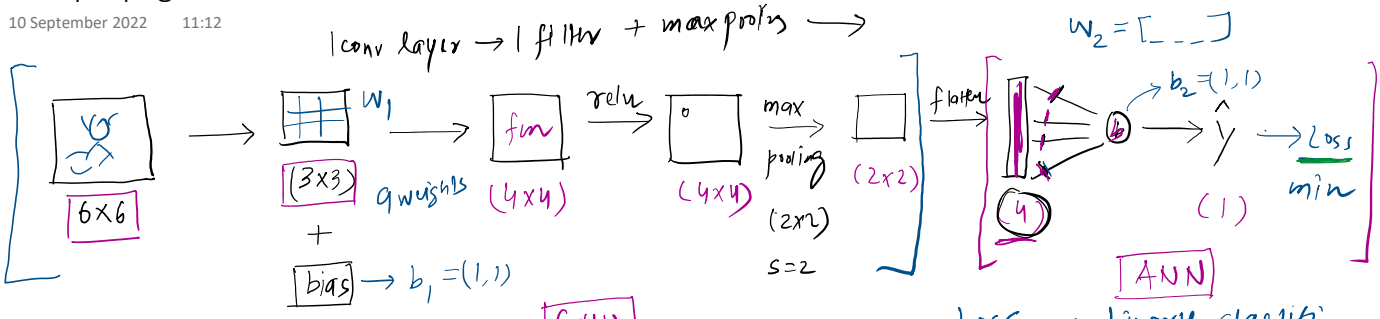
Backpropagation in CNN

10 September 2022 11:12



Trainable Parameters

$W_1 = (3,3)$ $W_2 = (1,4)$ = 15 trainable parameters

$b_1 = (1,1)$ $b_2 = (1,1)$

Logical Flow

Loss → binary classif

$$L = -y_i \log(\hat{y}_i) - (1-y_i)\log(1-\hat{y}_i)$$

$A_2 = \hat{y}$

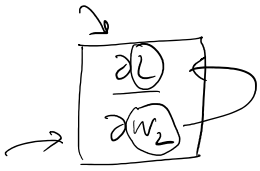
Forward Prop

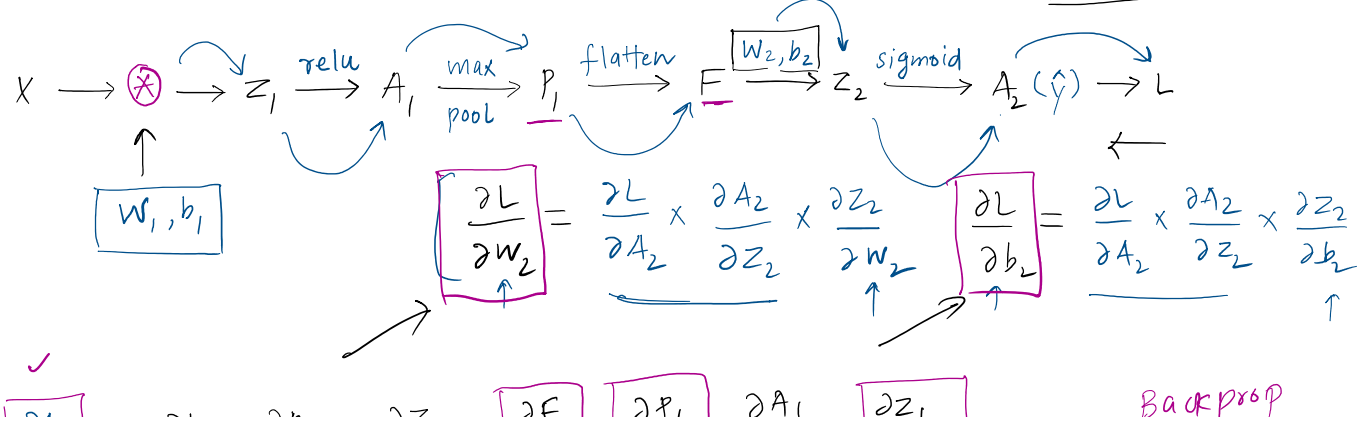
$$Z_1 = Conv(X, W_1) + b_1$$
$$A_1 = relu(Z_1)$$
$$P_1 = maxpool(A_1)$$
$$F = flatten(P_1)$$
$$Z_2 = W_2 F + b_2$$
$$A_2 = \sigma(Z_2)$$

