

# **An Empirical Model of Equilibrium Selection in Symmetric Normal-Form Games<sup>1</sup>**

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**“One cannot, without empirical evidence, deduce what understandings can be perceived in a nonzero-sum game of maneuver any more than one can prove, by purely formal deduction, that a particular joke is bound to be funny.”**

(Schelling 1960, p. 164)

## **Abstract**

In recent years, experiments on a wide variety of games with multiple equilibria have consistently rejected the saliency of existing deductive equilibrium selection principles. With the demise of the eductive refinement program of game theory, hope has arisen that dynamic learning models might be able to resolve the predictive impotency of equilibrium theories. The goal of this two-essay work is to provide social scientists with an empirically successful model of human strategic behavior in dynamic settings. There are two major ingredients to a complete theory of equilibrium selection: (i) a dynamic learning theory and (ii) a theory of initial conditions - the starting point of the dynamics. A robust empirical characterization of first period play, combined with learning dynamics, would make a powerful equilibrium prediction technique in a large class of coordination games.

In this essay, the first of a two-essay work, we characterize initial period play in 3×3 coordination games using an econometric model that allows for players who apply various deductive equilibrium selection principles as well as bounded rational behavior. We present experimental results on 20 3×3 games which capture and distinguish a variety of equilibrium selection principles.

In the second essay of the series, we will add learning dynamics. We will consequently develop an empirical equilibrium prediction technique based on our ability to characterize the heterogeneity in the first-period play in a systematic manner and apply learning dynamics to the predicted first-period play. We will report findings on the predictive power of our empirical approach compared to various deductive and inductive equilibrium selection principles.

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<sup>1</sup> This is a preliminary draft.

## 1. Introduction

Economies with multiple equilibria have been of great interest to economists for a long time. In economies with multiple equilibria, the rational agent formulating beliefs using deductive equilibrium concepts is uncertain as to which equilibrium strategy will be selected by other agents. Games of multiple equilibria are central to a variety of models in industrial organization and macroeconomics. These include models of network externalities (Katz and Shapiro, 1985), adoption of new technologies (Farrel and Saloner, 1985), product warranties under bilateral moral hazard (Cooper and Ross, 1985), Keynesian macroeconomics (Cooper and John, 1988), team production (Byrant, 1983), imperfect competition (Heller, 1986 and Kiyotaki, 1988), limit pricing and entry (Milgrom and Roberts, 1982), bank runs (Diamond and Dybvig, 1983), and search (Diamond, 1982).

Often in games of multiple equilibria some Nash equilibria can be dismissed by methods of *equilibrium refinement* under the argument that they are not self-enforcing. Equilibrium refinements predict whether or not an anticipated equilibrium will be played. Common equilibrium refinement ideas are the elimination of unreasonable actions, sequential rationality, perfectness (Selten, 1974), properness (Myerson, 1978), and strategic stability (Kohlberg and Mertens, 1986).

In contrast to equilibrium refinement, the *equilibrium selection* literature attempts to explain and predict which of the equilibria surviving refinements should be expected in different classes of games. The existing literature provides two general approaches to equilibrium selection: the deductive approach and the inductive approach. The deductive approach selects equilibrium points based on the description of the game. In this approach history of the game and learning dynamics have no bearing on the equilibrium selected. In contrast, the inductive approach selects equilibrium points based on the decision-makers' experiences using some model of adaptive dynamics.

Until recently, deductive principles have dominated equilibrium selection literature. A common conjecture in the deductive equilibrium selection literature is that decision makers apply some selection principle to identify a specific equilibrium point in situations involving multiple equilibria (Schelling, 1960). A prevalent deductive selection principle, when applicable, is the principle of payoff-dominance (Harsanyi and Selten, 1988, p. 81; also see Schelling, 1960, p. 291). Applying this principle, one would expect the equilibrium outcome in a coordination game to be the highest Pareto-ranking equilibrium. The greatest criticism of payoff-dominance is its failure to take into consideration out-of-equilibrium payoffs. To remedy this deficiency, equilibrium selection principles have been developed that are based on "riskiness," the most famous of which is Harsanyi and Selten's (1988) selection principle of risk-dominance.

Schelling (1960) was the first to note that the salience of a selection principle used in a particular game is largely an empirical question. His support of experimental methods came from his conviction that "... some essential part of the study of mixed motive games is empirical." And further, that "... the principles relevant to *successful* play, the *strategic* principles, the propositions of a *normative* theory, cannot be derived by purely theoretical means from a priori considerations" (Schelling 1960, p. 162).

Results from experimental works [Cooper, DeJong, Forsythe, and Ross (1990), Van Huyck, Battalio and Beil (1990, 1991; henceforth, VHBB), Van Huyck, Cook, and Battalio (1994, 1995; henceforth, VHCB), and Straub, 1995] do not seem to favor

deductive principles. A possible explanation for the failure of deductive principles is that they assume decision-makers possess beliefs consistent with some equilibrium without attempting to explain the process by which decision-makers acquire these equilibrium beliefs. Other experimental data [Stahl-Wilson (1994, 1995; henceforth, SW), Stahl (1994, 1996), Haruvy, Stahl and Wilson (1996; henceforth, HSW), and Haruvy (1997)] reject the hypothesis that all experimental subjects generally begin with equilibrium beliefs. Hence, it would seem that an equilibrium outcome is generally not the result of decision-makers with equilibrium beliefs but rather the result of a dynamic process starting with a first period play by less than super-rational decision-makers.

The failure of deductive principles has shifted interest in recent years to learning and evolutionary dynamics as a possible tool for equilibrium prediction. The basis for such theories is the idea that in cases where decision-makers initially fail to coordinate on some equilibrium, repeated interaction may allow them to learn to coordinate. Having some experience in the game provides a decision-maker with observed facts that can be used to reason about the equilibrium selection problem in the continuation game. This experience may influence the outcome of the continuation game by focusing expectations on a specific equilibrium point.

It has been found in some experimental works on games with multiple equilibria that very simple adaptive learning dynamics often yield inductive equilibrium selection principles. In these experiments, knowing the initial distribution of play was shown to be sufficient to predict the equilibrium outcome (see VHBB and VHCB for saliency of best response to first period play). However, even with a salient inductive selection principle, it would be more satisfying for a theorist to be able to predict the equilibrium outcome without having to first observe initial distribution of play. This calls for a complete theory that would characterize the first period distribution of play and then follow the rules of induction to arrive at the equilibrium.

Recent experimental work on player diversity in the one-period play sheds some light on what so far has been thought of as an ‘accidental’ initial distribution of play. The ‘level-n theory’ of SW has organized behavior into well-defined sub-populations of players possessing different models of other players’ strategies. This multi-modal theory includes a hierarchical conception of bounded rationality where differences between the sub-populations result from different depths of reasoning by an iterative self-referential process. The models developed by the theory have been extensively tested on experimental data and their parameters estimated. Furthermore, the results were shown to be robust out of sample (SW, HSW). The basic difference between models of bounded rationality and deductive equilibrium theories is that bounded rationality does not impose on players Nash equilibrium beliefs but rather allows players to have beliefs coming from a class of boundedly rational priors. This means that non-equilibrium and even dominated strategies influence the outcome and cannot be dismissed.

We begin by reviewing the level-n bounded rationality theory as a starting point for predicting initial play. We then discuss and compare competing deductive equilibrium selection principles; namely, payoff-dominance, security, risk-dominance, and alternative selection principles. We explain the differences and similarities between the selection rules and the types of predictions that they make.

We then introduce a discrete choice econometric model that merges equilibrium selection principles with the level-n theory. Specifically, our model allows for sub-

populations of players who apply different equilibrium selection principles as well as bounded rational behavior. Using experimental results, we estimate parameters that characterize the initial distribution of play. Since the ultimate goal of this two-essay series is to develop an empirical equilibrium selection technique, our ability to characterize the heterogeneity in the first-period play is an essential component of the complete theory.

In the second essay, competing models of adaptive dynamics will be formulated, each model will then be statistically evaluated as a nested hypothesis, and out-of-sample predictive power will be compared across models. Both individual data and aggregated experimental data will be used to test the theories. We will apply these models to our characterization of initial period play and compare the predictive power of these dynamic approaches combined with our initial play predictions to existing methods of equilibrium selection in a variety of games.

## 2. Level-n characterization of initial play

Though VHBB (1991) and VHCB (1995) refer to the initial period's play as a "historical accident," recent experimental evidence suggests otherwise. In particular, recent experimental works on heterogeneity in initial period play shed light on what so far has been thought of as 'accidental' initial distribution of play. Theory derived from such experimental work has organized behavior into sub-populations of players using different behavioral rules to map game characteristics into actions. The latest trends posit theories that players are made up of different sub-populations with finite depths of reasoning. Examples include Stahl's (1993) model of strategic intelligence with player types drawn from a hierarchy of 'smartness' and Nagel's (1995) evidence for a hierarchical model of bounded rationality in a mean-guessing game.

SW introduced an econometric approach to modeling heterogeneity. Their methodology allows for a characterization of first period play. Using this approach, one can test various bounded rationality behavioral rules as well as deductive selection rules to see which can help in explaining and predicting first-period play.

We embrace SW's conjecture that different boundedly rational<sup>2</sup> rules are due to different depths of reasoning by a self-referential process starting with a uniform prior over other players' strategies. In particular, the level-1 bounded rationality rule postulates that a given player, due to insufficient reason, holds a uniform prior probability distribution over other players' strategies. This will translate to a prior of (1/3, 1/3, 1/3) in any 3-by-3 game. The level-2 rule calls for a probability of near 1 to the best response to the uniform prior. Because of our desire to incorporate equilibrium selection principles as well as boundedly rational rules, we adopt the evidence-based econometric approach of the level-n theory (Stahl, 1997).

