSC413/A1' - in order to download and store the data. If you're not using colaboratory, then set the path to wherever you want the contents to be stored at locally.

You can also manually download and unzip the data from [http://www.cs.toronto.edu/~jba/a1_data.tar.gz] and put them in the same folder as where you store this notebook.

Feel free to use a different way to access the files *data.pk* , *partially_trained.pk*, and *raw_sentences.txt*.

The file *raw_sentences.txt* contains the sentences that we will be using for this assignment. These sentences are fairly simple ones and cover a vocabulary of only 250 words (+ 1 special `[MASK]` token word).

```
######################################################################
# Setup working directory
######################################################################
# Change this to a local path if running locally
%mkdir -p /content/CSC413/A1/
%cd /content/CSC413/A1

######################################################################
# Helper functions for loading data
######################################################################
# adapted from
# https://github.com/fchollet/keras/blob/master/keras/datasets/cifar10.py

def get_file(fname,
             origin,
```

```
             untar=False,
             extract=False,
             archive_format='auto',
             cache_dir='data'):
    datadir = os.path.join(cache_dir)
    if not os.path.exists(datadir):
        os.makedirs(datadir)

    if untar:
        untar_fpath = os.path.join(datadir, fname)
        fpath = untar_fpath + '.tar.gz'
    else:
        fpath = os.path.join(datadir, fname)

    print('File path: %s' % fpath)
    if not os.path.exists(fpath):
        print('Downloading data from', origin)

        error_msg = 'URL fetch failure on {}: {} -- {}'
        try:
            try:
                urlretrieve(origin, fpath)
            except URLError as e:
                raise Exception(error_msg.format(origin, e.errno, e.reason))
            except HTTPError as e:
                raise Exception(error_msg.format(origin, e.code, e.msg))
        except (Exception, KeyboardInterrupt) as e:
            if os.path.exists(fpath):
                os.remove(fpath)
            raise

    if untar:
        if not os.path.exists(untar_fpath):
            print('Extracting file.')
            with tarfile.open(fpath) as archive:
                archive.extractall(datadir)
        return untar_fpath

    if extract:
        _extract_archive(fpath, datadir, archive_format)

    return fpath
```

```
/content/CSC413/A1
```

```
# Download the dataset and partially pre-trained model
get_file(fname='a1_data',
                         origin='http://www.cs.toronto.edu/~jba/a1_data.tar.gz',
                         untar=True)
drive_location = 'data'
PARTIALLY_TRAINED_MODEL = drive_location + '/' + 'partially_trained.pk'
data_location = drive_location + '/' + 'data.pk'
```

```
File path: data/a1_data.tar.gz
Downloading data from http://www.cs.toronto.edu/~jba/a1_data.tar.gz
Extracting file.
```

We have already extracted the 4-grams from this dataset and divided them into training, validation, and test sets. To inspect this data, run the following:

```
data = pickle.load(open(data_location, 'rb'))
print(data['vocab'][0]) # First word in vocab is [MASK]
print(data['vocab'][1])
print(len(data['vocab'])) # Number of words in vocab
print(data['vocab']) # All the words in vocab
print(data['train_inputs'][:10]) # 10 example training instances
```

```
[MASK]
all
251
['[MASK]', 'all', 'set', 'just', 'show', 'being', 'money', 'over', 'both',
[[ 28  26  90 144]
 [184  44 249 117]
 [183  32  76 122]
 [117 247 201 186]
 [223 190 249   6]
 [ 42  74  26  32]
 [242  32 223  32]
 [223  32 158 144]
 [ 74  32 221  32]
 [ 42 192  91  68]]
```

Now `data` is a Python dict which contains the vocabulary, as well as the inputs and targets for all three splits of the data. `data['vocab']` is a list of the 251 words in the dictionary; `data['vocab'][0]` is the word with index 0, and so on. `data['train_inputs']` is a 372,500 x 4 matrix where each row gives the indices of the 4 consecutive context words for one of the 372,500 training cases. The validation and test sets are handled analogously.

Even though you only have to modify two specific locations in the code, you may want to read through this code before starting the assignment.

Part 1: GLoVE Word Representations (2pts)

In this part of the assignment, you will implement a simplified version of the GLoVE embedding (please see the handout for detailed description of the algorithm) with the loss defined as

$$L(\{\mathbf{w}_i, \tilde{\mathbf{w}}_i, b_i, \tilde{b}_i\}_{i=1}^V) = \sum_{i,j=1}^{V} (\mathbf{w}_i^\top \tilde{\mathbf{w}}_j + b_i + \tilde{b}_j - \log X_{ij})^2$$

.

Note that each word is represented by two d-dimensional embedding vectors $\mathbf{w}_i, \tilde{\mathbf{w}}_i$ and two scalar biases $b_i, \tilde{b}_i$.

Answer the following questions:

1.1. GLoVE Parameter Count [0pt]

Given the vocabulary size V and embedding dimensionality d, how many parameters does the GLoVE model have? Note that each word in the vocabulary is associated with 2 embedding vectors and 2 biases.

1.1 **Answer**: **4dV**

1.2. Expression for gradient $\frac{\partial L}{\partial \mathbf{w}_i}$ [1pt]

Write the expression for $\frac{\partial L}{\partial \mathbf{w}_i}$, the gradient of the loss function L with respect to one parameter vector $\mathbf{w}_i$. The gradient should be a function of $\mathbf{w}, \tilde{\mathbf{w}}, b, \tilde{b}, X$ with appropriate subscripts (if any).

1.2 **Answer**: $\frac{\partial L}{\partial s} = \sum_{j=1}^{V} 2(w_i^T \widetilde{w}_j + b_i + \widetilde{b_j} - logX_{ij})\widetilde{w}_j$

1.3. Implement the gradient update of GLoVE. [1pt]

See `YOUR CODE HERE` **Comment below for where to complete the code**

We have provided a few functions for training the embedding:

- `calculate_log_co_occurence` computes the log co-occurrence matrix of a given corpus
- `train_GLoVE` runs momentum gradient descent to optimize the embedding
- `loss_GLoVE`:
  - INPUT - $V \times d$ matrix `W` (collection of V embedding vectors, each d-dimensional); $V \times d$ matrix `W_tilde`; $V \times 1$ vector `b` (collection of V bias terms); $V \times 1$ vector `b_tilde`; $V \times V$ log co-occurrence matrix.
  - OUTPUT - loss of the GLoVE objective
- `grad_GLoVE`: **TO BE IMPLEMENTED.**
  - INPUT:
    - $V \times d$ matrix `W` (collection of V embedding vectors, each d-dimensional), embedding for first word;
    - $V \times d$ matrix `W_tilde`, embedding for second word;
    - $V \times 1$ vector `b` (collection of V bias terms);
    - $V \times 1$ vector `b_tilde`, bias for second word;
    - $V \times V$ log co-occurrence matrix.
  - OUTPUT:
    - $V \times d$ matrix `grad_W` containing the gradient of the loss function w.r.t. `W`;
    - $V \times d$ matrix `grad_W_tilde` containing the gradient of the loss function w.r.t. `W_tilde`;
    - $V \times 1$ vector `grad_b` which is the gradient of the loss function w.r.t. `b`.
    - $V \times 1$ vector `grad_b_tilde` which is the gradient of the loss function w.r.t. `b_tilde`.

Run the code to compute the co-occurence matrix. Make sure to add a 1 to the occurences, so there are no 0's in the matrix when we take the elementwise log of the matrix.

```
vocab_size = len(data['vocab']) # Number of vocabs

def calculate_log_co_occurence(word_data, symmetric=False):
  "Compute the log-co-occurence matrix for our data."
  log_co_occurence = np.zeros((vocab_size, vocab_size))
  for input in word_data:
    # Note: the co-occurence matrix may not be symmetric
    log_co_occurence[input[0], input[1]] += 1
    log_co_occurence[input[1], input[2]] += 1
    log_co_occurence[input[2], input[3]] += 1
    # If we want symmetric co-occurence can also increment for these.
    if symmetric:
      log_co_occurence[input[1], input[0]] += 1
      log_co_occurence[input[2], input[1]] += 1
      log_co_occurence[input[3], input[2]] += 1
  delta_smoothing = 0.5  # A hyperparameter.  You can play with this if you want
  log_co_occurence += delta_smoothing  # Add delta so log doesn't break on 0's.
  log_co_occurence = np.log(log_co_occurence)
  return log_co_occurence
```

```
asym_log_co_occurence_train = calculate_log_co_occurence(data['train_inputs'], s
asym_log_co_occurence_valid = calculate_log_co_occurence(data['valid_inputs'], s
```

- ☐ **TO BE IMPLEMENTED**: Calculate the gradient of the loss function w.r.t. the parameters W, $\tilde{W}$, $\mathbf{b}$, and $\mathbf{b}$. You should vectorize the computation, i.e. not loop over every word.

```
def loss_GLoVE(W, W_tilde, b, b_tilde, log_co_occurence):
  "Compute the GLoVE loss."
  n,_ = log_co_occurence.shape
  if W_tilde is None and b_tilde is None:
    return np.sum((W @ W.T + b @ np.ones([1,n]) + np.ones([n,1])@b.T - log_co_oc
  else:
    return np.sum((W @ W_tilde.T + b @ np.ones([1,n]) + np.ones([n,1])@b_tilde.T

def grad_GLoVE(W, W_tilde, b, b_tilde, log_co_occurence):
  "Return the gradient of GLoVE objective w.r.t W and b."
  "INPUT: W - Vxd; W_tilde - Vxd; b - Vx1; b_tilde - Vx1; log_co_occurence: VxV"
  "OUTPUT: grad_W - Vxd; grad_W_tilde - Vxd, grad_b - Vx1, grad_b_tilde - Vx1"
  n,_ = log_co_occurence.shape

  if not W_tilde is None and not b_tilde is None:
    loss = W @ W_tilde.T + b @ np.ones([1,n]) + np.ones([n,1])@b_tilde.T - log_c
    grad_W = 2 * loss @ W_tilde
    grad_W_tilde = 2 * loss.T @ W
    grad_b = 2 * (loss @ np.ones([n,1]))
    grad_b_tilde = 2 * (np.ones((1,n)) @ loss).T
  else:
    loss = (W @ W.T + b @ np.ones([1,n]) + np.ones([n,1])@b.T - 0.5*(log_co_occu
    grad_W = 4 *(W.T @ loss).T
    grad_W_tilde = None
    grad_b = 4 * (np.ones([1,n]) @ loss).T
    grad_b_tilde = None

  return grad_W, grad_W_tilde, grad_b, grad_b_tilde

def train_GLoVE(W, W_tilde, b, b_tilde, log_co_occurence_train, log_co_occurence
  "Traing W and b according to GLoVE objective."
  n,_ = log_co_occurence_train.shape
  learning_rate = 0.05 / n  # A hyperparameter.  You can play with this if you w
  for epoch in range(n_epochs):
    grad_W, grad_W_tilde, grad_b, grad_b_tilde = grad_GLoVE(W, W_tilde, b, b_til
    W = W - learning_rate * grad_W
    b = b - learning_rate * grad_b
    if not grad_W_tilde is None and not grad_b_tilde is None:
      W_tilde = W_tilde - learning_rate * grad_W_tilde
      b_tilde = b_tilde - learning_rate * grad_b_tilde
    train_loss, valid_loss = loss_GLoVE(W, W_tilde, b, b_tilde, log_co_occurence
    if do_print:
      print(f"Train Loss: {train_loss}, valid loss: {valid_loss}, grad_norm: {np
  return W, W_tilde, b, b_tilde, train_loss, valid_loss
```

1.4. Effect of embedding dimension d [0pt]

Train the both the symmetric and asymmetric GLoVe model with varying dimensionality d by running the cell below. Comment on:

1. Which d leads to optimal validation performance for the asymmetric and symmetric models?
2. Why does / doesn't larger d always lead to better validation error?
3. Which model is performing better, and why?