Gradient Descent

$$W_1 = W_1 - \eta \frac{\partial L}{\partial W_1} \quad \checkmark$$
$$W_2 = W_2 - \eta \frac{\partial L}{\partial W_2} \quad \checkmark$$
$$b_1 = b_1 - \eta \frac{\partial L}{\partial b_1} \quad \checkmark$$
$$b_2 = b_2 - \eta \frac{\partial L}{\partial b_2} \quad \checkmark$$

Loss is minimized

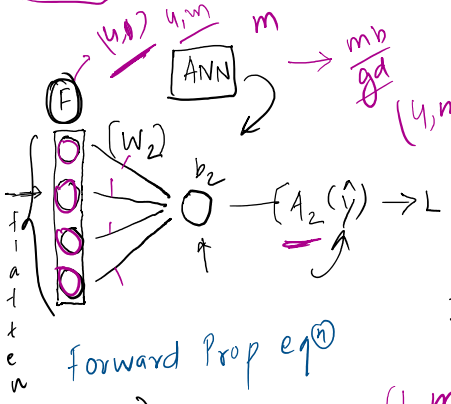




$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial A_2} \times \frac{\partial A_2}{\partial Z_2} \times \frac{\partial Z_2}{\partial F} \times \frac{\partial F}{\partial P_1} \times \frac{\partial P_1}{\partial A_1} \times \frac{\partial A_1}{\partial Z_1} \times \frac{\partial Z_1}{\partial w_1}$$

$$\frac{\partial L}{\partial b_1} = \frac{\partial L}{\partial A_2} \times \frac{\partial A_2}{\partial Z_2} \times \frac{\partial Z_2}{\partial F} \times \frac{\partial F}{\partial P_1} \times \frac{\partial P_1}{\partial A_1} \times \frac{\partial A_1}{\partial Z_1} \times \frac{\partial Z_1}{\partial b_1}$$

Backprop
- → Convolution
- → Flatten
- → Max pooling



$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial A_2} \frac{\partial A_2}{\partial Z_2} \frac{\partial Z_2}{\partial w_2} \qquad \frac{\partial L}{\partial b_2} = \frac{\partial L}{\partial A_2} \frac{\partial A_2}{\partial Z_2} \frac{\partial Z_2}{\partial b_2}$$

Forward Prop eqn

$$Z_2 = W_2 F + b_2$$
$$A_2 = \sigma(Z_2)$$

$$\frac{\partial L}{\partial a_2} = \frac{\partial}{\partial a_2}\left[-y_i \log(a_2) - (1-y_i)\log(1-a_2)\right]$$

$$(1,m) \quad Y = (1,m) \qquad = -\frac{y_i}{a_2} + \frac{(1-y_i)}{(1-a_2)} = \frac{-y_i(1-a_2) + a_2(1-y_i)}{a_2(1-a_2)}$$

$$\frac{\partial L}{\partial a_2} = \frac{-y_i + \cancel{y_i a_2} + a_2 - \cancel{a_2 y_i}}{a_2(1-a_2)} = \frac{(a_2 - y_i)}{a_2(1-a_2)}$$

$$\frac{\partial A_2}{\partial Z_2} = \sigma(Z_2)[1-\sigma(Z_2)] = a_2[1-a_2]$$

$$\frac{\partial Z_2}{\partial w_2} = F \qquad \frac{\partial Z_2}{\partial b_2} = 1$$

W_2 update

Shape =

$$\frac{\partial L}{\partial w_2} = \frac{(a_2 - y_i)}{\cancel{a_2(1-a_2)}} \times \cancel{a_2(1-a_2)} \times F = (a_2 - y_i) F = (A_2 - Y) F^T$$