### 2.1 An evidence-based econometric model

We assume that a player considers evidence for and against available actions and chooses the action that has the highest net favorable evidence. Given  $J$  actions in game  $g$ , the  $k^{\text{th}}$  kind of evidence is a  $J \times 1$  vector  $y_k^g$ . A kind of evidence, for example, could be the

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<sup>2</sup> The self-referential nature of the concept of "rationality" in interactive situations raises the question as to whether limits on computational powers of human beings impose fundamental constraints on their ability to arrive at rational strategies (Binmore, 1987, 1988).

expected payoff against a uniform prior or whether or not an action is in the support of the payoff dominant Nash equilibrium. Suppose there are  $K$  kinds of evidence:  $\{y_k^g, k = 1, \dots, K\} = y^g$ ; thus,  $y^g$  is a  $J \times K$  matrix of evidence. Let  $v = (v_1, \dots, v_K)'$  be a  $K \times 1$  vector of non-negative weights, which determine a  $J \times 1$  weighted evidence vector:

$$Y(g, v) = y^g v. \quad (1)$$

We choose the multinomial logit specification to map from the weighted evidence vector to choice probabilities:

$$P_j(g; v) = \exp [Y_j(g, v)] / \sum_{\ell} \exp [Y_{\ell}(g, v)], \quad (2)$$

A *behavioral rule* is characterized by the evidence weights  $v$ , which via equation (1) determine a mapping from the game to the space of probabilities on available actions. Some of our behavioral rules will be the boundedly rational “level- $n$ ” rules of SW; other behavioral rules will be based on equilibrium selection principles.

Starting with the level- $n$  rules, let  $y_1^g = U^g P_0$ , where  $P_0$  denotes the uniform distribution over the strategies. Hence,  $y_1^g$  is the evidence of the pure level-1 rule, since if  $v_k = 0 \forall k \neq 1$  and  $v_1 > 0$ , then  $P(g; v)$  is the level-1 choice probability vector. Similarly, let  $y_2^g = U^g b^1(P_0)$ , where  $b^1(\cdot)$  is the best response function. Hence  $y_2^g$  is the evidence of the pure level-2 rule.

We next consider the Nash equilibrium kinds of evidence. If all games had a unique Nash equilibrium (as is the case in SW), one could unambiguously define the Nash equilibrium evidence as  $y_3^g = U^g P^{NE}$ , where  $P_j^{NE} = 1$  if  $j$  indexes a pure Nash equilibrium strategy and  $P_j^{NE} = 0$  otherwise.

Before addressing how to extend this approach to games with multiple Nash equilibria, we present alternative Nash-like types of evidence more in the spirit of bounded rationality. We will then introduce equilibrium selection rules in section 3.

## 2.2 Alternative “Nash-like” types of evidence

### 2.2.1 The mutual best-response (MBR) evidence

The spirit of bounded rationality suggests a Nash-like type that does not depend on the super-rationality considerations of selection theories. In a symmetric game, ‘that an action is best under the assumption that everyone else also chooses it’ is a property that could be recognized by boundedly rational players with no concept of Nash equilibrium: call this the “mutual best response” (MBR) property. An individual could say to themselves: “this is an interesting property and is evidence in favor of choosing this action”. To capture this reasoning, we can define a kind of evidence that awards one point to an action with the MBR property and zero points otherwise. In other words, this

criterion implies an equal probability on all actions in the support of some pure Nash equilibrium. A player following this rule is indifferent between Nash equilibria. This is not a selection principle per se since it selects randomly among Nash equilibria. However, actions possessing the MBR property may have some focal appeal and this rule can be used to capture the appeal of Nash equilibria without assuming super-rationality or using any deductive selection rule.

### 2.2.2 The iterated mutual best-response (IMBR) evidence

In the spirit of the hierarchical structure of level- $n$  theory, a player's model of the other players could be that they behave as MBR types, in which case, the evidence would be given by the expected payoff to a uniform distribution over the subset of actions possessing the MBR property. Note that this solution is the result of  $n$  ( $n = 1, 2, \dots$ ) iterations on MBR; hence the name of iterated mutual best response (IMBR).

Alternatively, this rule can be interpreted as thinking that enough coordination failure will occur to have an equal number of players, on average, choose each Nash equilibrium strategy. This selection rule is similar in spirit to the risk-dominance idea. But whereas the risk dominance rule follows the belief that the other players' beliefs are uniformly distributed, the IMBR concept follows the belief that the other players' strategies are uniformly distributed over strategies in the support of some pure Nash equilibrium.

## 3. Deductive Equilibrium Selection Principles

In this section we briefly review each of the main deductive selection principles in the literature. These are payoff-dominance, risk-dominance, and security. The premise behind these selection principles is that players choose an action from the set of Nash equilibrium actions according to various criteria. If all players apply the same criterion, the equilibrium outcome can be predicted without any consideration of dynamics.

### 3.1 Payoff Dominance

It has been argued that in coordination games the payoff-dominant equilibrium is a natural focal point (Schelling, 1960, p. 291). An equilibrium point is said to be payoff dominant if it is not strictly Pareto-dominated by any other equilibrium point. This is equivalent to Luce and Raiffa's (1958, pp. 106-107) idea of jointly admissible strategies. According to the payoff-dominance principle, players faced with multiple self-enforcing equilibria which are Pareto-rankable should be expected to choose the highest-ranking equilibrium.

The payoff dominance principle relies on the idea that "rational individuals will cooperate in pursuing their common interests if the conditions permit them to do so" (Harsanyi and Selten, 1988, p.356). Under unlimited communication, it is easy to see why this would be the case (see Bernheim, Peleg and Whinston, 1987). One stage of "cheap talk" has been shown to be sufficient, under certain conditions in the one-shot game (Anderlini, 1995) and in the repeated game (Matsui, 1991), to uniquely determine the Pareto efficient outcome.

However, in the absence of an explicit mechanism to select equilibria as a group, collective rationality is much harder to justify and out-of-equilibrium payoffs become important. Experimental studies by Cooper et al. (1990, 1992), VHBB(1990, 1991), and

Straub (1995) on coordination games provide substantial evidence that players often fail to coordinate their actions to obtain a Pareto optimal equilibrium in experimental settings. They also provide evidence that suggests the importance of out-of-equilibrium payoffs in equilibrium selection. VHBB suggest that payoff-dominance is not salient in many strategic situations because of its failure to take into account out-of-equilibrium beliefs. Equilibrium selection principles based on “riskiness” attempt to remedy this deficiency.

### 3.2 Risk-based Selection Principles

Though solution concepts based on risk differ in many aspects, there are several common elements that link them together. The most important is their consideration of out-of-equilibrium payoffs. A related commonality is that these solution concepts in some sense can be thought of as minimizing a player’s “risk” in the face of uncertainty. They only differ in what they believe to be the best proxy for “risk.”

#### 3.2.1 The security selection principle (Van Huyck et al., 1990)

A *secure action* is that action which maximizes the minimum possible payoff. Thus, when each act is appraised by looking at the worst state for that act, the secure action is the action with the best worst state. This idea is the pure-strategy version of Von Neumann and Morgenstern’s (1947) maximin criterion. It is important to note that in games with actions that are not in the support of some Nash equilibrium (unlike Van Huyck et al.), there is no reason why the secure action should be in the support of some Nash equilibrium. Therefore, to make the security criterion an equilibrium selection principle it must be modified to exclude actions that are not in the support of some Nash equilibrium. There are two ways to go about such a restriction. Let  $U$  be the payoff matrix in a given game. One way to restrict the security criterion to equilibrium actions is by defining the *secure equilibrium action* as that equilibrium action which satisfies

$$\arg \max_{k \in NE} \min_j U_{kj}. \quad (3)$$

The alternative would be

$$\arg \max_{k \in NE} \min_{j \in NE} U_{kj}. \quad (3')$$

The first alternative appraises pure Nash equilibrium actions by the worst state for each action, while ignoring non-equilibrium actions. The second alternative applies the security criterion to the game remaining after deletion of non-equilibrium actions. We will only consider the first alternative in an effort to remain as close as possible to Von Neumann and Morgenstern’s original maximin criterion, which does not include deletion of non-equilibrium strategies.

In accordance with this restriction, the *security selection principle* is an equilibrium selection principle that, out of the set of actions consistent with some Nash equilibrium, predicts the action guaranteeing the best outcome in the worst possible scenario. While VHBB (1990, 1991) are inconclusive on the predictive power of the security criterion, data from experiments conducted by Straub (1995) seem to reject this principle.

### 3.2.2 The risk dominance selection principle

Harsanyi and Selten (1988) introduced a selection criterion named *risk-dominance*. This criterion is concerned with pair-wise comparisons between equilibrium points. The equilibrium with the highest Nash-product is selected out of each pair, where Nash-product is the term used for the product of the deviation losses of both players at a particular equilibrium.

In the *heuristic justification* to risk dominance, selection of the risk-dominant equilibrium results from postulating an initial state of uncertainty where the players have uniformly distributed second order beliefs; i.e., each player best responds to the belief that the other players' beliefs are uniformly distributed on the space of priors.

Unfortunately, due to the pair-wise nature of the ranking of equilibria, there are substantial difficulties in applying the risk-dominance principle to general  $n \times n$  games where  $n > 2$ . The main difficulty is that when  $n > 2$ , transitivity of risk-dominance relations between pairs of equilibria is not guaranteed. One solution is to assign weights of importance to the risk dominance relations according to *strategic net distance* (for definition, see Harsanyi and Selten, pp. 223-226). Another solution is to extend the heuristic idea of uniformly distributed second order beliefs to  $n$  dimensions. The latter solution is consistent with pair-wise predictions in  $2 \times 2$  games and ensures transitivity of risk-dominance relations in general 2-player  $n \times n$  games. We shall refer to the latter extension from now on when using the term *risk-dominance*. For a more detailed characterization of this extension, see Haruvy and Stahl (1998).

## 4. Incorporating equilibrium selection principles into the evidence-based choice model

In order to incorporate each of the deductive Nash equilibrium approaches described in section 3 into the evidence-based choice model of section 2.1 we must specify "equilibrium evidences." Specifically, we need to define an evidence vector for each of the four selection principles of payoff dominance, risk dominance, security, and iterated mutual best response. Since when configuring real choice data, it is virtually certain that choices inconsistent with these principles will be observed, it is necessary to supplement these pure selection principles with a model of errors or trembles. To ensure that our results are not artifacts of a particular error model, we explore three alternative approaches to modeling errors in the context of equilibrium selection: (i) prior-based; (ii) ranking-based; and (iii) uniform trembles.

### 4.1 Prior-based evidence

For games with a unique Nash equilibrium corresponding to the a particular selection principle (PD, RD, security, or IMBR), there is a unique prior corresponding to that selection principle ( $P^{PDNE}$ ,  $P^{RDNE}$ ,  $P^{SNE}$ ,  $P^{IMBR}$ ) which assigns probability 1 to the Nash equilibrium action selected by that principle. Then  $U p^{kNE}$  can be thought of as the prior-based evidence for selection principle  $k$ . Given  $v_k > 0$  and  $v_j = 0 \forall j \neq k$ , the behavioral rule is the probabilistic choice function given by equation (2).