1.4 Answer:
1.d=10.
2.Because larger d leads to a more flexible representation and may cause overfitting.
3.Assymetric, because the order of the words really matters in semantic meanings.

Train the GLoVE model for a range of embedding dimensions

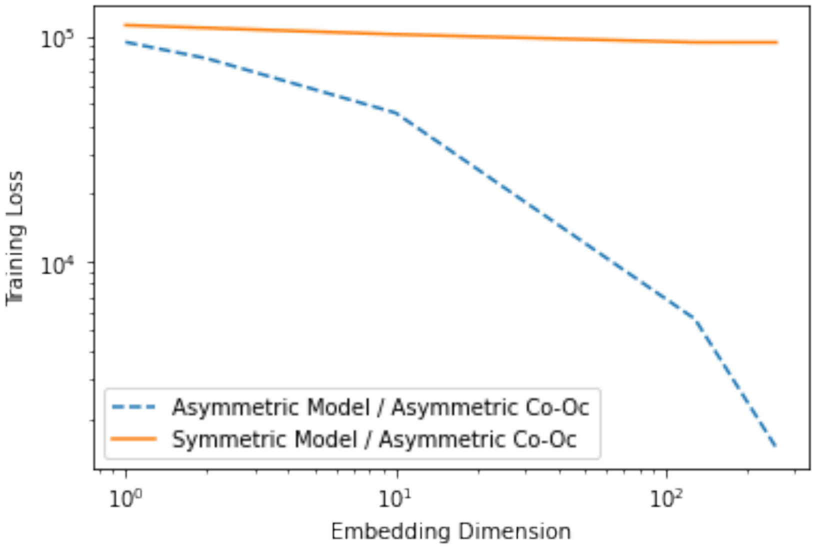
```
np.random.seed(1)
n_epochs = 500  # A hyperparameter.  You can play with this if you want.
embedding_dims = np.array([1, 2, 10, 128, 256])  # Play with this
# Store the final losses for graphing
asymModel_asymCoOc_final_train_losses, asymModel_asymCoOc_final_val_losses = [],
symModel_asymCoOc_final_train_losses, symModel_asymCoOc_final_val_losses = [], [
Asym_W_final_2d, Asym_b_final_2d, Asym_W_tilde_final_2d, Asym_b_tilde_final_2d =
W_final_2d, b_final_2d = None, None
do_print = False  # If you want to see diagnostic information during training

for embedding_dim in tqdm(embedding_dims):
  init_variance = 0.1  # A hyperparameter.  You can play with this if you want.
  W = init_variance * np.random.normal(size=(vocab_size, embedding_dim))
  W_tilde = init_variance * np.random.normal(size=(vocab_size, embedding_dim))
  b = init_variance * np.random.normal(size=(vocab_size, 1))
  b_tilde = init_variance * np.random.normal(size=(vocab_size, 1))
  if do_print:
    print(f"Training for embedding dimension: {embedding_dim}")

  # Train Asym model on Asym Co-Oc matrix
  Asym_W_final, Asym_W_tilde_final, Asym_b_final, Asym_b_tilde_final, train_loss
  if embedding_dim == 2:
    # Save a parameter copy if we are training 2d embedding for visualization la
    Asym_W_final_2d = Asym_W_final
    Asym_W_tilde_final_2d = Asym_W_tilde_final
    Asym_b_final_2d = Asym_b_final
    Asym_b_tilde_final_2d = Asym_b_tilde_final
  asymModel_asymCoOc_final_train_losses += [train_loss]
  asymModel_asymCoOc_final_val_losses += [valid_loss]
  if do_print:
    print(f"Final validation loss: {valid_loss}")

  # Train Sym model on Asym Co-Oc matrix
  W_final, W_tilde_final, b_final, b_tilde_final, train_loss, valid_loss = train
  if embedding_dim == 2:
    # Save a parameter copy if we are training 2d embedding for visualization la
    W_final_2d = W_final
    b_final_2d = b_final
  symModel_asymCoOc_final_train_losses += [train_loss]
  symModel_asymCoOc_final_val_losses += [valid_loss]
  if do_print:
    print(f"Final validation loss: {valid_loss}")
```

```
100%|██████████| 5/5 [00:24<00:00,  4.83s/it]
```

Plot the training and validation losses against the embedding dimension.

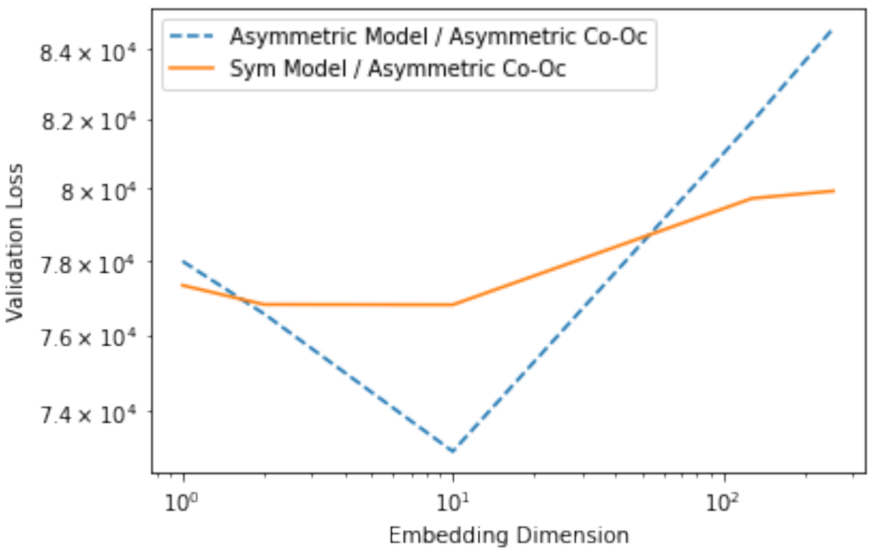
```
pylab.loglog(embedding_dims, asymModel_asymCoOc_final_train_losses, label="Asymm
pylab.loglog(embedding_dims, symModel_asymCoOc_final_train_losses , label="Symme
pylab.xlabel("Embedding Dimension")
pylab.ylabel("Training Loss")
pylab.legend()
```

```
<matplotlib.legend.Legend at 0x7f77817e5a90>
```




```
pylab.loglog(embedding_dims, asymModel_asymCoOc_final_val_losses, label="Asymmet
pylab.loglog(embedding_dims, symModel_asymCoOc_final_val_losses , label="Sym Mod
pylab.xlabel("Embedding Dimension")
pylab.ylabel("Validation Loss")
pylab.legend(loc="upper left")
```

```
<matplotlib.legend.Legend at 0x7f77811615f8>
```



Part 2: Network Architecture (2pts)

See the handout for the written questions in this part.

Answer the following questions

2.1. Number of parameters in neural network model [1pt]

Assume in general that we have V words in the dictionary and use the previous N words as inputs. Suppose we use a D-dimensional word embedding and a hidden layer with H hidden units. The trainable parameters of the model consist of 3 weight matrices and 2 sets of biases. What is the total number of trainable parameters in the model, as a function of V, N, D, H?

In the diagram given, which part of the model (i.e., `word_embbeding_weights`, `embed_to_hid_weights`, `hid_to_output_weights`, `hid_bias`, or `output_bias`) has the largest number of trainable parameters if we have the constraint that $V \gg H > D > N$? Note: The symbol ≫ means ``much greater than'' Explain your reasoning.

2.1 Answer: *word_embedding_weights* is V x D, *embed_to_hid_weights* is ND x H, *hid_bias* is H x 1, *hid_to_output_weights* is H x V, *output_bias* is V x 1. So there are in total VD + NDH + H + HV + V parameters, and since VH > VD > complexity of other parameters, *hid_to_output_weights* part accounts for most of the parameters.

2.2 Number of parameters in n-gram model [1pt]

Another method for predicting the next words is an n-gram model, which was mentioned in Lecture 3. If we wanted to use an n-gram model with the same context length N as our network, we'd need to store the counts of all possible $(N+1)$-grams. If we stored all the counts explicitly, how many entries would this table have?

2.2 Answer: Each letter in the $(N+1)$-gram has V difference choices, so in total there will be V^{N+1} entries.

2.3. Comparing neural network and n-gram model scaling [0pt]

How do the parameters in the neural network model scale with the number of context words N versus how the number of entries in the n-gram model scale with N? [0pt]

2.3 Answer: Parameter size in neural network model scales exponentially with N, whereas entry size in n-gram model scales linearly with N.

Part 3: Training the model (3pts)

We will modify the architecture slightly from the previous section, inspired by BERT \citep{devlin2018bert}. Instead of having only one output, the architecture will now take in $N = 4$ context words, and also output predictions for $N = 4$ words. See Figure 2 diagram in the handout for the diagram of this architecture.

During training, we randomly sample one of the N context words to replace with a `[MASK]` token. The goal is for the network to predict the word that was masked, at the corresponding output word position. In practice, this `[MASK]` token is assigned the index 0 in our dictionary. The weights $W^{(2)}$ = `hid_to_output_weights` now has the shape $NV \times H$, as the output layer has NV neurons, where the first V output units are for predicting the first word, then the next V are for predicting the second word, and so on. We call this as *concatenating* output uniits across all word positions, i.e. the $(j + nV)$-th column is for the word j in vocabulary for the n-th output word position. Note here that the softmax is applied in chunks of V as well, to give a valid probability distribution over the V words. Only the output word positions that were masked in the input are included in the cross entropy loss calculation:

There are three classes defined in this part: `Params`, `Activations`, `Model`. You will make changes to `Model`, but it may help to read through `Params` and `Activations` first.

$$C = -\sum_{i}^{B} \sum_{n}^{N} \sum_{j}^{V} m_n^{(i)} (t_{n,j}^{(i)} \log y_{n,j}^{(i)}),$$

Where $y_{n,j}^{(i)}$ denotes the output probability prediction from the neural network for the i-th training example for the word j in the n-th output word, and $t_{n,j}^{(i)}$ is 1 if for the i-th training example, the word j is the n-th word in context. Finally, $m_n^{(i)} \in \{0, 1\}$ is a mask that is set to 1 if we are predicting the n-th word position for the i-th example (because we had masked that word in the input), and 0 otherwise.