(1,4) (4,1) (1,1) (1,1) (1,1) (1,4) (1,4)

$$\frac{\partial L}{\partial b_2} = \frac{(a_2 - y_i)}{\cancel{a_2(1-a_2)}} \cancel{a_2(1-a_2)} \times 1 = (A_2 - Y)$$

m images

$$\frac{\partial L}{\partial w_2} = (A_2 - Y) F^T \qquad \frac{\partial L}{\partial b_2} = (A_2 - Y)$$

$\frac{\partial L}{\partial w_2} = (A_2 - Y) F'$ | $\frac{\partial L}{\partial b_2} =$

$(1,m) - (1,m)$

$(1,m)\ (m,4) \rightarrow (1,4)$

1 image

batch of image

$W_2 \rightarrow (1,4)$

Backpropogation in CNN

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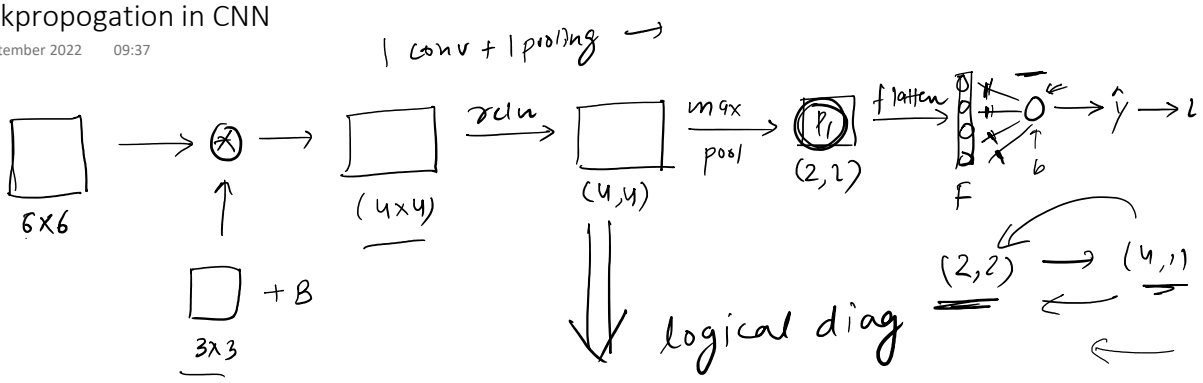


Forward Prop

$Z_1 = conv(X, W_1) + b_1$

$A_1 = relu(Z_1)$

$P_1 = maxpool(A_1)$

$F = flatten(P_1)$

flatten

$Z_2 = W_2 F + b_2$

$A_2 = \sigma(Z_2)$

$L = \frac{1}{m} \sum_{i=1}^{m} [-y_i \log(A_2) - (1-y_i)\log(1-A_2)]$

$(A_2 - Y)$

$$\frac{\partial L}{\partial W_1} = \frac{\partial L}{\partial A_2} \times \frac{\partial A_2}{\partial Z_2} \cdot \frac{\partial Z_2}{\partial F} \cdot \frac{\partial F}{\partial P_1} \cdot \frac{\partial P_1}{\partial A_1} \cdot \frac{\partial A_1}{\partial Z_1} \cdot \frac{\partial Z_1}{\partial W_1}$$

5

$$\frac{\partial L}{\partial b_1} = \frac{\partial L}{\partial A_2} \cdot \frac{\partial A_2}{\partial Z_2} \cdot \frac{\partial Z_2}{\partial F} \cdot \frac{\partial F}{\partial P_1} \cdot \frac{\partial P_1}{\partial A_1} \cdot \frac{\partial A_1}{\partial Z_1} \cdot \frac{\partial Z_1}{\partial b_1}$$

1 2 3 4 6

reshape(P_1.shape)