The SW interpretation of the probabilistic choices as arising from calculation errors may not seem reasonable in the context of equilibrium selection rules that presuppose that the set of Nash equilibria can be calculated exactly to begin with.



However, an alternative interpretation of ‘trembles’ is that they come from unobserved idiosyncratic non-Nash considerations by the player. One unfortunate consequence of this specification of Nash evidence is that the tremble probability on a non-Nash strategy can be higher than the tremble probability on a Nash strategy.

#### 4.2 Uniform trembles

An alternative specification of choice probabilities that avoids this last problem is simply to give positive evidence (say 1) to the selected Nash strategy and 0 evidence to all other strategies. This is equivalent to specifying uniform trembles

$$P_j = (1-\varepsilon) P^{NE} + \varepsilon P_0 \quad (4)$$

for some  $\varepsilon > 0$ . The obvious shortcomings of uniform trembles are that (1) trembles to non-equilibrium strategies are just as likely as trembles to equilibrium strategies, and (2) the ranking of the Nash equilibria by the selection criterion is not reflected in the tremble probabilities.

#### 4.3 Ranking-based evidence

In a coordination game with three Pareto ranked Nash equilibria, it is not unreasonable to want the tremble probabilities, given the payoff dominance selection criterion, to respect the Pareto ranking. To achieve this, it is natural to specify the payoff-dominance evidence of a Nash equilibrium strategy (say  $j$ ) to be the equilibrium payoff  $U_{jj}$ . To guarantee that non-Nash trembles are strictly less likely, we can specify their evidence to be strictly less than the minimum payoff-dominance evidence by some positive parameter (to be estimated from the data). One way to justify this definition of evidence is to note that each selection principle maximizes some implicit objective function over actions and hence it is not unreasonable to allow the value of that objective function for a particular row action to be the evidence in favor of that action. In the payoff dominance case, the objective function is:

$$Q^{PD}(j) = \{U_{jj}, j \in NE\}. \quad (5)$$

In the security case, the objective function is

$$Q^{SEC}(j) = \{ \min_{k \in NE} U_{jk}, j \in NE \}. \quad (6)$$

For any selection rule, if  $k \notin NE$ , then

$$Q(k) = \min_{i \in NE} Q(i) - \delta, \quad (7)$$

for some  $\delta > 0$ .

This approach is intuitive in that it closely follows the idea that players choose among Nash equilibria using some deductive criterion. The security type should then be more likely to tremble to a slightly less “secure” Nash equilibrium than to a risky Nash

equilibrium, whereas the payoff dominant type is more likely to tremble to the second Pareto-ranked Nash equilibrium than to the third Pareto-ranked Nash equilibrium. Since this approach preserves the ranking over Nash equilibria implied by each criterion, we will call it the *ranking-based* approach, to distinguish from the *prior-based* approach we described. Later in the paper, we will test the ranking-based approach against the prior-based approach.

#### 4.4 A mixture model

We can construct an econometric model that allows for sub-populations of players, each using a different behavioral rule. Such a model would allow us to:

1. Conditionally test for the existence of a sub-population that uses a given behavioral rule.
2. Measure the extent of use of one rule versus another in the first period play if neither is rejected; i.e., measure the relative size of a sub-population.
3. Test alternative ways of modeling a specific type.

It is natural to define archetypal rules that put positive weight on one and only one kind of evidence. Accordingly, we define the level-1 archetype rule ( $v_k = 0 \forall k \neq 1$  and  $v_1 > 0$ ), the level-2 rule ( $v_k = 0 \forall k \neq 2$  and  $v_2 > 0$ ), the payoff dominance selection rule and/or the MBR property ( $v_k = 0 \forall k \neq 3$  and  $v_3 > 0$ ), the RD rule ( $v_k = 0 \forall k \neq 4$  and  $v_4 > 0$ ), security ( $v_k = 0 \forall k \neq 5$  and  $v_5 > 0$ ), etc. In addition, we specify the level-0 rule ( $v_k = 0 \forall k$ ).

Next, we classify players into sub-populations. In addition to *behavioral rules*, we now define *behavioral types*. While a *behavioral rule* is a prescriptive way to map the game into the action space, a behavioral type refers to a sub-population of players using the same behavioral rule. We allow one behavior type for each archetypal rule specified above, as well as a ‘worldly’ behavioral type who takes into consideration a convex combination of evidences.

Let  $i$  index the player and  $a(i, g) \in \{1, 2, 3\}$  denote the action of player  $i$  in game  $g$ . Then, assuming a player used the same selection principle over all games, the probability of player  $i$ ’s joint choices if he belongs to sub-population  $t$  is

$$P_t^i = \prod_{g \in G} P(a(i, g) | g, t). \quad (8)$$

Then the unconditional likelihood of player  $i$ ’s joint choices is

$$L_i = \sum_{t=0}^4 \alpha_t P_t^i, \quad (9)$$

where  $\alpha_t$  is the ex ante probability of being a type  $t$  player. The log likelihood to be maximized over the entire sample of  $N$  players is then

$$LLF = \sum_{i=1}^N \log L_i. \quad (10)$$

## 5. Experimental Design

Twenty symmetric 3×3 game matrices were selected. Fourteen of these games were coordination games with two to three symmetric pure Nash equilibria. The coordination game matrices were designed such no two selection principles predict the same action for all 14 games. The three main selection principles captured by the coordination games are payoff-dominance, risk dominance, and security. Two of the matrices were specifically designed to separate out two different extensions to risk dominance. The six non-coordination games were designed to separate out various boundedly rational types. The payoff matrices for the row player are presented in figure II; the transposes of these matrices give the payoffs for the column player. Note that the numbering of the games does not correspond to the order in which they appear in figure II. Rather, the numbering corresponds to the order in which games appeared in the actual experiments. The order of display in the paper groups together games belonging to the same category.

Each participant “played the field”; i.e., each player faced the empirical distribution of choices of all other participants. This procedure is also known as mean matching. In the coordination literature, the mean of players’ actions is a common example of an abstract market process; see Cooper and John (1988). As the number of players increase, the influence of an individual player on the mean goes to zero and in the limit an individual player cannot influence the market outcome.

The experiment was conducted in a computer laboratory and each participant was assigned a computer terminal. During the experiment, each game matrix was presented on the computer screen (see figure I) once a participant clicked on a button corresponding to that game. Participants had an on-screen calculator available to them to calculate hypothetical payoffs. On the Main screen, players viewed the game matrices, entered hypotheses on the other players’ distribution of play, calculated payoffs to each action given a hypothesis, and chose a row action for each game matrix. In addition to the Main screen, players had available to them an Instruction screen, containing a copy of the instructions. Each player played each of the 20 games with no feedback until all games have been played. The total amount of time allotted for all 20 games was 30 minutes. Within the time allotted, players could revisit any game and revise their choices.

After the instructions were read, a quiz was given to the participants. The quiz required the subjects to use the main screen to determine the payoffs to various combinations of their individual choice and some distribution of the rest of the participants’ play. The game matrix used for the purpose of the quiz (see figure II) contained no pure Nash equilibrium in order that the practice questions not unintentionally induce coordination in the actual games. Reading aloud the instructions was intended to assure the subjects that all participants had *common information*. The quiz and the announcement at the end of the quiz that all participants had received a perfect score suggested some degree of *common knowledge*.

Two sessions were conducted, with 23 subjects in the first session, and 24 subjects in the second session. The subject pool consisted of upper division and graduate students in the fields of social sciences and natural sciences (economics graduate students

were not permitted to participate) at the University of Texas at Austin. Participants in previous experiments were screened out. Each session lasted about one hour and 30 minutes. Average payment in the first experiment was \$22.48 with a standard deviation of \$4.44, minimum payment of \$15 and a maximum payment of \$31. Average payment in the second experiment was \$23.46 with a standard deviation of \$4.62, minimum payment of \$18 and a maximum of payment of \$31. These payment statistics include a \$5 show-up payment.

## 6. Results on the characterization of initial conditions

### 6.1 Comparison of homogeneous models

An initial inspection of player choices (presented above the game matrices in figure II) reveals a significant amount of coordination failure in first period play. We observe that in some games, players display a great degree of heterogeneity in actions (game matrices 3,15,18,19 in figure II) whereas in other games there seems to be almost a consensus (game matrices 2,5,8,10,14,17 in figure II).

We define the “hit-rate” of a particular selection principle as the percentage of players who chose the action predicted by that selection principle. The hit-rates for payoff dominance, risk dominance, and security selection are 28%, 44%, and 53%, respectively, for the entire set of fourteen coordination games.

Looking at hit rates of deductive selection principles in each of the coordination games (figure III), we notice that no one deductive selection principle can explain the majority of choices in more than eight games. We define the “success rate” of a selection principle as the percentage of games for which the rule predicted successfully the action played by the majority of players (i.e., the percentage of games for which the hit rate was greater than 50%). We have success rates of 27%, 50%, and 73% for payoff dominance, risk dominance, and security selection, respectively. These statistics are unsatisfactory for real predictive ability. Furthermore, each of the deductive selection principles had at least one game for which it predicted correctly 0% of the actions taken by players. Yet, if we were forced to choose a unique deductive selection principle on the basis of which to make predictions, it would seem that security is the strongest candidate in terms of hit-rates and success rates out of all deductive selection principles.

Given the disappointing performance of deductive equilibrium principles, we turn to bounded rationality theories as a possible prediction tool for initial conditions. We first consider a homogeneous model consisting of only a level-1 type ( $v_k=0 \forall k \neq 1$ ,  $v_1$  estimated to be 0.105) The hit rate of level-1 bounded rationality was 83%. The success rate of level-1 bounded rationality in predicting the action played by the majority of players was 100%. The lowest hit rate for level-1 occurred in game 19 with a hit rate of 62%. This is much better than the 0% lowest hit rate witnessed in selection principles.

Though hit rates and success rates are powerful indicators of predictive power, any rigorous comparison must be made econometrically. Recall that econometric modeling (section 4) of equilibrium selection principles is problematic due to the need for a model of errors. Three alternatives were discussed; namely, the ranking-based specification, the prior-based specification, and the uniform tremble specification. Comparing selection principles under each of these alternative specifications is a challenging task due to the non-nested nature of the comparison.