There are three classes defined in this part: `Params`, `Activations`, `Model`. You will make changes to `Model`, but it may help to read through `Params` and `Activations` first.

```
class Params(object):
    """A class representing the trainable parameters of the model. This class ha

           word_embedding_weights, a matrix of size V x D, where V is the number
                   and D is the embedding dimension.
           embed_to_hid_weights, a matrix of size H x ND, where H is the number
                   columns represent connections from the embedding of the first
                   for the second context word, and so on. There are N context w
           hid_bias, a vector of length H
           hid_to_output_weights, a matrix of size NV x H
           output_bias, a vector of length NV"""

    def __init__(self, word_embedding_weights, embed_to_hid_weights, hid_to_outp
                 hid_bias, output_bias):
        self.word_embedding_weights = word_embedding_weights
```

```
        self.embed_to_hid_weights = embed_to_hid_weights
        self.hid_to_output_weights = hid_to_output_weights
        self.hid_bias = hid_bias
        self.output_bias = output_bias

    def copy(self):
        return self.__class__(self.word_embedding_weights.copy(), self.embed_to_
                              self.hid_to_output_weights.copy(), self.hid_bias.c

    @classmethod
    def zeros(cls, vocab_size, context_len, embedding_dim, num_hid):
        """A constructor which initializes all weights and biases to 0."""
        word_embedding_weights = np.zeros((vocab_size, embedding_dim))
        embed_to_hid_weights = np.zeros((num_hid, context_len * embedding_dim))
        hid_to_output_weights = np.zeros((vocab_size * context_len, num_hid))
        hid_bias = np.zeros(num_hid)
        output_bias = np.zeros(vocab_size * context_len)
        return cls(word_embedding_weights, embed_to_hid_weights, hid_to_output_w
                   hid_bias, output_bias)

    @classmethod
    def random_init(cls, init_wt, vocab_size, context_len, embedding_dim, num_hi
        """A constructor which initializes weights to small random values and bi
        word_embedding_weights = np.random.normal(0., init_wt, size=(vocab_size,
        embed_to_hid_weights = np.random.normal(0., init_wt, size=(num_hid, cont
        hid_to_output_weights = np.random.normal(0., init_wt, size=(vocab_size *
        hid_bias = np.zeros(num_hid)
        output_bias = np.zeros(vocab_size * context_len)
        return cls(word_embedding_weights, embed_to_hid_weights, hid_to_output_w
                   hid_bias, output_bias)

    ###### The functions below are Python's somewhat oddball way of overloading
    ###### we can do arithmetic on Params instances. You don't need to understan

    def __mul__(self, a):
        return self.__class__(a * self.word_embedding_weights,
                              a * self.embed_to_hid_weights,
                              a * self.hid_to_output_weights,
                              a * self.hid_bias,
                              a * self.output_bias)

    def __rmul__(self, a):
        return self * a

    def __add__(self, other):
        return self.__class__(self.word_embedding_weights + other.word_embedding
                              self.embed_to_hid_weights + other.embed_to_hid_wei
                              self.hid_to_output_weights + other.hid_to_output_w
```

```
                                 self.hid_bias + other.hid_bias,
                                 self.output_bias + other.output_bias)

    def __sub__(self, other):
        return self + -1. * other


class Activations(object):
    """A class representing the activations of the units in the network. This cl

        embedding_layer, a matrix of B x ND matrix (where B is the batch size, D
                and N is the number of input context words), representing the ac
                layer on all the cases in a batch. The first D columns represent
                first context word, and so on.
        hidden_layer, a B x H matrix representing the hidden layer activations f
        output_layer, a B x V matrix representing the output layer activations f

    def __init__(self, embedding_layer, hidden_layer, output_layer):
        self.embedding_layer = embedding_layer
        self.hidden_layer = hidden_layer
        self.output_layer = output_layer

def get_batches(inputs, batch_size, shuffle=True):
    """Divide a dataset (usually the training set) into mini-batches of a given
    'generator', i.e. something you can use in a for loop. You don't need to und
    works to do the assignment."""

    if inputs.shape[0] % batch_size != 0:
        raise RuntimeError('The number of data points must be a multiple of the
    num_batches = inputs.shape[0] // batch_size

    if shuffle:
        idxs = np.random.permutation(inputs.shape[0])
        inputs = inputs[idxs, :]

    for m in range(num_batches):
        yield inputs[m * batch_size:(m + 1) * batch_size, :]
```

In this part of the assignment, you implement a method which computes the gradient using backpropagation. To start you out, the *Model* class contains several important methods used in training:

- `compute_activations` computes the activations of all units on a given input batch
- `compute_loss` computes the total cross-entropy loss on a mini-batch
- `evaluate` computes the average cross-entropy loss for a given set of inputs and targets

You will need to complete the implementation of two additional methods which are needed for training, and print the outputs of the gradients.

3.1 Implement gradient with respect to output layer inputs [1pt]

`compute_loss_derivative` computes the derivative of the loss function with respect to the output layer inputs.

In other words, if C is the cost function, and the softmax computation for the j-th word in vocabulary for the n-th output word position is:

$$y_{n,j} = \frac{e^{z_{n,j}}}{\sum_l e^{z_{n,l}}}$$

This function should compute a $B \times NV$ matrix where the entries correspond to the partial derivatives $\partial C / \partial z_j^n$. Recall that the output units are concatenated across all positions, i.e. the $(j + nV)$-th column is for the word j in vocabulary for the n-th output word position.

3.2 Implement gradient with respect to parameters [1pt]

`back_propagate` is the function which computes the gradient of the loss with respect to model parameters using backpropagation. It uses the derivatives computed by *compute_loss_derivative*. Some parts are already filled in for you, but you need to compute the matrices of derivatives for `embed_to_hid_weights`, `hid_bias`, `hid_to_output_weights`, and `output_bias`. These matrices have the same sizes as the parameter matrices (see previous section).

In order to implement backpropagation efficiently, you need to express the computations in terms of matrix operations, rather than *for* loops. You should first work through the derivatives on pencil and paper. First, apply the chain rule to compute the derivatives with respect to individual units, weights, and biases. Next, take the formulas you've derived, and express them in matrix form. You should be able to express all of the required computations using only matrix multiplication, matrix transpose, and elementwise operations --- no *for* loops! If you want inspiration, read through the code for *Model.compute_activations* and try to understand how the matrix operations correspond to the computations performed by all the units in the network.