6 derivatives

$\left[\frac{\partial Z_2}{\partial F}\right] = W_2 \rightarrow$

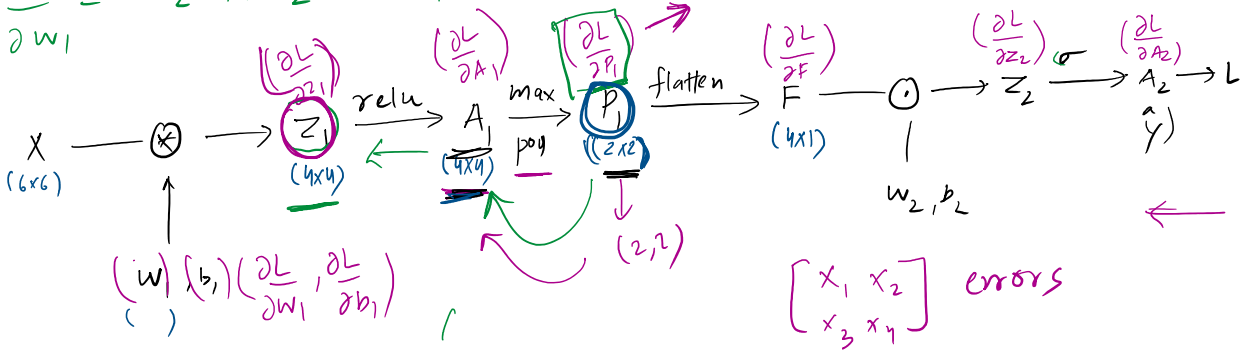
Shape ? → (F)

$\frac{\partial F}{\partial P_1}$ no trainable parameters

$conv\left(X, \frac{\partial L}{\partial Z_1}\right)$

$\frac{\partial L}{\partial W_1} = (A_2 - Y) W_2 . reshape(P_1.shape)$

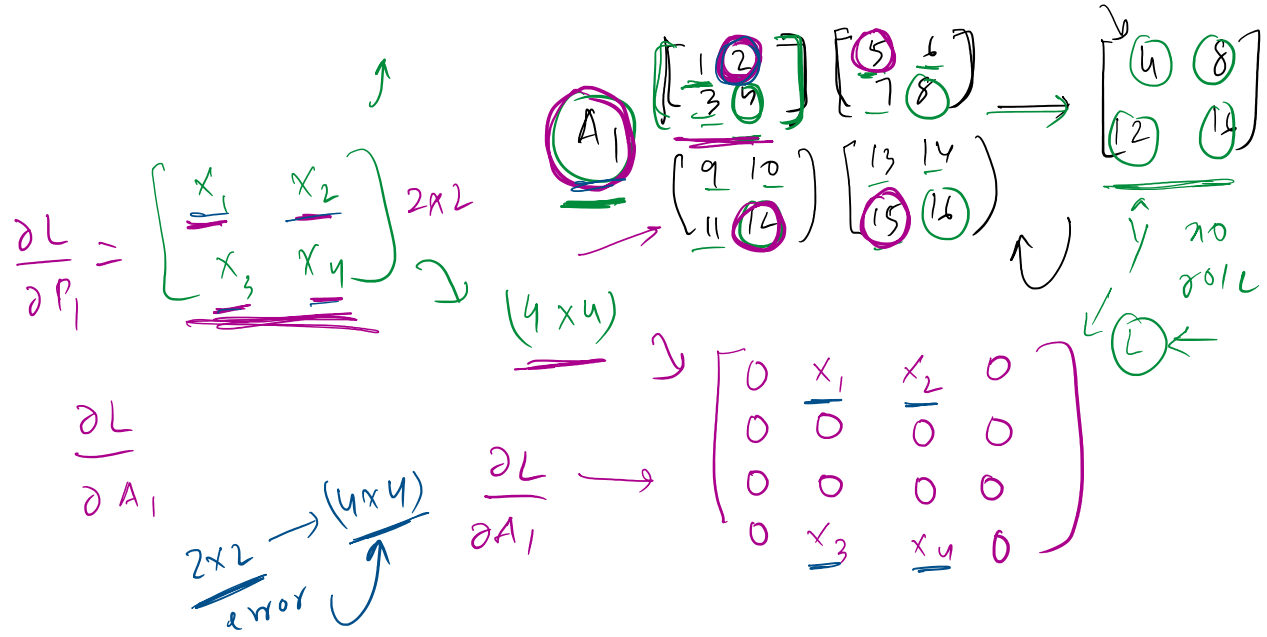
$\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$