However, note that each of the selection principles, under the ranking-based specification, has nested within it the equilibrium principle of mutual best response. This is because for each selection principle model under the ranking-based approach we can let  $v$  approach 0 and  $\delta$  approach  $\infty$ . This would effectively place a zero weight on non-Nash actions and a positive weight on the Nash actions. The product of  $v$  and  $\delta$  in any of the selection principle models is then equivalent to the  $v$  parameter of the mutual best response model. Hence we do the nested comparisons between each selection principle, under the ranking-based specification, and the mutual best response principle (using the likelihood ratio test). We conclude that only the security selection principle produces a significant improvement over mutual best response at any acceptable significance level (see figure IV for log-likelihood and parameter estimates for each selection principles under each specification). Though under the prior-based and uniform tremble specifications no nested comparison is available, security produces a log likelihood that is at least 93 points higher than any other principle under the prior-based approach and at least 44.56 points higher than any other principle under the uniform trembles approach. The superior performance of the security selection principle in likelihood terms is consistent with its success in terms of hit rates and success rates.

Turning our attention to bounded rationality rules, we note that the log likelihood of -676.365 (figure IV) for the homogeneous model of level-1 bounded rationality is a great improvement over -868.94 (figure IV) produced by the ranking-based security model. When specifying an evidence-based rule, we are not limited to a single type of evidence. Recall equation (1). In that equation  $K$  kinds of evidence entered each rule. Each rule weighed these  $K$  types of evidence differently. So far, we have limited the scope of the discussion to rules that place zero weights on all but one type of evidence. We shall relax this restriction and examine a rule that allows positive weights on both the level-1 evidence and the mutual best-response evidence. This particular rule is consistent with SW's (1995) boundedly rational "worldly" type of behavior. The "worldly" model produces a log-likelihood of -664.01 (figure IV), significantly better than the level-1 model at the 5% level.

The combined evidence of hit-rates and econometric fit demonstrates that no reasonable characterization of first-period play should be made without allowing for bounded rationality behavioral rules, and specifically, that level-1 bounded rationality qualifies as a good bounded rationality rule to include in first period play characterization. Moreover, the success of the level-1 rule sheds some light on our comparison of deductive selection rules: It often happens that the level-1 prediction coincides with the security prediction. The security row action has the highest lowest-in-the-row (worst-case) cell, whereas the level-1 action has the highest sum of cells. These properties are highly related and hence their predictions often coincide. Similarly, level-1 is related to risk-dominance. Note that in  $2 \times 2$  games with two strong Nash equilibrium points the level-1 prediction and the risk-dominance prediction will always coincide. In contrast to risk-dominance and security, no particularly strong reason exists for a significant correlation between level-1 and payoff dominance predictions. The relationships between level-1 and the selection principles are most likely responsible for the relative performance of the selection principles vis-a-vis each other in the homogeneous population models. The payoff dominance, which is not closely related to level-1, does the poorest, whereas the security, most closely coinciding with the level-1

predictions, does the best. It may be of interest to compare the saliency of deductive equilibrium selection models after taking account of the level-1 influence. This can be done in the context of models of heterogeneous populations as described in the next section.

In addition to comparing the performance of selection rules within each econometric modeling approach, we would like to compare the modeling approaches themselves against each other. We could note off-hand that for any given selection principle, the ranking-based approach produces a higher likelihood than the prior-based and uniform-tremble approaches. However, without a strong prior on the comparison at hand, the non-nested nature of the comparisons does not allow us to say whether the better fit of the ranking-based approach is statistically significant. For example, consider the comparison of the ranking-based approach to the prior-based approach. Even if we knew the distribution of the likelihood ratio conditional on the ranking-based model being the true underlying specification and the distribution conditional on the prior-based model being the true underlying specification, it is conceivable that the observed likelihood-ratio falls within the latter distribution but not within the former. A Monte Carlo procedure of the kind described in HSW can estimate the two likelihood ratio distributions needed for the comparison. However, the oversimplification entailed in homogeneous models of deductive selection may render the bootstrap results inconclusive. In other words, the observed likelihood ratio is not likely to fall within the 95% confidence interval of either estimated distribution. We shall therefore postpone the comparison of econometric modeling of deductive selection to the next section, where a heterogeneous specification is applied.

## 6.2 Comparisons of heterogeneous models

By allowing for heterogeneity in behavior, we may be able to get a substantial improvement in our fit since players displayed various degrees of heterogeneity in several of the games. Comparison of heterogeneous models of behavior cannot be done using hit rates and success rates as before since more than one action may be predicted for a given game. Econometric techniques, however, can be used to compare goodness of fit.

To investigate heterogeneity, we estimate the mixture model of sec 4.4. For the Nash-like evidence, there are several advantages to the ranking-based specification. Beside being consistent with the implicit equilibrium selection rankings, it allows for nested comparisons between any selection principle and the mutual best-response principle. This comparison will allow us to decide on the optimal selection principle (or principles) to incorporate into the heterogeneous model.

First note that any of the heterogeneity models we examine (figure V) produce great improvements over homogenous characterizations of first-period play (figure IV).

Second, looking at figure V, we see that once added to bounded rational evidence, neither security nor risk-dominance perform any better than the non-selection concept of mutual best-response. Since under the ranking-based specification, any selection principle has nested within the mutual best-response principle, we can reject risk dominance and security in favor of mutual best-response. Only payoff dominance performs significantly better than mutual best-response. Furthermore, neither adding security only, nor adding both security and risk-dominance, to the heterogeneous model with payoff dominance evidence adds significant improvement (these hypotheses are

nested). Hence we conclude that (1) the population is heterogeneous, and (2) payoff dominance is the unique selection principle that adds significantly to the model's fit. The parameter estimates and the 95% confidence interval for the final model of heterogeneity are in figure VI.

Monte Carlo simulations of the likelihood ratio distributions, under the null that the ranking-based specification is correct and under the null that the prior-based specification is correct, confirm that our use of the ranking-based approach to modeling equilibrium selection is optimal (see figure VII).

### 6.3 Robustness Tests

For the purpose of testing for the prediction power of the above model out-of-sample, we re-estimate the model on a subset of games and use these estimates to predict the behavior on the rest of the games. For that purpose, the composition of the two subsets has to be similar and the subset used to estimate the parameters for prediction must be large enough to have a reasonable efficiency of parameter estimates. For the predicted subset (Subset II), we picked eight representative games (game matrices 7, 16, 11, 17, 1, 3, 9, and 15 in figure II ) out of the 20 games selected for the experiment. These eight games were picked a priori to be representative of the entire set of 20 games; i.e., a representative subset (of 1-2 games) was picked from each of the 6 categories of games. Specifically, we selected two selection-principle test games, two Cooper et al. games, one risk-dominance test game, one dominance solvable game, one mixed Nash equilibrium game, and one Litmus-test game. Furthermore, we ensured that the games in each subset provided a satisfactory temporal sampling. We refer to the remaining subset of 12 games as subset I.

To test the robustness of the model with respect to the games used, we conducted two tests using subsets I and II above. The log-likelihood ratio of subset I relative to the full set of games is  $(-352.036 - (-358.034)) = 5.998$ , and the log-likelihood ratio of subset II relative to the full set is  $(-248.73 - (-251.619)) = 2.889$ ; these are distributed chi-square with 13 degrees of freedom with p-values of 0.946 and 0.998 respectively. Thus, we cannot reject the null hypothesis that the parameter estimates from the full set of games are valid for subsets I and II. Further, the log-likelihood ratio from predicting subset II from subset I is  $(-248.73 - (-260.171)) = 11.441$ , which is distributed chi-square with 13 degrees of freedom and a p-value of 0.574. Thus, we cannot reject the null hypothesis that the parameter estimates from subset I are valid for subset II. In other words, the parameter estimates are stable across these subsets of games, demonstrating the out-of-sample predictive power of the model.

## 7. Conclusions and Preview of Coming Attractions

This paper's starting point was the conjecture, often made by works in adaptive dynamics literature, that given initial conditions, the path of play and hence the subsequent resulting equilibrium could be predicted with a high level of confidence. All that was left, given the conjecture, was to design a model that could empirically characterize the initial-period behavior of a population over different games. Natural candidates for such a model were various equilibrium selection principles. Deductive selection principles were shown to be inadequate in predicting first-period play. However, simple models of bounded rationality, specifically level-1 bounded rationality

as well as “worldly” bounded rationality, displayed much promise and good prediction ability over all games. Following this finding, we allowed for heterogeneous populations which apply equilibrium selection principles as well as boundedly rational rules. We proceeded to show that this relaxation of the homogeneity assumption improved the fit significantly. Moreover, the estimated parameters were shown to be robust over subsets of games.

While the literature has produced mixed to unfavorable support for the payoff dominance selection principle, our results, using the heterogeneous model, provide strong evidence in favor of payoff dominance while rejecting the risk-dominance and security criteria. It appears that once the presence of boundedly rational behavior (specifically level-1) is accounted for in the econometric specification, payoff dominance emerges as the most (and the only statistically significant) informative deductive selection principle.

We further applied Monte Carlo techniques to provide a rigorous analysis of the non-nested comparisons of the ranking-based versus prior-based approaches to econometric modeling of equilibrium selection principles. We concluded in favor of the ranking-based specification.

Our results to date strongly indicate that despite claims to the contrary, first period behavior is not a ‘historical accident.’ This fact and ‘path dependence’ allow us to take our fitted models of first period play and combine them with adaptive dynamics to arrive at a prediction of the dynamic path of play. In particular, for each alternative model [a first-period model coupled with a dynamic model], we can forecast the population distribution of play in any future period and hence the likelihood of any potential data sample. We will also do this for the more sophisticated “rule-learning” approach of Stahl (1997) with modifications needed for allowing multiple equilibria.



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Figure I The Computer Screen

Participant 1

**INSTRUCTIONS** **RECORD** Enter password:

**Choices Of Others**

	A	B	C	Payoff
<b>Your Choice</b> A	40	15	70	47
B	22	80	0	24.8
C	30	100	55	54

Matrix 1  
Matrix 2  
Matrix 3  
Matrix 4  
Matrix 5  
Matrix 6  
Matrix 7  
Matrix 8  
Matrix 9  
Matrix 10  
Matrix 11  
Matrix 12  
Matrix 13  
Matrix 14  
Matrix 15  
Matrix 16  
Matrix 17  
Matrix 18  
Matrix 19  
Matrix 20

A:  B:  C:  Total is:  **CALCULATOR**

## Figure II The Game Matrices and Player Choices

Below are the game matrices of the experiment sorted into categories but numbered according to the order in which they were presented. Above each column of each matrix is the number of players who chose the corresponding row of that matrix.