To make your life easier, we have provided the routine `checking.check_gradients`, which checks your gradients using finite differences. You should make sure this check passes before continuing with the assignment.

```
class Model(object):
    """A class representing the language model itself. This class contains vario
```

```
    the model and visualizing the learned representations. It has two fields:
        params, a Params instance which contains the model parameters
        vocab, a list containing all the words in the dictionary; vocab[0] is th
              0, and so on."""

    def __init__(self, params, vocab):
        self.params = params
        self.vocab = vocab

        self.vocab_size = len(vocab)
        self.embedding_dim = self.params.word_embedding_weights.shape[1]
        self.embedding_layer_dim = self.params.embed_to_hid_weights.shape[1]
        self.context_len = self.embedding_layer_dim // self.embedding_dim
        self.num_hid = self.params.embed_to_hid_weights.shape[0]

    def copy(self):
        return self.__class__(self.params.copy(), self.vocab[:])

    @classmethod
    def random_init(cls, init_wt, vocab, context_len, embedding_dim, num_hid):
        """Constructor which randomly initializes the weights to Gaussians with
        and initializes the biases to all zeros."""
        params = Params.random_init(init_wt, len(vocab), context_len, embedding_
        return Model(params, vocab)

    def indicator_matrix(self, targets, mask_zero_index=True):
        """Construct a matrix where the (k + j*V)th entry of row i is 1 if the j
         for example i is k, and all other entries are 0.

         Note: if the j-th target word index is 0, this corresponds to the [MASK
              and we set the entry to be 0.
        """
        batch_size, context_len = targets.shape
        expanded_targets = np.zeros((batch_size, context_len * len(self.vocab)))
        targets_offset = np.repeat((np.arange(context_len) * len(self.vocab))[np
        targets += targets_offset

        for c in range(context_len):
          expanded_targets[np.arange(batch_size), targets[:,c]] = 1.
          if mask_zero_index:
            # Note: Set the targets with index 0, V, 2V to be zero since it corr
            expanded_targets[np.arange(batch_size), targets_offset[:,c]] = 0.
        return expanded_targets

    def compute_loss_derivative(self, output_activations, expanded_target_batch,
        """Compute the derivative of the multiple target position cross-entropy

            For example:
```

```
         [y_{0} ....  y_{V-1}] [y_{V}, ..., y_{2*V-1}] [y_{2*V} ... y_{i,3*V-1}]

         Where for colum j + n*V,

            y_{j + n*V} = e^{z_{j + n*V}} / \sum_{m=0}^{V-1} e^{z_{m + n*V}}, fo

        This function should return a dC / dz matrix of size [batch_size x (voca
        where each row i in dC / dz has columns 0 to V-1 containing the gradient
        context word from i-th training example, then columns vocab_size to 2*vo
        output context word of the i-th training example, etc.

        C is the loss function summed acrossed all examples as well:

            C = -\sum_{i,j,n} mask_{i,n} (t_{i, j + n*V} log y_{i, j + n*V}), fo

        where mask_{i,n} = 1 if the i-th training example has n-th context word
        otherwise mask_{i,n} = 0.

        The arguments are as follows:

            output_activations - A [batch_size x (context_len * vocab_size)] ten
                for the activations of the output layer, i.e. the y_j's.
            expanded_target_batch - A [batch_size (context_len * vocab_size)] te
                where expanded_target_batch[i,n*V:(n+1)*V] is the indicator vect
                the n-th context target word position, i.e. the (i, j + n*V) ent
                i'th example, the context word at position n is j, and 0 otherwi
            target_mask - A [batch_size x context_len x 1] tensor, where target_
                if for the i'th example the n-th context word is a target positi

        Outputs:
            loss_derivative - A [batch_size x (context_len * vocab_size)] matrix
                where loss_derivative[i,0:vocab_size] contains the gradient
                dC / dz_0 for the i-th training example gradient for 1st output
                context word, and loss_derivative[i,vocab_size:2*vocab_size] for
                the 2nd output context word of the i-th training example, etc.
        """

        V = self.params.word_embedding_weights.shape[0]
        loss_derivative = np.repeat(target_mask, V, axis=1)[:,:,0] * (output_act
        return loss_derivative

    def compute_loss(self, output_activations, expanded_target_batch):
        """Compute the total loss over a mini-batch. expanded_target_batch is th
        by calling indicator_matrix on the targets for the batch."""
        return -np.sum(expanded_target_batch * np.log(output_activations + TINY)

    def compute_activations(self, inputs):
```

```
        """Compute the activations on a batch given the inputs. Returns an Activ
        You should try to read and understand this function, since this will giv
        how to implement back_propagate."""

        batch_size = inputs.shape[0]
        if inputs.shape[1] != self.context_len:
            raise RuntimeError('Dimension of the input vectors should be {}, but
                self.context_len, inputs.shape[1]))

        # Embedding layer
        # Look up the input word indies in the word_embedding_weights matrix
        embedding_layer_state = np.zeros((batch_size, self.embedding_layer_dim))
        for i in range(self.context_len):
            embedding_layer_state[:, i * self.embedding_dim:(i + 1) * self.embed
                self.params.word_embedding_weights[inputs[:, i], :]

        # Hidden layer
        inputs_to_hid = np.dot(embedding_layer_state, self.params.embed_to_hid_w
                        self.params.hid_bias
        # Apply logistic activation function
        hidden_layer_state = 1. / (1. + np.exp(-inputs_to_hid))

        # Output layer
        inputs_to_softmax = np.dot(hidden_layer_state, self.params.hid_to_output
                            self.params.output_bias

        # Subtract maximum.
        # Remember that adding or subtracting the same constant from each input
        # softmax unit does not affect the outputs. So subtract the maximum to
        # make all inputs <= 0. This prevents overflows when computing their exp
        inputs_to_softmax -= inputs_to_softmax.max(1).reshape((-1, 1))

        # Take softmax along each V chunks in the output layer
        output_layer_state = np.exp(inputs_to_softmax)
        output_layer_state_shape = output_layer_state.shape
        output_layer_state = output_layer_state.reshape((-1, self.context_len, l
        output_layer_state /= output_layer_state.sum(axis=-1, keepdims=True) # S
        output_layer_state = output_layer_state.reshape(output_layer_state_shape

        return Activations(embedding_layer_state, hidden_layer_state, output_lay

    def back_propagate(self, input_batch, activations, loss_derivative):
        """Compute the gradient of the loss function with respect to the trainab
        of the model. The arguments are as follows:

             input_batch - the indices of the context words
             activations - an Activations class representing the output of Model
             loss_derivative - the matrix of derivatives computed by compute_los
```

```
        Part of this function is already completed, but you need to fill in the
        computations for hid_to_output_weights_grad, output_bias_grad, embed_to_
        and hid_bias_grad. See the documentation for the Params class for a desc
        these matrices represent."""

        hid_deriv = np.dot(loss_derivative, self.params.hid_to_output_weights) \
                    * activations.hidden_layer * (1. - activations.hidden_layer)
        hid_to_output_weights_grad = loss_derivative.T.dot(activations.hidden_la
        output_bias_grad = loss_derivative.sum(axis=0)
        embed_to_hid_weights_grad = hid_deriv.T.dot(activations.embedding_layer)
        hid_bias_grad = hid_deriv.sum(axis=0)
        # The matrix of derivatives for the embedding layer
        embed_deriv = np.dot(hid_deriv, self.params.embed_to_hid_weights)

        # Embedding layer
        word_embedding_weights_grad = np.zeros((self.vocab_size, self.embedding_
        for w in range(self.context_len):
            # TODO: double check the mask_zero_index part
            word_embedding_weights_grad += np.dot(self.indicator_matrix(input_ba
                                                   embed_deriv[:, w * self.embedd
        return Params(word_embedding_weights_grad, embed_to_hid_weights_grad, hi
                      hid_bias_grad, output_bias_grad)

    def sample_input_mask(self, batch_size):
        """Samples a binary mask for the inputs of size batch_size x context_len
        For each row, at most one element will be 1.
        """
        mask_idx = np.random.randint(self.context_len, size=(batch_size,))
        mask = np.zeros((batch_size, self.context_len), dtype=np.int)# Convert t
        mask[np.arange(batch_size), mask_idx] = 1
        return mask

    def evaluate(self, inputs, batch_size=100):
        """Compute the average cross-entropy over a dataset.

            inputs: matrix of shape D x N"""

        ndata = inputs.shape[0]

        total = 0.
        for input_batch in get_batches(inputs, batch_size):
            mask = self.sample_input_mask(batch_size)
            input_batch_masked = input_batch * (1 - mask)
            activations = self.compute_activations(input_batch_masked)
            target_batch_masked = input_batch * mask
            expanded_target_batch = self.indicator_matrix(target_batch_masked)
            cross_entropy = -np.sum(expanded_target_batch * np.log(activations.o
```

```
            total += cross_entropy
        return total / float(ndata)

    def display_nearest_words(self, word, k=10):
        """List the k words nearest to a given word, along with their distances.

        if word not in self.vocab:
            print('Word "{}" not in vocabulary.'.format(word))
            return

        # Compute distance to every other word.
        idx = self.vocab.index(word)
        word_rep = self.params.word_embedding_weights[idx, :]
        diff = self.params.word_embedding_weights - word_rep.reshape((1, -1))
        distance = np.sqrt(np.sum(diff ** 2, axis=1))

        # Sort by distance.
        order = np.argsort(distance)
        order = order[1:1 + k]  # The nearest word is the query word itself, ski
        for i in order:
            print('{}: {}'.format(self.vocab[i], distance[i]))

    def word_distance(self, word1, word2):
        """Compute the distance between the vector representations of two words.

        if word1 not in self.vocab:
            raise RuntimeError('Word "{}" not in vocabulary.'.format(word1))
        if word2 not in self.vocab:
            raise RuntimeError('Word "{}" not in vocabulary.'.format(word2))

        idx1, idx2 = self.vocab.index(word1), self.vocab.index(word2)
        word_rep1 = self.params.word_embedding_weights[idx1, :]
        word_rep2 = self.params.word_embedding_weights[idx2, :]
        diff = word_rep1 - word_rep2
        return np.sqrt(np.sum(diff ** 2))
```

3.3 Print the gradients [1pt]

To make your life easier, we have provided the routine `check_gradients`, which checks your gradients using finite differences. You should make sure this check passes before continuing with the assignment. Once `check_gradients()` passes, call `print_gradients()` and include its output in your write-up.

```
def relative_error(a, b):
```

```
    return np.abs(a - b) / (np.abs(a) + np.abs(b))

def check_output_derivatives(model, input_batch, target_batch):
    def softmax(z):
        z = z.copy()
        z -= z.max(-1, keepdims=True)
        y = np.exp(z)
        y /= y.sum(-1, keepdims=True)
        return y

    batch_size = input_batch.shape[0]
    z = np.random.normal(size=(batch_size, model.context_len, model.vocab_size))
    y = softmax(z).reshape((batch_size, model.context_len * model.vocab_size))
    z = z.reshape((batch_size, model.context_len * model.vocab_size))

    expanded_target_batch = model.indicator_matrix(target_batch)
    target_mask = expanded_target_batch.reshape(-1, model.context_len, len(model
    loss_derivative = model.compute_loss_derivative(y, expanded_target_batch, ta

    if loss_derivative is None:
        print('Loss derivative not implemented yet.')
        return False

    if loss_derivative.shape != (batch_size, model.vocab_size * model.context_le
        print('Loss derivative should be size {} but is actually {}.'.format(
            (batch_size, model.vocab_size), loss_derivative.shape))
        return False

    def obj(z):
        z = z.reshape((-1, model.context_len, model.vocab_size))
        y = softmax(z).reshape((batch_size, model.context_len * model.vocab_size
        return model.compute_loss(y, expanded_target_batch)

    for count in range(1000):
        i, j = np.random.randint(0, loss_derivative.shape[0]), np.random.randint

        z_plus = z.copy()
        z_plus[i, j] += EPS
        obj_plus = obj(z_plus)

        z_minus = z.copy()
        z_minus[i, j] -= EPS
        obj_minus = obj(z_minus)

        empirical = (obj_plus - obj_minus) / (2. * EPS)
        rel = relative_error(empirical, loss_derivative[i, j])
        if rel > 1e-4:
            print('The loss derivative has a relative error of {}, which is too
```

```
            return False

    print('The loss derivative looks OK.')
    return True


def check_param_gradient(model, param_name, input_batch, target_batch):
    activations = model.compute_activations(input_batch)
    expanded_target_batch = model.indicator_matrix(target_batch)
    target_mask = expanded_target_batch.reshape(-1, model.context_len, len(model
    loss_derivative = model.compute_loss_derivative(activations.output_layer, ex
    param_gradient = model.back_propagate(input_batch, activations, loss_derivat

    def obj(model):
        activations = model.compute_activations(input_batch)
        return model.compute_loss(activations.output_layer, expanded_target_batc

    dims = getattr(model.params, param_name).shape
    is_matrix = (len(dims) == 2)

    if getattr(param_gradient, param_name).shape != dims:
        print('The gradient for {} should be size {} but is actually {}.'.format
            param_name, dims, getattr(param_gradient, param_name).shape))
        return

    for count in range(1000):
        if is_matrix:
            slc = np.random.randint(0, dims[0]), np.random.randint(0, dims[1])
        else:
            slc = np.random.randint(dims[0])

        model_plus = model.copy()
        getattr(model_plus.params, param_name)[slc] += EPS
        obj_plus = obj(model_plus)

        model_minus = model.copy()
        getattr(model_minus.params, param_name)[slc] -= EPS
        obj_minus = obj(model_minus)

        empirical = (obj_plus - obj_minus) / (2. * EPS)
        exact = getattr(param_gradient, param_name)[slc]
        rel = relative_error(empirical, exact)
        if rel > 3e-4:
            import pdb; pdb.set_trace()
            print('The loss derivative has a relative error of {}, which is too
            return False

    print('The gradient for {} looks OK.'.format(param_name))
```

```
def load_partially_trained_model():
    obj = pickle.load(open(PARTIALLY_TRAINED_MODEL, 'rb'))
    params = Params(obj['word_embedding_weights'], obj['embed_to_hid_weights'],
                               obj['hid_to_output_weights'], obj['hid_bias']
                               obj['output_bias'])
    vocab = obj['vocab']
    return Model(params, vocab)


def check_gradients():
    """Check the computed gradients using finite differences."""
    np.random.seed(0)

    np.seterr(all='ignore')  # suppress a warning which is harmless

    model = load_partially_trained_model()
    data_obj = pickle.load(open(data_location, 'rb'))
    train_inputs = data_obj['train_inputs']
    input_batch = train_inputs[:100, :]
    mask = model.sample_input_mask(input_batch.shape[0])
    input_batch_masked = input_batch * (1 - mask)
    target_batch_masked = input_batch * mask

    if not check_output_derivatives(model, input_batch_masked, target_batch_mask
        return

    for param_name in ['word_embedding_weights', 'embed_to_hid_weights', 'hid_to
                       'hid_bias', 'output_bias']:
        input_batch_masked = input_batch * (1 - mask)
        target_batch_masked = input_batch * mask
        check_param_gradient(model, param_name, input_batch_masked, target_batch


def print_gradients():
    """Print out certain derivatives for grading."""
    np.random.seed(0)

    model = load_partially_trained_model()
    data_obj = pickle.load(open(data_location, 'rb'))
    train_inputs = data_obj['train_inputs']
    input_batch = train_inputs[:100, :]

    mask = model.sample_input_mask(input_batch.shape[0])
    input_batch_masked = input_batch * (1 - mask)
    activations = model.compute_activations(input_batch_masked)
    target_batch_masked = input_batch * mask
```

```
    expanded_target_batch = model.indicator_matrix(target_batch_masked)
    target_mask = expanded_target_batch.reshape(-1, model.context_len, len(model
    loss_derivative = model.compute_loss_derivative(activations.output_layer, ex
    param_gradient = model.back_propagate(input_batch, activations, loss_derivat

    print('loss_derivative[2, 5]', loss_derivative[2, 5])
    print('loss_derivative[2, 121]', loss_derivative[2, 121])
    print('loss_derivative[5, 33]', loss_derivative[5, 33])
    print('loss_derivative[5, 31]', loss_derivative[5, 31])
    print()
    print('param_gradient.word_embedding_weights[27, 2]', param_gradient.word_em
    print('param_gradient.word_embedding_weights[43, 3]', param_gradient.word_em
    print('param_gradient.word_embedding_weights[22, 4]', param_gradient.word_em
    print('param_gradient.word_embedding_weights[2, 5]', param_gradient.word_emb
    print()
    print('param_gradient.embed_to_hid_weights[10, 2]', param_gradient.embed_to_
    print('param_gradient.embed_to_hid_weights[15, 3]', param_gradient.embed_to_
    print('param_gradient.embed_to_hid_weights[30, 9]', param_gradient.embed_to_
    print('param_gradient.embed_to_hid_weights[35, 21]', param_gradient.embed_to
    print()
    print('param_gradient.hid_bias[10]', param_gradient.hid_bias[10])
    print('param_gradient.hid_bias[20]', param_gradient.hid_bias[20])
    print()
    print('param_gradient.output_bias[0]', param_gradient.output_bias[0])
    print('param_gradient.output_bias[1]', param_gradient.output_bias[1])
    print('param_gradient.output_bias[2]', param_gradient.output_bias[2])
    print('param_gradient.output_bias[3]', param_gradient.output_bias[3])
```

```
# Run this to check if your implement gradients matches the finite difference wi
# Note: this may take a few minutes to go through all the checks
check_gradients()
```

```
The loss derivative looks OK.
The gradient for word_embedding_weights looks OK.
The gradient for embed_to_hid_weights looks OK.
The gradient for hid_to_output_weights looks OK.
The gradient for hid_bias looks OK.
The gradient for output_bias looks OK.
```

```
# Run this to print out the gradients
print_gradients()
```

```
loss_derivative[2, 5] 0.0
loss_derivative[2, 121] 0.0
loss_derivative[5, 33] 0.0
loss_derivative[5, 31] 0.0

param_gradient.word_embedding_weights[27, 2] 0.0
param_gradient.word_embedding_weights[43, 3] 0.011596892511489458
param_gradient.word_embedding_weights[22, 4] -0.0222670623817297
param_gradient.word_embedding_weights[2, 5] 0.0

param_gradient.embed_to_hid_weights[10, 2] 0.3793257091930164
param_gradient.embed_to_hid_weights[15, 3] 0.01604516132110917
param_gradient.embed_to_hid_weights[30, 9] -0.4312854367997419
param_gradient.embed_to_hid_weights[35, 21] 0.06679896665436337

param_gradient.hid_bias[10] 0.023428803123345148
param_gradient.hid_bias[20] -0.024370452378874197

param_gradient.output_bias[0] 0.000970106146902794
param_gradient.output_bias[1] 0.16868946274763222
param_gradient.output_bias[2] 0.0051664774143909235
param_gradient.output_bias[3] 0.15096226471814364
```