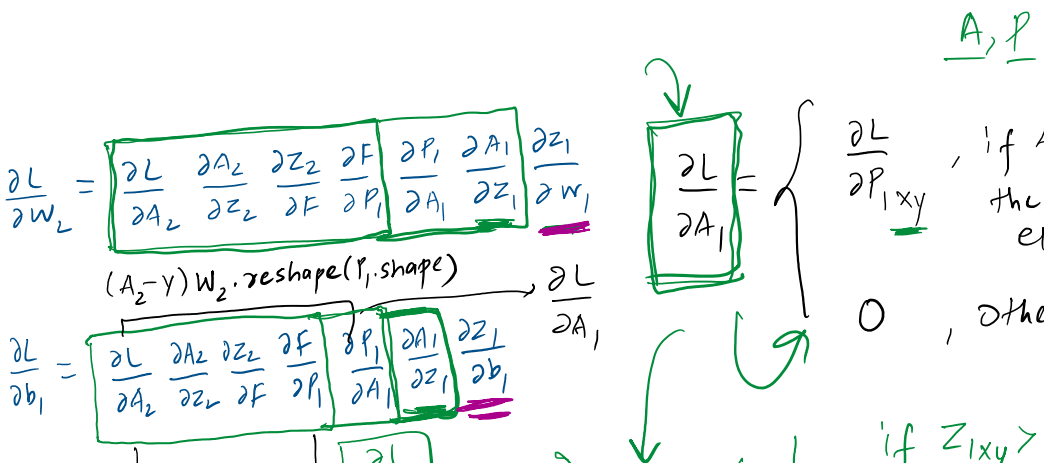


$\frac{\partial L}{\partial A_1} = (4,4)$

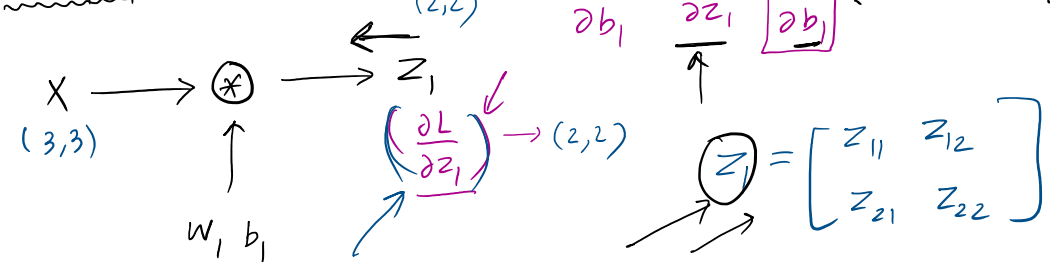
$\frac{\partial L}{\partial A_1} = (4,4)$