### Game 4

<b>9</b>	<b>35</b>	<b>3</b>	
<b>70</b>	<b>30</b>	<b>20</b>	Payoff-dominance Nash equilibrium
<b>60</b>	<b>60</b>	<b>30</b>	Level-1 and Risk-dominant Nash equilibrium
<b>45</b>	<b>45</b>	<b>40</b>	Security Nash equilibrium

### Game 7

<b>39</b>	<b>1</b>	<b>7</b>	
<b>60</b>	<b>30</b>	<b>60</b>	Level-1, Risk-dominant, and Security Nash equilibrium
<b>50</b>	<b>25</b>	<b>55</b>	Non-cooperative dominated strategy
<b>30</b>	<b>20</b>	<b>70</b>	Payoff-dominance Nash equilibrium

### Game 10

<b>1</b>	<b>2</b>	<b>44</b>	
<b>25</b>	<b>45</b>	<b>55</b>	Non-cooperative dominated strategy
<b>35</b>	<b>50</b>	<b>35</b>	Security Nash equilibrium
<b>30</b>	<b>45</b>	<b>60</b>	Level-1, Risk-dominant, and Payoff-dominant Nash equilibrium

### Game 13

<b>32</b>	<b>6</b>	<b>9</b>	
<b>60</b>	<b>60</b>	<b>30</b>	Level-1 / security strategy
<b>30</b>	<b>70</b>	<b>20</b>	Payoff-dominant and Risk-dominant Nash equilibrium
<b>70</b>	<b>25</b>	<b>35</b>	Level-2 strategy, Security Nash equilibrium

**Game 16**

<b>37</b>	<b>7</b>	<b>3</b>	
<b>20</b>	<b>0</b>	<b>60</b>	Level-1 strategy
<b>0</b>	<b>60</b>	<b>0</b>	Risk-dominant and payoff-dominant Nash equilibrium
<b>10</b>	<b>25</b>	<b>25</b>	Non-cooperative dominated strategy

**Games from Cooper et al.****Game 2 (Cooper et al., 1990, game 3, converged to row 1)**

<b>0</b>	<b>4</b>	<b>43</b>	
<b>60</b>	<b>0</b>	<b>0</b>	Cooperative dominated strategy
<b>0</b>	<b>55</b>	<b>25</b>	Payoff-dominant and Risk-dominant Nash equilibrium
<b>100</b>	<b>35</b>	<b>35</b>	Level-1 and Security Nash equilibrium

**Game 5 (Cooper et al., 1990, game 4, converged to row 1)**

<b>44</b>	<b>0</b>	<b>3</b>	
<b>35</b>	<b>70</b>	<b>35</b>	Level-1 and Security Nash equilibrium
<b>0</b>	<b>60</b>	<b>0</b>	Cooperative dominated strategy
<b>25</b>	<b>0</b>	<b>55</b>	Payoff-dominant and Risk-dominant Nash equilibrium

**Game 8 (Cooper et al., 1990, game 5, converged to row 2)**

<b>0</b>	<b>46</b>	<b>1</b>	
<b>35</b>	<b>35</b>	<b>70</b>	Security Nash equilibrium
<b>25</b>	<b>55</b>	<b>100</b>	Level-1, Risk-dominant and payoff-dominant Nash equilibrium
<b>0</b>	<b>0</b>	<b>60</b>	Cooperative dominated strategy

**Game 11 (Cooper et al., 1990, game 6, converged to row 2)**

30	16	1	
55	25	65	Level-1, Risk-dominant and payoff-dominant Nash equilibrium
35	35	70	Security Nash equilibrium
0	0	60	Cooperative dominated strategy

**Game 14 (Cooper et al., 1990, game 7, converged to row 2)**

1	41	5	
50	0	0	Non-cooperative dominated strategy
70	35	35	Level-1 and Security Nash equilibrium
0	25	55	Payoff-dominant and Risk-dominant Nash equilibrium

**Game 17 (Cooper et al., 1990, game 8, converged to row 2)**

0	3	44	
50	0	0	Non-cooperative dominated strategy
0	55	25	Payoff-dominant and Risk-dominant Nash equilibrium
100	35	35	Level-1 and security Nash equilibrium

**Game 20 (Extension of game 8 of Cooper et al., 1990)**

0	3	44	
55	0	25	Payoff-dominant and Risk-dominant Nash equilibrium
50	50	30	Non-dominated non-cooperative strategy
35	100	35	Level-1 and security Nash equilibrium



## Games for the Comparison of Extended and Pair-wise Risk Dominance

### Game 1

#### (Comparison of Extended and Pair-wise Risk Dominance)

36      1      10

70	60	90
60	80	50
40	20	100

Extended risk-dominance (level-1 and security) Nash equilibrium

Pair-wise risk-dominance Nash equilibrium ( $B \succ A \succ C \prec B$ )

Payoff dominant Nash equilibrium

### Game 19

#### (Comparison of Extended and Pair-wise Risk Dominance)

9      29      9

80	60	50
60	70	90
0	0	100

Pair-wise risk-dominance Nash equilibrium ( $A \succ B \succ C \prec A$ )

Extended risk-dominance (level-1 and security) Nash equilibrium

Payoff dominant Nash equilibrium (highly risky)

## Non-coordination Games

### Game 3 (Strict Dominance Solvable, Stahl-Wilson 1995, game 1)

20      25      2

25	30	100
40	45	65
31	0	40

Level-1 strategy

Unique Nash equilibrium

Dominated strategy

**Game 6 (Strict Dominance Solvable, Stahl-Wilson 1995, game 12)**

14	1	32	
40	15	70	Unique Nash equilibrium
22	80	0	Dominated strategy
30	100	55	Level-1 strategy

**Game 9 (Unique mixed strategy NE, Stahl-Wilson 1995, game 4)**

38	9	0	
30	50	100	Level-1 strategy
40	45	10	Level-2 strategy
35	60	0	

**Game 12 (Unique mixed strategy NE, Stahl-Wilson 1995, game 11)**

28	3	16	
30	100	22	Level-1 strategy
35	0	45	
51	50	20	Level-2 strategy

**Game 15 (Unique pure NE; not dominance solvable; NE, level-1 and level-2 distinct, Stahl-Wilson 1995, game 2)**

9	29	9	
75	40	45	Unique Nash equilibrium
70	15	100	Level-1 strategy
70	60	0	Level-2 strategy

**Game 18 (Unique pure NE; not dominance solvable; NE, level-1 and level-2 distinct, Stahl-Wilson 1995, game 6)**

<b>25</b>	<b>14</b>	<b>8</b>	
<b>25</b>	<b>30</b>	<b>100</b>	Level-1 strategy
<b>60</b>	<b>31</b>	<b>51</b>	Unique Nash equilibrium
<b>95</b>	<b>30</b>	<b>0</b>	Level-2 strategy

**Practice Games**

**Quiz Game/ Practice game I (Unique mixed strategy NE, Stahl-Wilson 1995, game 7)**

<b>30</b>	<b>100</b>	<b>50</b>	Level-1 strategy
<b>40</b>	<b>0</b>	<b>90</b>	
<b>50</b>	<b>75</b>	<b>29</b>	Level-2 strategy

**Practice game II (“Litmus test” game)**

<b>78</b>	<b>48</b>	<b>0</b>	Nash equilibrium strategy
<b>5</b>	<b>37</b>	<b>99</b>	Level-2 strategy
<b>29</b>	<b>96</b>	<b>69</b>	Level-1 strategy

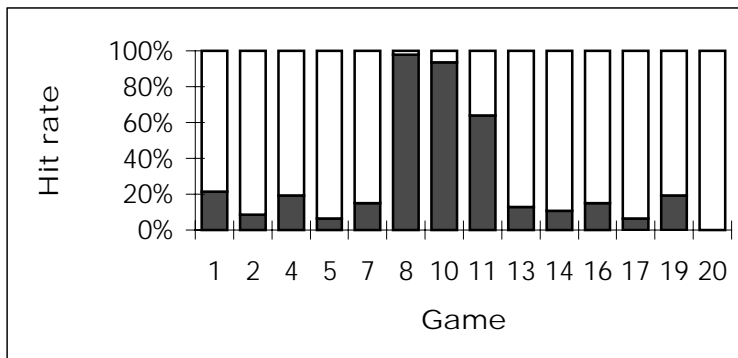
**Practice game III (“Litmus test” game)**

<b>16</b>	<b>99</b>	<b>53</b>	Level-1 strategy
<b>79</b>	<b>64</b>	<b>0</b>	Level-2 strategy
<b>13</b>	<b>24</b>	<b>85</b>	Nash equilibrium strategy

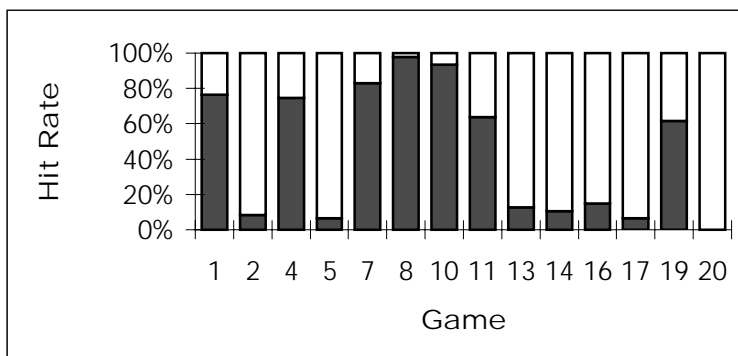
### Figure III Performance of pure rules of behavior in first-period play

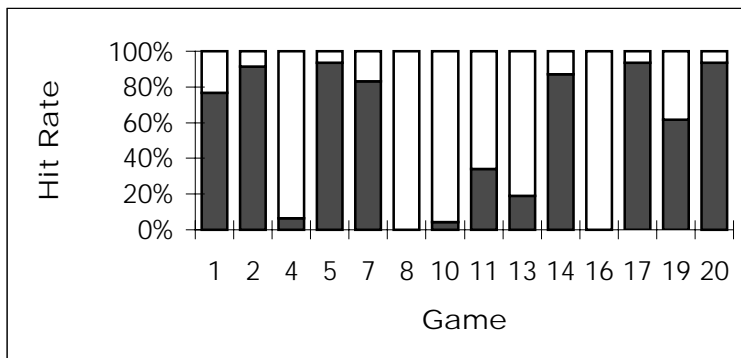
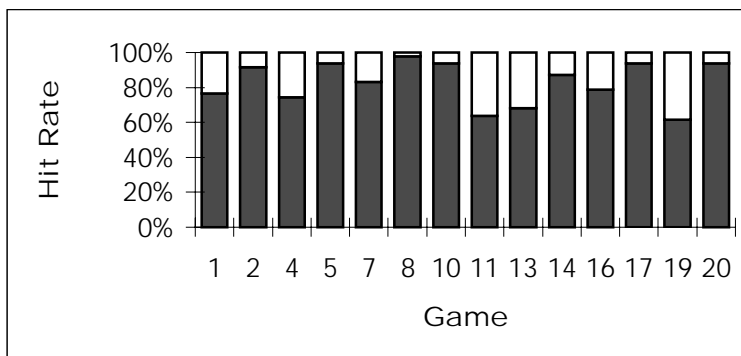
The following charts show the hit rate per game of the three Nash equilibrium selection rules of payoff dominance, risk-dominance, and security, compared to the bounded rationality rule of SW's level-1. The x-axis lists the 14 coordination games. On the y-axis is the hit-rate of a given rule per game. The hit rate is the percentage of players that chose the action in a given game that the person using a given rule without error would choose with probability 1.