3.4 Run model trainin [0pt]

Once you've implemented the gradient computation, you'll need to train the model. The function *train* implements the main training procedure. It takes two arguments:

- `embedding_dim`: The number of dimensions in the distributed representation.
- `num_hid`: The number of hidden units

As the model trains, the script prints out some numbers that tell you how well the training is going. It shows:

- The cross entropy on the last 100 mini-batches of the training set. This is shown after every 100 mini-batches.
- The cross entropy on the entire validation set every 1000 mini-batches of training.

At the end of training, this function shows the cross entropies on the training, validation and test sets. It will return a *Model* instance.

```
_train_inputs = None
_train_targets = None
_vocab = None
```

```
DEFAULT_TRAINING_CONFIG = {'batch_size': 100,  # the size of a mini-batch
                           'learning_rate': 0.1,  # the learning rate
                           'momentum': 0.9,  # the decay parameter for the momen
                           'epochs': 50,  # the maximum number of epochs to run
                           'init_wt': 0.01,  # the standard deviation of the ini
                           'context_len': 4,  # the number of context words used
                           'show_training_CE_after': 100,  # measure training er
                           'show_validation_CE_after': 1000,  # measure validati
                           }


def find_occurrences(word1, word2, word3):
    """Lists all the words that followed a given tri-gram in the training set an
    times each one followed it."""

    # cache the data so we don't keep reloading
    global _train_inputs, _train_targets, _vocab
    if _train_inputs is None:
        data_obj = pickle.load(open(data_location, 'rb'))
        _vocab = data_obj['vocab']
        _train_inputs, _train_targets = data_obj['train_inputs'], data_obj['trai

    if word1 not in _vocab:
        raise RuntimeError('Word "{}" not in vocabulary.'.format(word1))
    if word2 not in _vocab:
        raise RuntimeError('Word "{}" not in vocabulary.'.format(word2))
    if word3 not in _vocab:
        raise RuntimeError('Word "{}" not in vocabulary.'.format(word3))

    idx1, idx2, idx3 = _vocab.index(word1), _vocab.index(word2), _vocab.index(wo
    idxs = np.array([idx1, idx2, idx3])

    matches = np.all(_train_inputs == idxs.reshape((1, -1)), 1)

    if np.any(matches):
        counts = collections.defaultdict(int)
        for m in np.where(matches)[0]:
            counts[_vocab[_train_targets[m]]] += 1

        word_counts = sorted(list(counts.items()), key=lambda t: t[1], reverse=T
        print('The tri-gram "{} {} {}" was followed by the following words in th
            word1, word2, word3))
        for word, count in word_counts:
            if count > 1:
                print('    {} ({} times)'.format(word, count))
            else:
                print('    {} (1 time)'.format(word))
```

```
    else:
        print('The tri-gram "{} {} {}" did not occur in the training set.'.forma


def train(embedding_dim, num_hid, config=DEFAULT_TRAINING_CONFIG):
    """This is the main training routine for the language model. It takes two pa

        embedding_dim, the dimension of the embedding space
        num_hid, the number of hidden units."""
    # For reproducibility
    np.random.seed(123)

    # Load the data
    data_obj = pickle.load(open(data_location, 'rb'))
    vocab = data_obj['vocab']
    train_inputs = data_obj['train_inputs']
    valid_inputs = data_obj['valid_inputs']
    test_inputs = data_obj['test_inputs']

    # Randomly initialize the trainable parameters
    model = Model.random_init(config['init_wt'], vocab, config['context_len'], e

    # Variables used for early stopping
    best_valid_CE = np.infty
    end_training = False

    # Initialize the momentum vector to all zeros
    delta = Params.zeros(len(vocab), config['context_len'], embedding_dim, num_h

    this_chunk_CE = 0.
    batch_count = 0
    for epoch in range(1, config['epochs'] + 1):
        if end_training:
            break

        print()
        print('Epoch', epoch)

        for m, (input_batch) in enumerate(get_batches(train_inputs, config['batc
            batch_count += 1

            # For each example (row in input_batch), select one word to mask out
            mask = model.sample_input_mask(config['batch_size'])
            input_batch_masked = input_batch * (1 - mask) # We only zero out one
            target_batch_masked = input_batch * mask # We want to predict the ma

            # Forward propagate
            activations = model.compute_activations(input_batch_masked)
```

```
            # Compute loss derivative
            expanded_target_batch = model.indicator_matrix(target_batch_masked)
            loss_derivative = model.compute_loss_derivative(activations.output_l
            loss_derivative /= config['batch_size']

            # Measure loss function
            cross_entropy = model.compute_loss(activations.output_layer, expande
            this_chunk_CE += cross_entropy
            if batch_count % config['show_training_CE_after'] == 0:
                print('Batch {} Train CE {:1.3f}'.format(
                    batch_count, this_chunk_CE / config['show_training_CE_after'
                this_chunk_CE = 0.

            # Backpropagate
            loss_gradient = model.back_propagate(input_batch, activations, loss_

            # Update the momentum vector and model parameters
            delta = config['momentum'] * delta + loss_gradient
            model.params -= config['learning_rate'] * delta

            # Validate
            if batch_count % config['show_validation_CE_after'] == 0:
                print('Running validation...')
                cross_entropy = model.evaluate(valid_inputs)
                print('Validation cross-entropy: {:1.3f}'.format(cross_entropy))

                if cross_entropy > best_valid_CE:
                    print('Validation error increasing!  Training stopped.')
                    end_training = True
                    break

                best_valid_CE = cross_entropy

    print()
    train_CE = model.evaluate(train_inputs)
    print('Final training cross-entropy: {:1.3f}'.format(train_CE))
    valid_CE = model.evaluate(valid_inputs)
    print('Final validation cross-entropy: {:1.3f}'.format(valid_CE))
    test_CE = model.evaluate(test_inputs)
    print('Final test cross-entropy: {:1.3f}'.format(test_CE))

    return model
```