Flatten → no trainable parameters





Backprop on Convolution



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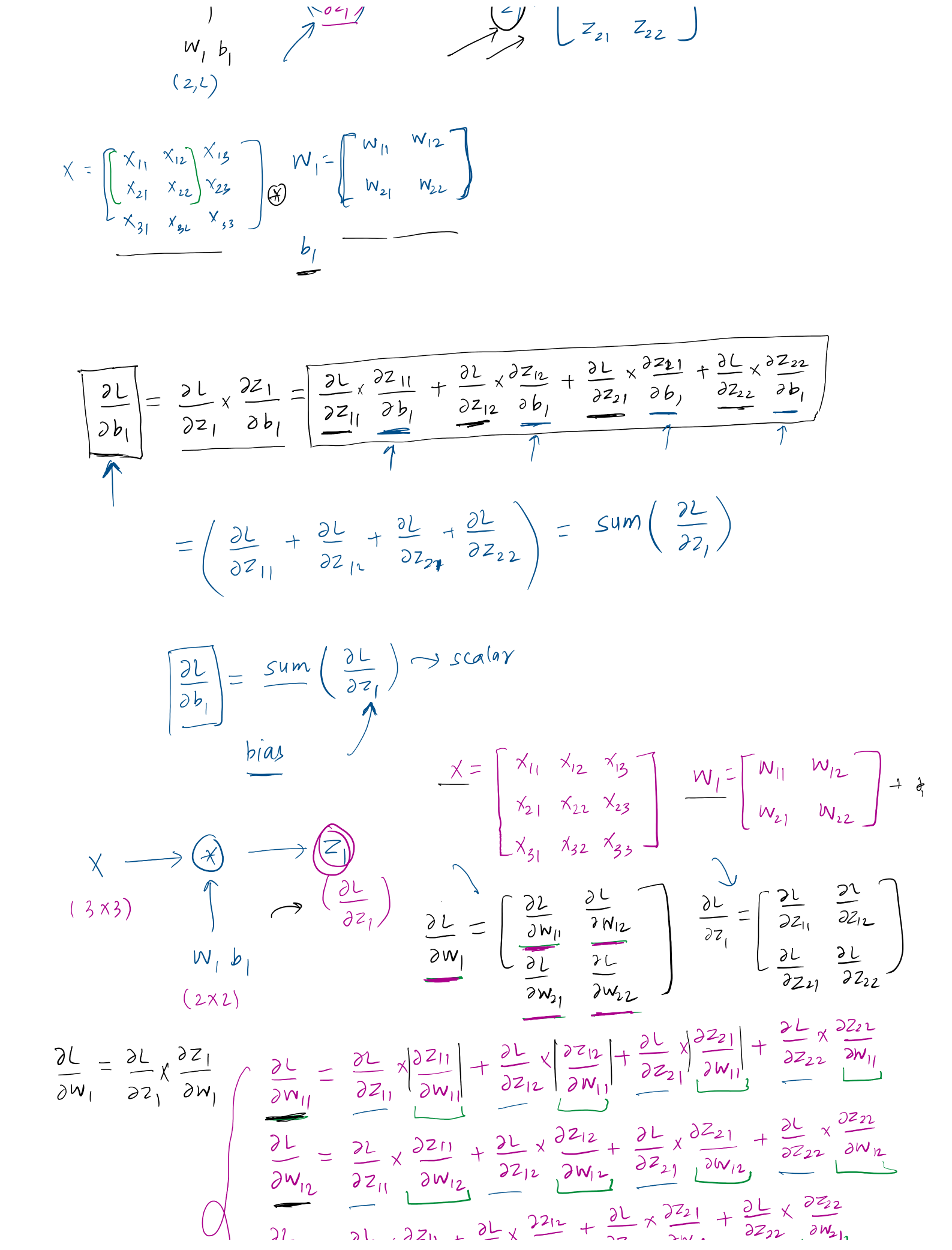
W_1 b_1
(2,2)
Z_{21} Z_{22}
$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$ $W_1 = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$
b_1
$\frac{\partial L}{\partial b_1} = \frac{\partial L}{\partial z_1} \times \frac{\partial z_1}{\partial b_1} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial b_1} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial b_1} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial b_1} + \frac{\partial L}{\partial z_{22}} \times \frac{\partial z_{22}}{\partial b_1}$
$= \left(\frac{\partial L}{\partial z_{11}} + \frac{\partial L}{\partial z_{12}} + \frac{\partial L}{\partial z_{21}} + \frac{\partial L}{\partial z_{22}} \right) = sum\left(\frac{\partial L}{\partial z_1} \right)$
$\frac{\partial L}{\partial b_1} = sum\left(\frac{\partial L}{\partial z_1} \right) \rightarrow$ scalar
bias
$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$ $W_1 = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$ + b
X → (*) → Z_1
(3x3)
W_1 b_1
(2x2)
$\left(\frac{\partial L}{\partial z_1} \right)$
$\frac{\partial L}{\partial W_1} = \begin{bmatrix} \frac{\partial L}{\partial w_{11}} & \frac{\partial L}{\partial w_{12}} \\ \frac{\partial L}{\partial w_{21}} & \frac{\partial L}{\partial w_{22}} \end{bmatrix}$ $\frac{\partial L}{\partial z_1} = \begin{bmatrix} \frac{\partial L}{\partial z_{11}} & \frac{\partial L}{\partial z_{12}} \\ \frac{\partial L}{\partial z_{21}} & \frac{\partial L}{\partial z_{22}} \end{bmatrix}$
$\frac{\partial L}{\partial W_1} = \frac{\partial L}{\partial z_1} X \frac{\partial z_1}{\partial W_1}$
$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial w_{11}} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial w_{11}} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial w_{11}} + \frac{\partial L}{\partial z_{22}} \times \frac{\partial z_{22}}{\partial w_{11}}$
$\frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial w_{12}} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial w_{12}} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial w_{12}} + \frac{\partial L}{\partial z_{22}} \times \frac{\partial z_{22}}{\partial w_{12}}$
$\frac{\partial L}{\partial w_{21}} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial w_{21}} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial w_{21}} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial w_{21}} + \frac{\partial L}{\partial z_{22}} \times \frac{\partial z_{22}}{\partial w_{21}}$