#### A. Payoff-dominance rule



#### B. Risk-dominance rule



**C. Security selection rule****D. Level-1 bounded rationality**

## FIGURE IV ESTIMATION OF HOMOGENEOUS MODELS

For comparison, the log-likelihood for the homogeneous model with random behavior is -1,032.70.

### Models using Ranking-Based Evidence

Rule Used	$\nu$ estimate	$\delta$ estimate	log-likelihood
Payoff-dominance	$0.9 \times 10^{-7}$	$0.999 \times 10^7$	-976.14
Risk-dominance	0.005	168.4	-974.81
Security	0.051	11.55	-868.94
MBR	0.109	-----	-976.14
IMBR	0.018	47.66	-971.68

### Models using Prior-Based Evidence

Rule Used	$\nu$ estimate	log-likelihood
Payoff-dominance	0.015	-1004.84
Risk-dominance	0.018	-993.32
Security	0.052	-900.32
IMBR	0.018	-993.32

### Uniform Tremble Models

Rule Used	$\varepsilon$ estimate	log-likelihood
Payoff-dominance	0.71	-1028.97
Risk-dominance	0.59	-1021.15
Security	0.48	-976.59
IMBR	0.59	-1021.15

### Bounded Rational Models

Rule Used	$\nu$ estimate	log-likelihood
Level-1 bounded rational	0.105	<b>-676.365</b>
Level-2 bounded rational	0.031	-934.955
Worldly <sup>3</sup>	0.104 0.538	<b>-664.010</b>

<sup>3</sup> A homogenous evidence-based model allowing for a mix between the level-1 and the mutual best response evidences. The first parameter estimate is the estimated weight associated with level-1 evidence; the second parameter estimate is the estimated weight associated with the mutual best response evidence.

## FIGURE V

### ESTIMATION OF HETEROGENEOUS EVIDENCE-BASED MODELS

Estimation of evidence-based heterogeneity models with level-0, level-1, level-2, the Nash equilibrium selection principles of payoff-dominance (PD), risk-dominance (RD), and security, and the non-selection Nash equilibrium principle of mutual best-response (MBR). The first four columns allow only one Nash equilibrium evidence in addition to the boundedly rational evidence. In these columns, the unique Nash evidence enters the model both in the unique Nash rule and in the worldly rule. The last two columns allow for a combination of PD, RD, and security. In these two columns, the PD evidence is used in the worldly rule.

	MBR	PD	RD	Security	All selection principles	PD and Security
$v^1$	0.421	0.418	0.421	0.421	0.429	0.426
$v^2$	0.108	0.108	0.108	0.100	0.108	0.109
$v^3$	2.105	0.035	-----	-----	0.034	0.034
$v^4$	-----	-----	0.147E-03	-----	0.085	-----
$v^5$	-----	-----	-----	0.932E-04	0.039	0.039
$v^{W1}$	0.107	0.113	0.107	0.107	0.113	0.113
$v^{W2}$	0.177E-16	0.0023	0.646E-05	0.359E-04	0.003	0.003
$v^{W3}$	0.684	0.0153	0.137E-04	0.151E-04	0.016	0.016
$v^3 \delta^3$	-----	1.924	-----	-----	1.943	1.927
$v^4 \delta^4$	-----	-----	2.110	-----	0.290	-----
$v^5 \delta^5$	-----	-----	-----	2.104	0.390E-03	0.101E-02
$v^{W3} \delta^W$	0.684	0.634	0.682	0.683	0.604	0.608
$\alpha_0$	0.039	0.042	0.039	0.039	0.000	0.001
$\alpha_1$	0.295	0.295	0.295	0.295	0.289	0.297
$\alpha_2$	0.111	0.108	0.111	0.111	0.110	0.104
$\alpha_3$	0.061	0.052	-----	-----	0.054	0.052
$\alpha_4$	-----	-----	0.060	-----	0.000	-----
$\alpha_5$	-----	-----	-----	0.060	0.044	0.047
$\alpha_W$	0.495	0.502	0.495	0.495	0.503	0.501
<b>Log-likelihood</b>	<b>-586.29</b>	<b>-581.28</b>	<b>-586.29</b>	<b>-586.42</b>	<b>-577.98</b>	<b>-578.01</b>

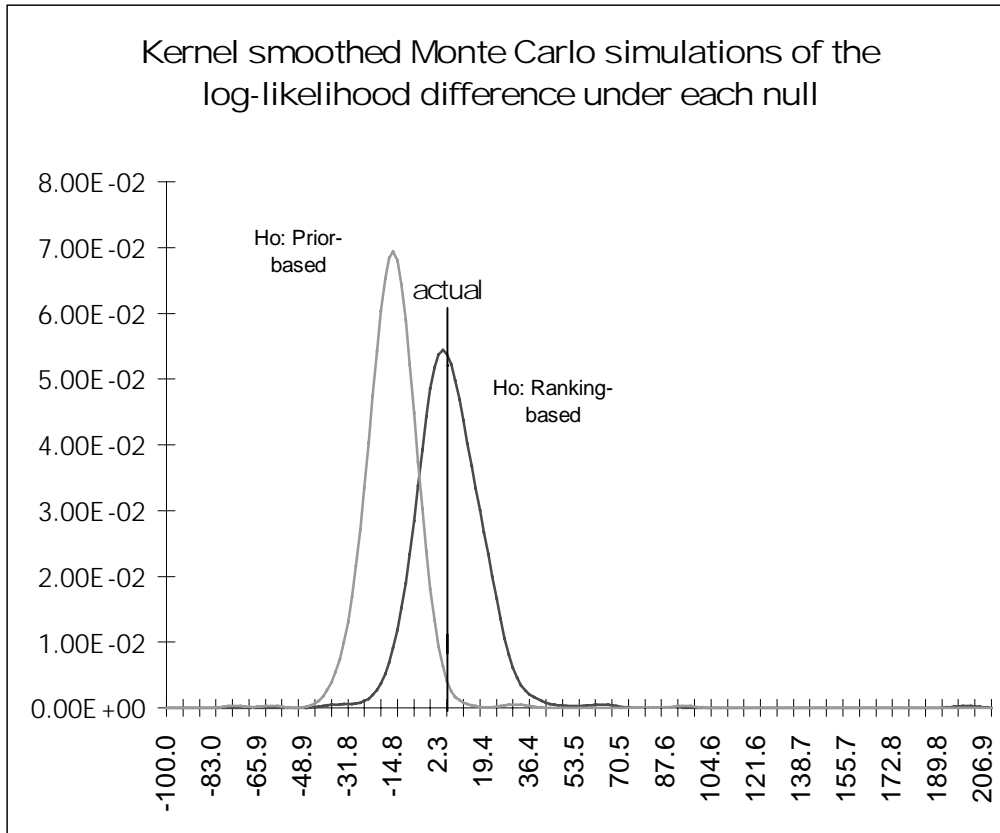
**Key:**  $v^i$  is the scaling parameter for evidence vector  $i$ ;  $\delta^i$  is the scalar subtracted from the lowest Nash evidence in evidence vector  $i$  to yield the evidence for non-equilibrium strategies in evidence vector  $i$ ;  $\alpha_i$  is the proportion of participants that can best be classified as using evidence vector  $i$  in a given model, where  $i=1$  is the level-1 evidence,  $i=2$  is the level-2 evidence,  $i=3$  is the payoff-dominance evidence,  $i=4$  is the risk-dominance evidence,  $i=5$  is the security Nash evidence,  $i=W1$  is the level-1 evidence component in the worldly evidence,  $i=W2$  is the level-2 evidence component in the worldly evidence,  $i=W3$  is the Nash evidence component in the worldly evidence.

**FIGURE VI Estimates and a 95% confidence interval for the heterogeneity model chosen to characterize initial period play.**

	<b>PD</b>	<b>95% Confidence interval</b>	
$v^1$	0.418	0.3203	0.6145
$v^2$	0.108	0.1	0.1694
$v^3$	0.035	3.89E-04	0.1195
$v^{w1}$	0.113	9.57E-02	0.1351
$v^{w2}$	0.0023	8.36E-96	9.85E-03
$v^{w3}$	0.0153	5.78E-03	2.99E-02
$v_{\delta}^{3\delta^3}$	1.951	0.3400	50.000
$v_{\delta}^{w3\delta^w}$	0.664	0.3433	0.9987
$\alpha_0$	0.042	1.04E-24	0.1069
$\alpha_1$	0.295	0.1582	0.4418
$\alpha_2$	0.108	2.21E-02	0.2065
$\alpha_3$	0.052	6.94E-11	0.1296
$\alpha_w$	0.502	0.3492	0.6552
<b>Log-likelihood</b>	<b>-581.28</b>		



**Figure VII Monte Carlo Simulations of the log-likelihood difference between the ranking-based and prior-based specifications under the null hypotheses that each is true. Kernel smoothing of the densities was done using the plug-in method.**



## Appendix A Instructions to the Participants in the 3×3 Experiment

Introduce self and helpers. Say how everyone was recruited.

Explain: "This session will be tape-recorded for scientific purposes."

During the experiment, you will be making choices using the computer mouse and keyboard. You may reposition the mouse pad so it is comfortable to you. Your mouse cursor should move when you slide the mouse on the pad. If not, please raise your hand.

Click page down now.

I will read the instructions that appear on your computer screen aloud. After going over the instructions, we will administer a quiz which you **MUST PASS** in order to continue. Hence, it is very important that you pay close attention.