Run the training.

```
embedding_dim = 16
```

```
num_hid = 128
trained_model = train(embedding_dim, num_hid)
```

```
Batch 9700 Train CE 3.200
Batch 9800 Train CE 3.229
Batch 9900 Train CE 3.189
Batch 10000 Train CE 3.179
Running validation...
Validation cross-entropy: 3.175
Batch 10100 Train CE 3.168
Batch 10200 Train CE 3.163
Batch 10300 Train CE 3.166
Batch 10400 Train CE 3.197
Batch 10500 Train CE 3.174
Batch 10600 Train CE 3.173
Batch 10700 Train CE 3.143
Batch 10800 Train CE 3.178
Batch 10900 Train CE 3.185
Batch 11000 Train CE 3.101
Running validation...
Validation cross-entropy: 3.146
Batch 11100 Train CE 3.159

Epoch 4
Batch 11200 Train CE 3.156
Batch 11300 Train CE 3.134
Batch 11400 Train CE 3.140
Batch 11500 Train CE 3.155
Batch 11600 Train CE 3.128
Batch 11700 Train CE 3.121
Batch 11800 Train CE 3.161
Batch 11900 Train CE 3.111
Batch 12000 Train CE 3.141
Running validation...
Validation cross-entropy: 3.121
Batch 12100 Train CE 3.136
Batch 12200 Train CE 3.132
Batch 12300 Train CE 3.120
Batch 12400 Train CE 3.105
Batch 12500 Train CE 3.078
Batch 12600 Train CE 3.136
Batch 12700 Train CE 3.120
Batch 12800 Train CE 3.125
Batch 12900 Train CE 3.080
Batch 13000 Train CE 3.107
Running validation...
Validation cross-entropy: 3.104
Batch 13100 Train CE 3.116
Batch 13200 Train CE 3.088
Batch 13300 Train CE 3.091
Batch 13400 Train CE 3.093
Batch 13500 Train CE 3.069
Batch 13600 Train CE 3.074
Batch 13700 Train CE 3.084
Batch 13800 Train CE 3.075
```

```
Batch 13800 Train CE 3.075
Batch 13900 Train CE 3.081
Batch 14000 Train CE 3.089
Running validation...
Validation cross-entropy: 3.088
Batch 14100 Train CE 3.090
Batch 14200 Train CE 3.108
Batch 14300 Train CE 3.127
Batch 14400 Train CE 3.075
```

To convince us that you have correctly implemented the gradient computations, please include the following with your assignment submission:

- [ ] You will submit `a1-code.ipynb` through MarkUs. You do not need to modify any of the code except the parts we asked you to implement.
- [ ] In your writeup, include the output of the function `print_gradients`. This prints out part of the gradients for a partially trained network which we have provided, and we will check them against the correct outputs. **Important:** make sure to give the output of `print_gradients`, **not** `check_gradients`.

This is worth 4 points:

- 1 for the loss derivatives,
- 1 for the bias gradients, and
- 2 for the weight gradients.

Since we gave you a gradient checker, you have no excuse for not getting full points on this part.

Part 4: Arithmetics and Analysis (2pts)

In this part, you will perform arithmetic calculations on the word embeddings learned from previous models and analyze the representation learned by the networks with t-SNE plots.

4.1 t-SNE

You will first train the models discussed in the previous sections; you'll use the trained models for the remainder of this section.

Important: if you've made any fixes to your gradient code, you must reload the a1-code module and then re-run the training procedure. Python does not reload modules automatically, and you don't want to accidentally analyze an old version of your model.

These methods of the Model class can be used for analyzing the model after the training is done:

- `tsne_plot_representation` creates a 2-dimensional embedding of the distributed representation space using an algorithm called t-SNE. (You don't need to know what this is for the assignment, but we may cover it later in the course.) Nearby points in this 2-D space are meant to correspond to nearby points in the 16-D space.
- `display_nearest_words` lists the words whose embedding vectors are nearest to the given word
- `word_distance` computes the distance between the embeddings of two words

Plot the 2-dimensional visualization for the trained model from part 3 using the method `tsne_plot_representation`. Look at the plot and find a few clusters of related words. What do the words in each cluster have in common? Plot the 2-dimensional visualization for the GloVe model from part 1 using the method `tsne_plot_GLoVe_representation`. How do the t-SNE embeddings for both models compare? Plot the 2-dimensional visualization using the method `plot_2d_GLoVe_representation`. How does this compare to the t-SNE embeddings? Please answer in 2 sentences for each question and show the plots in your submission.

4.1 **Answer**:

Q: Plot the 2-dimensional visualization for the trained model from part 3 using the method `tsne_plot_representation`. Look at the plot and find a few clusters of related words. What do the words in each cluster have in common?

A: In the 2-dimensional TSNE plot for trained model, words in the same cluster have similar meanings/usages(although some of them shouldn't be close). For example, most, many, much all means 'a lot'; what, how, where, who are all query words.

Q: Plot the 2-dimensional visualization for the GloVe model from part 1 using the method `tsne_plot_GLoVe_representation`. How do the t-SNE embeddings for both models compare?

A: In both TSNE plots, words with similar meanings are put in the same cluster.TSNE plot for GloVE not only has this attribute, but it also puts word phrases into the same cluster. For example, "new", "york" and "city" are close to each other, "one" and "another" are also close to each other.

Q: Plot the 2-dimensional visualization using the method `plot_2d_GLoVe_representation`. How does this compare to the t-SNE embeddings?

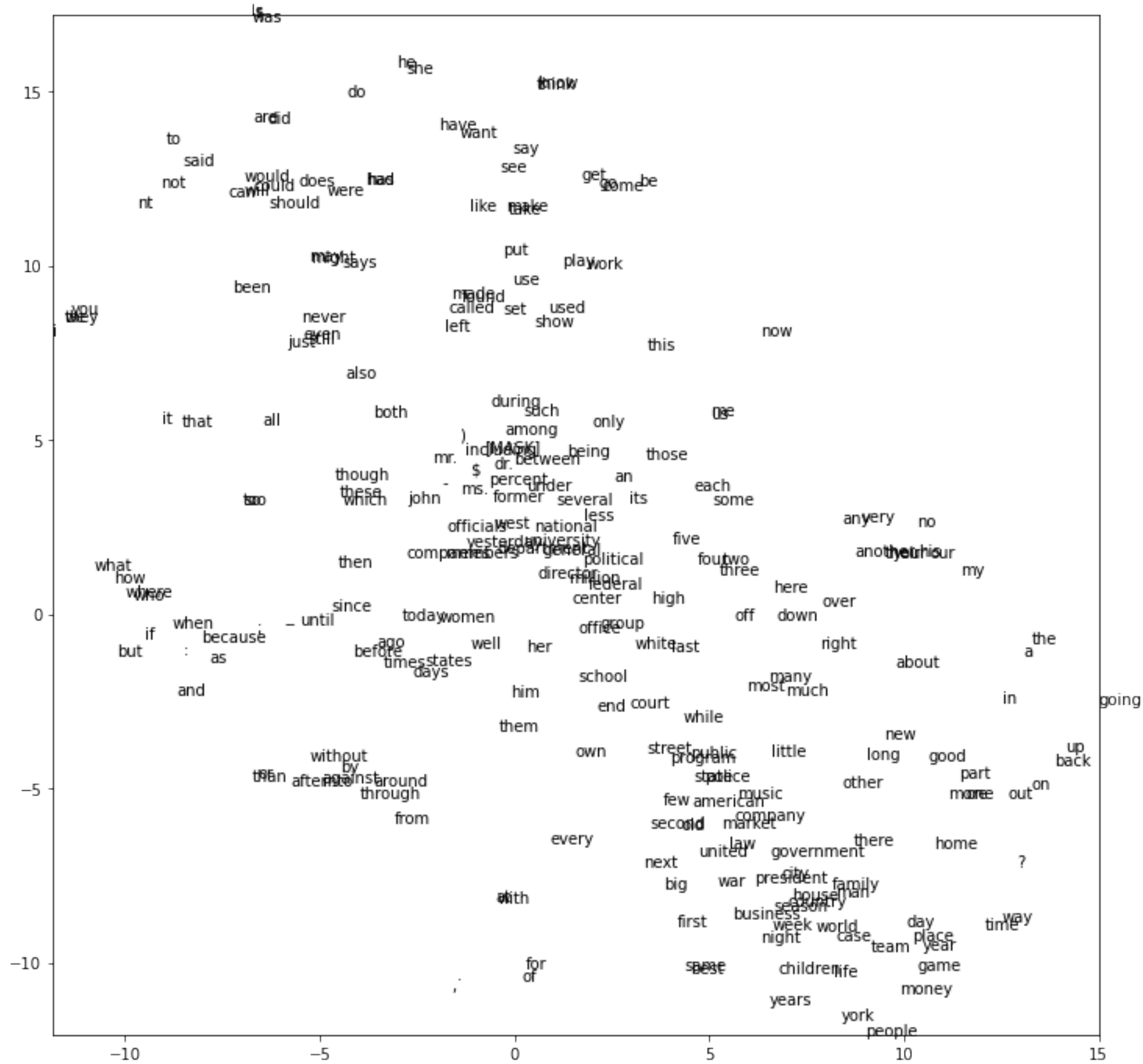
A: t-SNE embedding visualizations are more evenly spread out across the figure, whereas in 2-dimensional visualization, most of the words are clustered together in the bottom region. The latter makes it hard for the audience to identify the relationship between word embeddings.

```
from sklearn.manifold import TSNE

def tsne_plot_representation(model):
    """Plot a 2-D visualization of the learned representations using t-SNE."""
    print(model.params.word_embedding_weights.shape)
    mapped_X = TSNE(n_components=2).fit_transform(model.params.word_embedding_we
    pylab.figure(figsize=(12,12))
    for i, w in enumerate(model.vocab):
        pylab.text(mapped_X[i, 0], mapped_X[i, 1], w)
    pylab.xlim(mapped_X[:, 0].min(), mapped_X[:, 0].max())
    pylab.ylim(mapped_X[:, 1].min(), mapped_X[:, 1].max())
    pylab.show()

def tsne_plot_GLoVE_representation(W_final, b_final):
    """Plot a 2-D visualization of the learned representations using t-SNE."""
    mapped_X = TSNE(n_components=2).fit_transform(W_final)
    pylab.figure(figsize=(12,12))
    data_obj = pickle.load(open(data_location, 'rb'))
    for i, w in enumerate(data_obj['vocab']):
        pylab.text(mapped_X[i, 0], mapped_X[i, 1], w)
    pylab.xlim(mapped_X[:, 0].min(), mapped_X[:, 0].max())
    pylab.ylim(mapped_X[:, 1].min(), mapped_X[:, 1].max())
    pylab.show()

def plot_2d_GLoVE_representation(W_final, b_final):
    """Plot a 2-D visualization of the learned representations."""
    mapped_X = W_final
    pylab.figure(figsize=(12,12))
    data_obj = pickle.load(open(data_location, 'rb'))
    for i, w in enumerate(data_obj['vocab']):
        pylab.text(mapped_X[i, 0], mapped_X[i, 1], w)
    pylab.xlim(mapped_X[:, 0].min(), mapped_X[:, 0].max())
    pylab.ylim(mapped_X[:, 1].min(), mapped_X[:, 1].max())
    pylab.show()
```