$$\frac{\partial L}{\partial w_{21}} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial w_{21}} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial w_{21}} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial w_{21}} + \frac{\partial L}{\partial z_{22}} \times \frac{\partial z_{22}}{\partial w_{21}}$$

$$\frac{\partial L}{\partial w_{22}} = \frac{\partial L}{\partial z_{11}} \times \frac{\partial z_{11}}{\partial w_{22}} + \frac{\partial L}{\partial z_{12}} \times \frac{\partial z_{12}}{\partial w_{24}} + \frac{\partial L}{\partial z_{21}} \times \frac{\partial z_{21}}{\partial w_{22}} + \frac{\partial L}{\partial z_{32}} \times \frac{\partial z_{22}}{\partial w_{22}}$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial z_{11}} x_{11} + \frac{\partial L}{\partial z_{12}} x_{12} + \frac{\partial L}{\partial z_{21}} x_{21} + \frac{\partial L}{\partial z_{22}} x_{22}$$

$$\frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial z_{11}} x_{12} + \frac{\partial L}{\partial z_{12}} x_{13} + \frac{\partial L}{\partial z_{21}} x_{22} + \frac{\partial L}{\partial z_{22}} x_{23}$$

$$\frac{\partial L}{\partial w_{21}} = \frac{\partial L}{\partial z_{11}} x_{21} + \frac{\partial L}{\partial z_{12}} x_{22} + \frac{\partial L}{\partial z_{21}} x_{31} + \frac{\partial L}{\partial z_{22}} x_{32}$$

$$\frac{\partial L}{\partial w_{22}} = \frac{\partial L}{\partial z_{11}} x_{22} + \frac{\partial L}{\partial z_{12}} x_{23} + \frac{\partial L}{\partial z_{21}} x_{32} + \frac{\partial L}{\partial z_{22}} x_{33}$$

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \qquad \frac{\partial L}{\partial z_1} = \begin{bmatrix} \frac{\partial L}{\partial z_{11}} & \frac{\partial L}{\partial z_{12}} \\ \frac{\partial L}{\partial z_{21}} & \frac{\partial L}{\partial z_{22}} \end{bmatrix}$$

$$\frac{\partial L}{\partial w_1} = \text{conv}\left(X, \frac{\partial L}{\partial z_1}\right)$$

$$\frac{\partial L}{\partial w_1} = \text{conv}\left(X, \frac{\partial L}{\partial z_1}\right)$$

$$\frac{\partial L}{\partial b_1} = \text{sum}\left(\frac{\partial L}{\partial z_1}\right)$$