### WELCOME

This is an experiment about economic decision making. If you follow the instructions carefully you might earn a considerable amount of money. This money will be paid at the end of the experiment in private and in cash.

Wave cash.

It is important that during the experiment you remain **SILENT**. If you have any questions, or need assistance of any kind, **RAISE YOUR HAND** but **DO NOT SPEAK**. One of the experiment administrators will come to you and you may whisper your question to him. If you talk, laugh, or exclaim out loud, you will be asked to leave and will not be paid. We expect and appreciate your cooperation.

This experiment will be conducted in two stages. In Stage I, you and all other participants in this room will each make a number of decisions. Based on your choices and the choices of the other participants, you will earn **TOKENS**. In Stage II, you will have the opportunity to receive **DOLLARS** based on the number of **TOKENS** you earned in Stage I. We describe Stage II first, so you can better understand how the number of **TOKENS** that you earn in Stage I affect the number of **DOLLARS** that you receive at the end of the experiment.

Click on the "page down" icon located below to display the next page.

PG 1 \*\*\*\*\*

You should be on page 2. If not, please raise your hand.

### DESCRIPTION OF STAGE II

At the beginning of Stage II, you will have earned between 0 and 100 **TOKENS** for each decision that you made in Stage I. The total number of **DOLLARS** that you receive at the end of the experiment will depend partly on the number of **TOKENS** earned in Stage I and partly on chance.

Chance enters into the experiment in Stage II. You will then be asked to roll three ten sided dice, once for each decision. Since there are 20 decisions in Stage I, you will roll the dice 20 times. By comparing the number of **TOKENS** that you earned for each decision and the corresponding roll of dice, we compute the amount of **DOLLARS** earned by you.

Specifically, three ten sided dice determine a three digit number on each roll. The first die will count tens, the second die will count ones, and the third die will count tenths. For example, if you rolled 5, 4, and 8, the dice number would be 54.8.

All the possible numbers from 00.0 to 99.9 are equally likely.

By comparing the three-digit number from the dice-roll to the number of TOKENS for each decision, we determine how many DOLLARS you receive. If the number of TOKENS for a decision is GREATER THAN the dice number, then you receive \$2.00 for that decision. If the number of TOKENS is LESS THAN OR EQUAL TO the dice number, then you get \$0 for that decision.

For example, if you earn 80 TOKENS for your seventh decision, and on the seventh roll your number was 47.6 in Stage II, since 80 is GREATER THAN 47.6, you would receive \$2.00 for that decision. This comparison is repeated 20 times to compute the total number of dollars you receive.

Note that if you earn 80 TOKENS for a decision, you have an 80% chance of winning the \$2.00 prize and if you earn 20 TOKENS for a decision you have a 20% chance of winning the \$2.00 prize.

HENCE, THE MORE TOKENS YOU EARN, THE GREATER WILL BE YOUR CHANCE OF WINNING THE \$2.00 PRIZE FOR ANY GIVEN DECISION.

Click page down now.

PG 2 \*\*\*\*\*

You should be on page 3. If not, please raise your hand.

## STAGE I

During Stage I, you and all other participants will make 20 decisions.

Each decision you face will be described by a MATRIX, consisting of nine numbers arranged in three rows and three columns as displayed at the front of the room now.

Please look up.

Point to the screen.

The rows indicate your possible choices; the columns indicate the possible choices of all other participants in this room. The numbers in the MATRIX, along with your choices and the choices of all OTHER participants in this session, determine your TOKEN earnings for each decision.

Each participant will receive exactly the same 20 matrices and will be asked to make decisions based on the same amount of information. Each of the 20 matrices will differ only by the nine numbers. Your decision on a matrix will be compared to the decisions of the other participants on that same matrix. Depending on your decision and those of all the other participants, you will earn TOKENS for each matrix.

**STOP! DO NOT PAGE DOWN UNTIL TOLD TO DO SO.**

PG 3

\*\*\*\*\*

Using the overhead projector in the front of the room, I will now demonstrate exactly how your token earnings will be computed. Please look at the overhead as I am working the numerical example.

First we write down three columns with headings "A," "B" and "C." Next we write under the headings the percentage of the other participants in the room choosing A, B, and C, respectively.

For example, suppose 20% chose A, 30% chose B, and 50% chose C.

Suppose you chose row A. Then we write down the numbers of row A underneath the percentages. We then multiply each column as follows:

[Point to the screen and do the multiplication of columns]

Finally, we add up the results:

[Point to the screen and do the addition]

	A	B	C	
<b>% of Others' Choices :</b>	<b>20%</b>	<b>30%</b>	<b>50%</b>	<b>100%</b>
<b>Your Row Choice:</b> A	<b>30</b>	<b>100</b>	<b>50</b>	
<b>Tokens Earned:</b>	<b>6</b>	<b>30</b>	<b>25</b>	<b>61</b>

That means your payoff for choosing A given the other participants' percentages would be **61** tokens, giving you a **61%** chance of winning \$2.00.

The payoffs you would have earned for the other row choices are calculated the same way. I will quickly work out the payoffs for the other two rows.

	A	B	C	
<b>% of Others' Choices :</b>	<b>20%</b>	<b>30%</b>	<b>50%</b>	<b>100%</b>
<b>Your Row Choice:</b> B	<b>40</b>	<b>0</b>	<b>90</b>	
<b>Tokens Earned:</b>	<b>8</b>	<b>0</b>	<b>45</b>	<b>53</b>

That means your payoff for choosing B given the other participants' percentages would be **53** tokens, giving you a **53%** chance of winning \$2.00.

	A	B	C	
<b>% of Others' Choices :</b>	<b>20%</b>	<b>30%</b>	<b>50%</b>	<b>100%</b>
<b>Your Row Choice:</b> C	<b>50</b>	<b>75</b>	<b>29</b>	
<b>Tokens Earned:</b>	<b>10</b>	<b>22.5</b>	<b>14.5</b>	<b>47</b>

That means your payoff for choosing C given the other participants' percentages would be **47** tokens, giving you a **47%** chance of winning \$2.00.

If you have any questions, please raise your hand

During the experiment you will have a computer interface available to you that will make these calculations for you. You will be able to enter hypotheses about the percentages of others making each choice. The computer can then calculate hypothetical payoffs for you.

We will now demonstrate this feature. Exit the Instructions page by clicking on QUIT.

## THE DEMO SCREEN

After you quit the Instructions page, the screen should be blue and have the word "Participant" and your assigned number on top. On the line just below the top there are two buttons labeled INSTRUCTIONS and RECORD. Anytime during the experiment you will be able to view these instructions by clicking on INSTRUCTIONS. The "RECORD" button will be used only at the end of the experiment to display your token payoffs.

Notice the button on the right of the screen with the word "DEMO" on it. During the actual experiment, there will be 20 such buttons on the right of the screen, where the demo button is now, each labeled by a number corresponding to a different matrix. Click on DEMO now to see the matrix that is now displayed on the overhead. The row labels A, B, and C correspond to your three possible choices whereas the column labels A, B, and C correspond to the possible choices of others.

The bottom panel contains, from left to right, three white boxes labeled A, B, C, one pale blue box labeled "Total is," an orange button with the word "CALCULATOR" inside, and a yellow box labeled "clock", in that order.

The three white boxes are where you will enter your hypotheses on the percentages of other participants making each choice. To enter a number, use your mouse to click on a white box then enter a number in it using the number keys in the top row of your keyboard in front of you. To change a number in a box, click on the box and use the "Back-Space" key to delete the previous number before entering a new one. You may move from one white box to another by pressing the Tab button to move right and Shift-Tab to move left. Or you may simply use the mouse to move from one box to the next by moving the cursor to the box you wish to enter a number in and clicking the left mouse button. Each time you enter or change a number in a white box, the total of the three numbers is computed and displayed in the "Total" box. When you enter a number in one of the white hypothesis boxes, the pale blue box displaying the total in the three hypothesis boxes will change color to red and will remain red until the total is 100. When the total of the three white boxes is EXACTLY 100, the box displaying the total will change color to green.

In our example, the actual choice percentages are 20, 30, and 50. To enter "20" for the percentage of the other participants who chose A, first move the mouse cursor to the white box preceded by the letter A. Click on that box using the left mouse button. Then type in the number 20 using the number keys at the top of the keyboard. If you make a typo, use the Backspace key to erase the number and enter the correct number.

Notice that as soon as you typed 20, the Total box turned red and 20 appeared in it. That number is the current total in the three white boxes. It is red because the total is not 100. If there are any questions, please raise your hand.

Enter the numbers 30 and 50 in the boxes corresponding to percentages of other participants choosing rows B and C.

[Repeat, then pause]

[Instruct experiment administrators to walk around and be prepared to help]

If you do not have 20, 30, and 50 in the white boxes, please raise your hand.

Notice that the number in the Total box is 100. Therefore, the Total box is now green and the calculator button is enabled. If your Total box is NOT green, please raise your hand.

When you click on CALCULATOR, the computer will calculate your hypothetical token earnings for each row. Click on CALCULATOR now.

[Pause. Administrators will go around making sure everyone is keeping up]

[Point to the board]

Three numbers will appear to the right of the matrix under the word PAYOFF, indicating your hypothetical token earnings for each of the possible choices. The numbers to the right of the matrix are **61, 53, and 47**, corresponding to your payoffs for each row exactly as we computed on the overhead. If you had chosen

row A, then your token earnings would have been 61, which is the maximum you could have earned given the percentages of others choosing each row. On the other hand, if you had chosen row C, you would have earned 47, which is the minimum you could have earned given the percentage of others choosing each row.

Suppose the actual choice percentages are as in this example (20, 30, and 50) but your hypothesis was that all other participants would choose C. Hence, you entered (0, 0, 100) as your hypothesis. You will do so now. Delete the existing numbers in the white boxes using the Backspace key. Then enter 0, 0, and 100 in the appropriate boxes.

[Pause]

The Total box should be green and have 100 in it. If not, please raise your hand.

[Pause]

Notice that the hypothetical payoffs to the right of the matrix have NOT changed. The hypothetical payoffs will NOT match your new hypothesis UNLESS you click on CALCULATOR. Click on CALCULATOR now.

[Pause]

Notice that the calculator reproduced column C as the hypothetical payoffs, indicating that choice B would give you the largest payoff (i.e. 90). In reality, entering this hypothesis cannot change anyone else's ACTUAL choices. Therefore, given the actual choices of everyone else your payoff from choosing B would be ONLY 53, not 90. The point is that **the more your hypothesis differ from the actual percentage of other participants, the more the computed hypothetical payoffs will differ from the actual token earnings, row by row.**

[Draw attention to overhead: column C versus payoffs.]