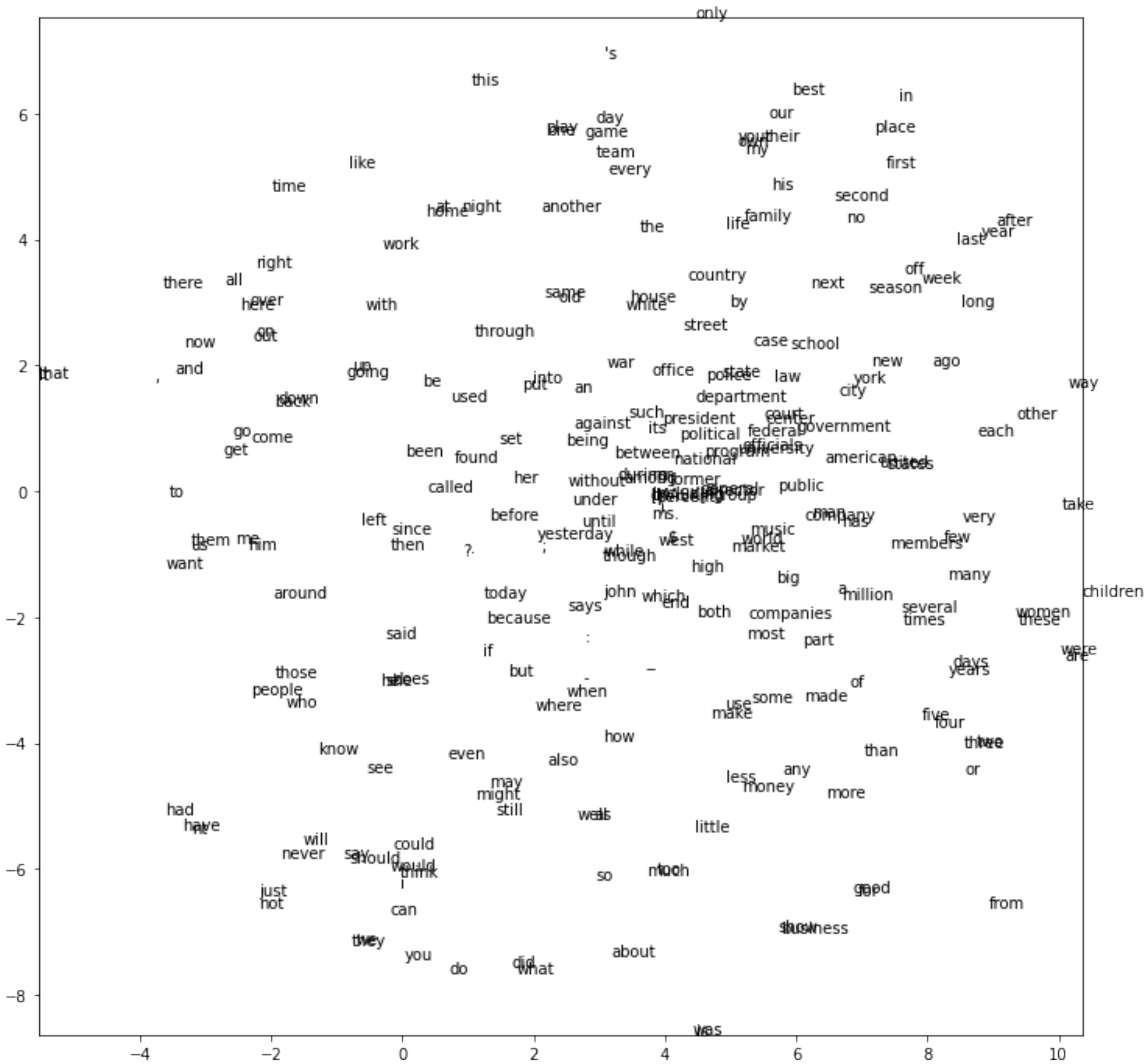
Double-click (or enter) to edit

```
tsne_plot_representation(trained_model)
```

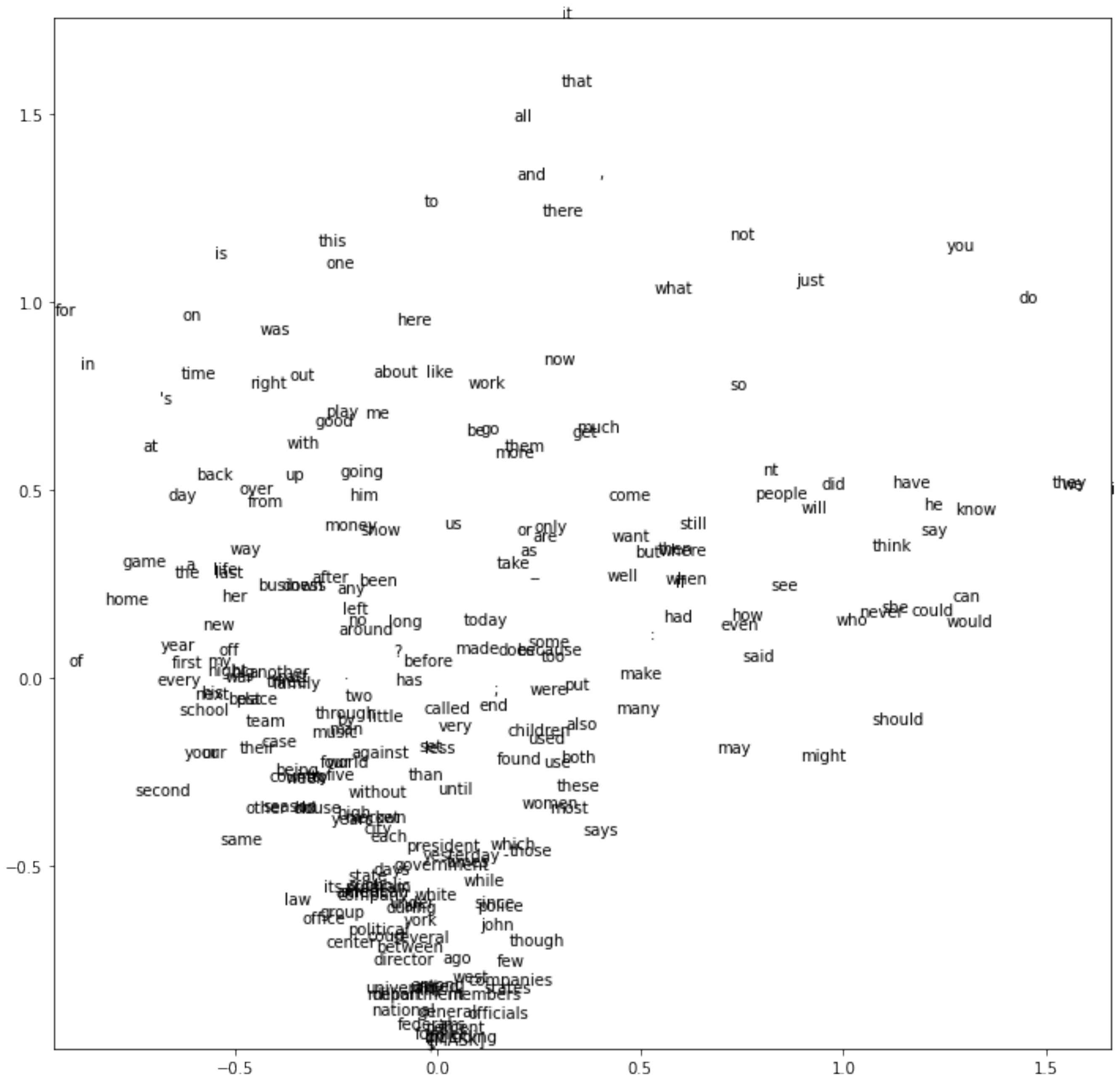
```
(251, 16)
```



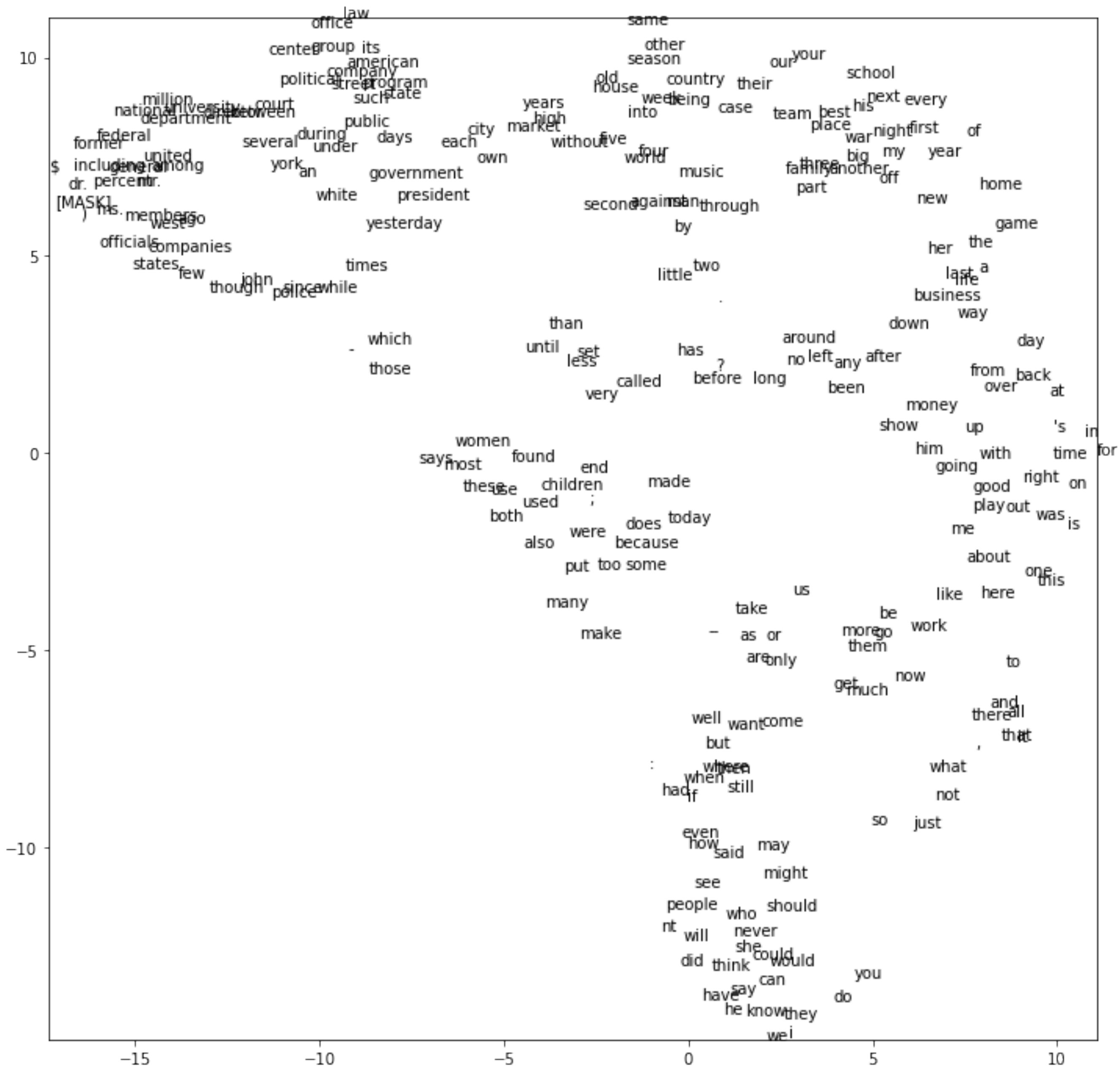
```
tsne_plot_GLoVE_representation(W_final, b_final)
```



```
plot_2d_GLoVE_representation(W_final_2d, b_final_2d)
```



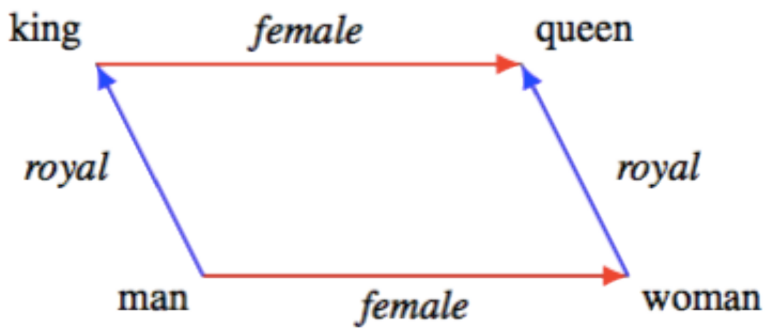
```
tsne_plot_GLoVE_representation(W_final_2d, b_final_2d)
```



4.2 Word Embedding Arithmetic

A word analogy f is an invertible transformation that holds over a set of ordered pairs S iff $\forall(x, y) \in s, f(x) = y \wedge f^{-1}(y) = x$. When f is of the form $\vec{\text{x}} \rightarrow \vec{\text{x}} + \vec{\text{r}}$, it is a linear word analogy.

