Problema 8:

(a) Asintoticamente $(1+o(1))^{w(1)} = 1 \Rightarrow (1+0)^{\infty} = 1^{\infty} = e \neq 1$

$\Rightarrow$ Falso

(b) Si $f(n) = \frac{(n+2)\cdot n}{2} \Rightarrow f(n) \in \Theta(n^2)$

$$\lim_{n\to\infty} \frac{f(n)}{n^2} \overset{!}{=} \lim_{n\to\infty} \frac{\frac{(n+2)n}{2}}{n^2} \overset{!}{=} \lim_{n\to\infty} \frac{n+1}{2n} \overset{!}{=} \lim_{n\to\infty} \frac{1}{2} = 1/2 \Rightarrow f(n) \in \Theta(n^2) \Rightarrow \text{Cierto}$$

(c) Si $f(n) = \frac{(n+2)\cdot n}{2} \Rightarrow f(n) \in \Theta(n^3)$

$$\lim_{n\to\infty} \frac{f(n)}{n^3} \overset{!}{=} \lim_{n\to\infty} \frac{\frac{(n+2)n}{2}}{n^3} \overset{!}{=} \lim_{n\to\infty} \frac{n+1}{3n^2} \overset{!}{=} \lim_{n\to\infty} \frac{1}{6n} = 0 \Rightarrow f(n) \in o(n^3) \Rightarrow \text{Falso}$$

(d) $n^{1.1} \in O(n(\lg n)^2)$

$$\lim_{n\to\infty} \frac{n^{1.1}}{n(\lg n)^2} = \lim_{n\to\infty} \frac{n^{0.1}}{(\lg n)^2} = \lim_{n\to\infty} \frac{n^{0.1}}{\left(\frac{\ln(n)}{\ln(10)}\right)^2} \overset{!}{=} \lim_{n\to\infty} \frac{0.1\cdot n^{-0.9}}{\frac{2\ln(n)}{n\ln^2(10)}} = \lim_{n\to\infty} \frac{0.1\cdot\ln^2(10)\cdot n^{0.1}}{2\ln(n)}$$

$$\overset{!}{=} \lim_{n\to\infty} \frac{0.1\cdot\ln^2(10)\cdot 0.1\cdot n^{-0.9}}{2\cdot 1/n} = \lim_{n\to\infty} \frac{\cdots n}{2} = \infty \Rightarrow n^{1.1} \in w(n(\lg n))^2$$

$\Rightarrow$ Falso

(e) $n^{0.01} \in w((\lg(n))^2)$

$$\lim_{n\to\infty} \frac{n^{0.01}}{\left(\frac{\ln(n)}{\ln(10)}\right)^2} \overset{!}{=} \lim \frac{0{,}01\cdot n^{-0{,}99}}{\frac{2\ln(n)}{n\ln^2(10)}} = \lim_{n\to\infty} \frac{0{,}01\cdot\ln^2(10)\cdot n^{0{,}01}}{2\ln(n)} \overset{!}{=} \lim_{n\to\infty} \frac{0{,}01\cdot\ln^2(10)\cdot 0{,}01\cdot n^{-0{,}99}}{2/n}$$

$$= \infty \Rightarrow n^{0.01} \in w((\lg n)^2) \Rightarrow \text{Cierto}$$