### **So then, what are the hypothesis boxes good for?**

1. By entering different hypotheses and calculating hypothetical payoffs to these hypotheses, you can explore how the actual choices (including your own) will affect your token earnings. In other words, you can answer "what if" questions.
2. You can enter your best guess about the percentage of others choosing each row and use the computed token earnings to guide your choice.

If you have any questions, please raise your hand.

**CAUTION: The numbers used in this example were selected arbitrarily and are in no way intended to suggest how participants might respond.**

\*\*\*\*\*

We will now demonstrate how you make a choice. Move the mouse cursor to the row you wish to choose in the yellow matrix and click the left mouse button. The row you clicked on will change color to an orange/pink color indicating your choice.

Make a choice NOW by clicking on ANY row of the yellow matrix.

Change your choice NOW by clicking on ANY OTHER row of the matrix.

Notice that it is not necessary for you to do any hypothetical calculations before making a choice.

Notice that once you made a choice, the DEMO button turned green. This indicates that you have made a decision for that matrix. Also, on the bottom of the screen it should now say "0 Matrices remaining." These two devices will prove very important later when you have more than one matrix, to remind you how many decisions you have left and for which matrices.

We will now pass out the quiz. Make sure you put your participant number on the quiz. Your participant number is located on the very top of your screen. You must answer all questions correctly to continue. Please read the questions carefully, follow the directions exactly, and raise your hand if you need help. Once you are done with the quiz, raise your hand. An experiment administrator will come by to check your answers.

\*\*\*\*\*

### QUIZ

Participant No. \_\_\_\_\_

Follow the instructions. Fill in the blanks when appropriate. Use the computer screen and keyboard to answer the questions. Raise your hand if you need assistance. Raise your hand when you have completed the quiz to have your questions checked.

1. Consider the Demo matrix displayed on the screen. Suppose of the other participants in the room, 40 % chose A, 30% chose B, and 30% chose C. What would be your token earnings if you had chosen B?

\_\_\_\_\_43\_\_\_\_\_

2. Given the choice percentages in question 1, what would be your token earnings if you had chosen C?

\_\_\_\_\_51.2\_\_\_\_\_

3. Next click on the **INSTRUCTIONS** button. Then click on **QUIT INSTRUCTIONS** and restore the Main Screen.

4. Suppose that in the next period the choice percentages of the other players in the room are the same as in question 1 EXCEPT that everyone who previously chose C now switches to B. What would be your token earnings if you had chosen A?

\_\_\_\_\_72\_\_\_\_\_

5. In question 4, how would your choice of C translate into dollars? (Circle one)

a) \$65 cash.

b) 65% of \$2 = \$1.04.

c) 2% chance of winning \$65.

d) 65% chance of winning \$2.

6. (a) Choose row B. What color is row B now?

\_\_\_\_\_pink\_\_\_\_\_

(b) Choose row C. What color is row B now?

\_\_\_\_\_yellow\_\_\_\_\_

7. Can you change your actual payoff from choosing C simply by entering some hypothesis? (Circle one)

- (a) Yes.                      (b) No.

8. Assume that the other participants in the room entered choices as in question 4 (still displayed on your screen). However, you “guessed” that everybody else chose C.

(a) What is the **hypothetical** payoff to choosing B given your “guess” (the payoff you would see on the screen if you entered the hypothesis that everyone else chose C)? \_\_\_\_90\_\_\_\_

(b) What would be your **actual** payoff to choosing B given the other players’ choices? \_\_\_\_16\_\_\_\_

9. Which is more accurate predictor of your actual token earnings? (circle one)

a) The displayed payoffs when my preferred row has the largest possible payoff (after trying different hypotheses).

b) The displayed payoffs when my preferred row has the smallest possible payoff (after trying different hypotheses).

c) The displayed payoffs when my hypothesis is the same as the actual choice percentages of the other participants in the room.

10. The farther my hypothesis is from the actual choice percentages of the other participants in the room, the (circle one) CLOSER / FARTHER will be my actual payoffs from the hypothetical payoffs displayed on the screen.

11. My actual token earnings will be determined by (circle one)

(a) the actual percentage of all other participants making each choice.

(b) the hypothesis I enter in the hypothesis boxes.

(c) my choice.

(d) both a and c.

12. What happens if the numbers in the hypothesis boxes do not add up to 100?

(a) The calculator is (circle one) ENABLED / DISABLED.

(b) The TOTAL box is (circle one) GREEN / RED / LIGHT BLUE.

\*\*\*\*\*

(Once done with the quiz):

In the main part of the experiment you will have to make 20 decisions in a format similar to the example we just went through. You will have a total of 30 minutes, which is about 1½ minutes for each decision. To give you some practice in making decisions under a time constraint, you will now have a 4½ - minute timed practice session. You will have 4½ minutes to make choices and practice making hypothetical calculations on 3 matrices. The clock at the bottom right of your screen will count down from 4 minutes and 30 seconds to 0 minutes and 0 seconds. A 60-Seconds warning will appear when only a minute remains for you to make decisions. Otherwise the screen will look exactly as during the Demo session. You should practice making hypothetical calculations, making choices, and revising choices.

To select a Matrix, click on the corresponding number in the Active Matrix bar on the right side of the screen where the Demo button is now. The corresponding matrix will appear and the letters in the



Active Matrix button you clicked will be red as long as you are visiting that matrix. You may return to matrices that you previously selected. When you do this, the screen will re-appear exactly as you left it.

Once you make a decision, the Matrix Button corresponding to that matrix will change color to green. This will remind you which of the matrices you have made choices for and for which you **MUST** still make a choice. Also, on the bottom of the screen it should now say how many matrices remain to make a choice for. These two devices remind you how many decisions you have left and for which matrices.

Enter "555" in the password box on top of your screen. Then click on the PRACTICE button. Begin as soon as the 3 active matrix buttons appear to the right of the screen. A matrix will appear only after you click on a matrix button.

Return to INSTRUCTIONS by clicking on the INSTRUCTIONS button at the top of your screen.

Click page down now to see a summary.

## SUMMARY

**To Select A Matrix:** click on a button in the Active Matrix bar. If a button is white, you still need to make a decision for that Matrix.

**To Enter or Change Hypothesis:** click inside a white box under the Matrix, use the keyboard to enter a number. If there is already a number in that white box and you wish to change it, use the Backspace key to delete the current number, then enter a new one. Remember that all hypotheses are in terms of percentages and hence must sum up to 100.

Remember: The hypothetical payoffs will NOT match your hypothesis UNLESS you click on CALCULATOR.

**To calculate hypothetical payoffs:** Once the white boxes contain your hypothesis, click on CALCULATOR.

**To make a choice:** Click on the desired row of the matrix. Once you make a choice, the matrix button for that matrix will turn green. Also, the message at the bottom of your screen telling you how many matrices are left will be updated. You must make a choice for each Matrix to earn any TOKENS.

**To Review Instructions:** click on INSTRUCTIONS. To return to the main screen, click on "QUIT INSTRUCTIONS" and move the mouse a bit.

### Warning:

**If you fail to make a choice for any matrix, you will forfeit your entire earnings for all the matrices.**

If you have any questions please raise your hand.

Click Page Down now.

## BEGINNING STAGE I

This concludes the instruction phase.

Every participant in this room will be presented with 20 Matrices just like the examples we went through. Each Matrix will also be displayed at the front of the room.

The experiment will take 30 minutes. A red warning-- "60 seconds remaining"-- will appear when there are 60 seconds left.

Just before the main experiment starts, your screen will be blank except for the words "WAIT TO PROCEED". You may begin as soon as your screen displays the Matrix Buttons. Your screen will be blank until you click on one of the Matrix buttons on the right side of the main screen.

You may proceed without further signals from me until you have made your 20 decisions. After that, you may review the Instructions or continue to review your choices. However, you may not talk nor stand up.

Quit the Instructions page again.

Enter "777" in the password box on top of your screen.

[Repeat]

Then click on the BEGIN STAGE I button. Once the buttons appear to the right of your screen you may begin by clicking on any of the Matrix buttons.

After phase I is over:

You are now looking at the record screen. The first column displays numbers from 1 to 20 corresponding to the 20 different decisions you made. The second column has your choice. The next column has your token earnings for that decision given the other participants' choices. The next three columns have the number of other participants making each choice.

We will now proceed to Stage II, where you will roll the dice to translate your token earnings into DOLLARS.

Make sure "777" is in the password box on top of your screen. Then click on the ROLL DICE button.

You are now looking at three dice. These dice are as described in the INSTRUCTIONS earlier. Namely, you will roll these dice 20 times, once for each decision. Every time your dice number is LOWER than your token earnings for the corresponding decision, you win \$2.

To the right of the dice you will see four column titles "Decision", "TOKENS", "Roll", and "Pay". The first column has numbers from 1 to 20 corresponding to the 20 matrices. The second column has your token earnings for each of the matrices. The third column will have the dice rolls for each matrix and will be updated each time you roll the dice. The last row will indicate every matrix for which you won \$2, i.e., your TOKENS earned exceed your dice roll for that matrix. At the bottom of the dice screen you will see the label "Total pay." To the right of "Total pay" will be your total earnings for the experiment. These will be updated after each dice roll until you are done with all 20 dice rolls. Again, for each game in which your token earnings exceed the dice roll, you will win \$2.00.

Begin rolling NOW! Repeat 20 times.

[Pause and wait for people to roll.]

Once you rolled the dice 20 times, quit the dice screen and click on the RECORD button at the top of your screen. The record button will give you a summary of your choices, the choices of others, your token earnings, your dice rolls, and your pay.

[Pass out receipts and post-experiment questionnaire]

Please make sure you are looking at the RECORD screen. If not, quit the dice screen and click on the RECORD button at the top of your screen.

Use a pencil to write your payoffs down on the receipt slip, which we are now passing around. Make sure you sign the receipt and fill out the relevant information, including your social security number. We are also passing a post-experiment questionnaire. Please take a moment to answer this questionnaire to help us improve future experiments.

(Then, when everyone is done) You will now be called one by one to receive your payments. Your payments will be in a closed envelope. Bring the receipt on which you have written down your payoffs with you when are called to receive your payments. Verify that the amount on your receipt matches the amount written on the envelope. Then count your money. You will then leave the room. Please maintain silence during the payment phase.

Thank for your time.