Arithmetic operators can be applied to vectors generated by language models. There is a famous example: $\overrightarrow{\text{king}} - \overrightarrow{\text{man}} + \overrightarrow{\text{women}} \approx \overrightarrow{\text{queen}}$. These linear word analogies form a parallelogram structure in the vector space (Ethayarajh, Duvenaud, & Hirst, 2019).



In this section, we will explore a property of *linear word analogies*. A linear word analogy holds exactly over a set of ordered word pairs S iff $|| \vec{\text{x}} - \vec{\text{y}} ||^2$ is the same for every word pair, $|| \vec{\text{a}} - \vec{\text{x}} ||^2 = || \vec{\text{b}} - \vec{\text{y}} ||^2$ for any two word pairs, and the vectors of all words in S are coplanar.

We will use the embeddings from the symmetric, asymmetrical GloVe model, and the neural network model from part 3 to perform arithmetics. The method to perform the arithmetic and retrieve the closest word embeddings is provided in the notebook using the method `find_word_analogy`:

- `find_word_analogy` returns the closest word to the word embedding calculated from the 3 given words.

```
np.random.seed(1)
n_epochs = 500  # A hyperparameter.  You can play with this if you want.
embedding_dims = 16

W_final_sym, W_tilde_final_asym, W_final_asym = None, None, None
init_variance = 0.1  # A hyperparameter.  You can play with this if you want.
W = init_variance * np.random.normal(size=(vocab_size, embedding_dim))
W_tilde = init_variance * np.random.normal(size=(vocab_size, embedding_dim))
b = init_variance * np.random.normal(size=(vocab_size, 1))
b_tilde = init_variance * np.random.normal(size=(vocab_size, 1))

# Symmetric model
W_final_sym, _, b_final_sym, _ , _, _ = train_GLoVE(W, None, b, None, asym_log_c
# Asymmetric model
W_final_asym, W_tilde_final_asym, b_final_asym, b_tilde_final_asym, _, _ = train
```

You will need to use different embeddings to evaluate the word analogy

```
def get_word_embedding(word, embedding_weights):
    assert word in data['vocab'], 'Word not in vocab'
    return embedding_weights[data['vocab'].index(word)]


# word4 = word1 - word2 + word3
def find_word_analogy(word1, word2, word3, embedding_weights):
    embedding1 = get_word_embedding(word1, embedding_weights)
    embedding2 = get_word_embedding(word2, embedding_weights)
    embedding3 = get_word_embedding(word3, embedding_weights)
    target_embedding = embedding1 - embedding2 + embedding3

    # Compute distance to every other word.
    diff = embedding_weights - target_embedding.reshape((1, -1))
    distance = np.sqrt(np.sum(diff ** 2, axis=1))

    # Sort by distance.
    order = np.argsort(distance)[:10]
    print("The top 10 closest words to emb({}) - emb({}) + emb({}) are:".format(
    for i in order:
        print('{}: {}'.format(data['vocab'][i], distance[i]))
```

In this part of the assignment, you will use the find_word_analogy function to analyze quadruplets from the vocabulary.

4.2.1 Specific example

Perform arithmetic on words *her*, *him*, *her*, using: (1) symmetric, (2) averaging asymmetrical GloVe embedding, (3) concatenating asymmetrical GloVe embedding, and (4) neural network word embedding from part 3. That is, we are trying to find the closet word embedding vector to the vector

$$emb(he) - emb(him) + emb(her)$$

For each sets of embeddings, you should list out: (1) what the closest word that is not one of those three words, and (2) the distance to that closest word. Is the closest word *she*? Compare the results with the tSNE plots.

4.2.1 **Answer**:

- **Symmetric GloVe embeddings: the closest word is 'she', distance is 1.4817**
- **Concatenation of W_final_asym, W_tilde_final_asym: the closest word is 'she', distance is 2.2666**
- **Averaging asymmetric GLoVE vectors: the closest word is 'she', distance is 1.03212**
- **Neural Netework Word Embeddings: the closest word is 'she', distance is 17.44**
  In the t-SNE plots, these four words "he", "him", "her" and "she" indeed forms a parallelogram.

```
## GloVe embeddings
embedding_weights = W_final_sym # Symmetric GloVe
find_word_analogy('he', 'him', 'her', embedding_weights)
```

```
The top 10 closest words to emb(he) - emb(him) + emb(her) are:
he: 1.4213098857979793
she: 1.48167433432594
said: 2.1025960106397767
then: 2.2720425987761406
does: 2.301964867719902
says: 2.318047293286045
who: 2.328984314854128
where: 2.334702431567161
did: 2.353623598835888
should: 2.4126428205989865
```

```
# Concatenation of W_final_asym, W_tilde_final_asym
embedding_weights = np.concatenate((W_tilde_final_asym, W_final_asym), axis=1)
find_word_analogy('he', 'him', 'her', embedding_weights)
```

```
The top 10 closest words to emb(he) - emb(him) + emb(her) are:
he: 2.039616071340694
she: 2.2666140980184095
i: 3.0939387870946673
we: 3.5467103314054147
they: 3.64678415208859
john: 4.78151104220341
you: 4.872888583359583
president: 4.998402177074601
never: 4.998603257310438
program: 5.034287825042142
```

```
# Averaging asymmetric GLoVE vectors
embedding_weights = (W_final_asym + W_tilde_final_asym)/2
find_word_analogy('he', 'him', 'her', embedding_weights)
```

```
The top 10 closest words to emb(he) - emb(him) + emb(her) are:
he: 1.0121693139948509
she: 1.0321156534651934
should: 1.5537719142839865
could: 1.6358936351370592
i: 1.6897024008395518
would: 1.706366985675714
did: 1.717163418741739
might: 1.7563467295540773
will: 1.786151305288381
does: 1.8138772243838674
```

```
## Neural Netework Word Embeddings
embedding_weights = trained_model.params.word_embedding_weights # Neural network
find_word_analogy('he', 'him', 'her', embedding_weights)
```

```
The top 10 closest words to emb(he) - emb(him) + emb(her) are:
he: 2.4284684644619032
she: 17.4415802699889
have: 25.921497697983263
they: 25.981587972296392
want: 26.437644546989542
we: 27.128094534488834
i: 27.215833550319473
but: 28.03028938337095
about: 28.163403568035555
this: 28.531350495330678
```

4.2.2 Finding another Quadruplet

Pick another quadruplet from the vocabulary which displays the parallelogram property (and also makes sense sementically) and repeat the above proceduces. Compare and comment on the results from arithmetic and tSNE plots.

```
# words = ['we','i','me'] #should produce 'us'
words = ['would','will','can'] #should produce 'could'
```

```
## GloVe embeddings
embedding_weights = W_final_sym # Symmetric GloVe
find_word_analogy(words[0],words[1],words[2], embedding_weights)
```

```
The top 10 closest words to emb(would) - emb(will) + emb(can) are:
would: 0.889337838475016
can: 0.8964817942184019
might: 1.0340965355687213
could: 1.1447023401622758
think: 1.1875990985221023
may: 1.3379067703858845
should: 1.3726894885752774
i: 1.433248518352924
say: 1.5715342999859456
also: 1.580922486838985
```

```
# Concatenation of W_final_asym, W_tilde_final_asym
embedding_weights = np.concatenate((W_tilde_final_asym, W_final_asym), axis=1)
find_word_analogy(words[0],words[1],words[2], embedding_weights)
```

```
The top 10 closest words to emb(would) - emb(will) + emb(can) are:
would: 1.4337146325692331
can: 1.5835482617721106
could: 1.6039662466268356
should: 1.7905802833643336
might: 2.5678340651690994
will: 2.6988553218811524
did: 2.9778253654705993
may: 2.9894091349308876
does: 3.755189746018247
says: 3.955099742650736
```

```
# Averaging asymmetric GLoVE vectors
embedding_weights = (W_final_asym + W_tilde_final_asym)/2
find_word_analogy(words[0],words[1],words[2], embedding_weights)
```

```
The top 10 closest words to emb(would) - emb(will) + emb(can) are:
would: 0.6149472754010692
can: 0.8403552114940229
could: 0.91636290023521
should: 0.9843943654179407
they: 1.134451198433816
might: 1.1505856066892406
i: 1.251320706030586
we: 1.3034912278595319
will: 1.3920001403551847
she: 1.422190146041118
```

```
## Neural Netework Word Embeddings
embedding_weights = trained_model.params.word_embedding_weights # Neural network
find_word_analogy(words[0],words[1],words[2], embedding_weights)
```

```
The top 10 closest words to emb(would) - emb(will) + emb(can) are:
can: 3.272836240569943
would: 4.148235222643199
could: 4.9658839126677154
will: 6.119355841743761
should: 6.908970186782535
were: 9.632037506872697
does: 9.700119500872312
never: 9.994980256632076
may: 10.238960024013927
might: 10.339289527512182
```

What you have to submit

For reference, here is everything you need to hand in. See the top of this handout for submission directions.

- A PDF file titled *a1-writeup.pdf* containing the following:
  - [ ] **Part 1**: Questions 1.1, 1.2, 1.3, 1.4. Completed code for `grad_GLoVE` function.
  - [ ] **Part 2**: Questions 2.1, 2.2, 2.3.
  - [ ] **Part 3**: Completed code for `compute_loss_derivative()` (3.1), `back_propagate()` (3.2) functions, and the output of `print_gradients()` (3.3)
  - [ ] **Part 4**: Questions 4.1, 4.2.1, 4.2.2
- Your code file `a1-code.ipynb`