HW1 part 2

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To find the marginal likelihood we assume a model for the data, which depends on a variable theta. As we are not sure on the exact value of theta, we choose it to follow a distribution p(theta|alpha) specified by a constant parameter alpha. So, now the distribution of X is directly determined by alpha . This way, we can calculate the marginal likelihood by integrating out the variable theta.

As of this; Marginal Likelihood $m(y) = \int_{\Theta} L(\theta)\pi(\theta)d\theta$,

Given our likelihood function of the chosen model(Poisson,2)

$$L(\theta) = \prod_{i=1}^{n} f(yi\,|\,\theta) = \prod_{i=1}^{n} \frac{\exp^{-\theta}\,\theta^{yi}}{yi\,!} = \frac{e^{-\theta n}\theta^{\sum_{i=1}^{n} yi}}{\prod_{i=1}^{n} yi\,!}$$

We choose theta to follow a Gamma(1,1) distribution. $\theta \approx Gamma(1, 1)$.

As, so $\pi(\theta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}$-> $\pi(\theta) = e^{-\theta}$

We integrate, over the support of the Gamma distribution->(0,Inf)

Solving the integral

$$m(y) = \int_0^{\infty} \frac{e^{-\theta n}\theta^{\sum_{i=1}^{n} yi}}{\prod_{i=1}^{n} yi\,!} e^{-\theta}d\theta = \frac{1}{\prod_{i=1}^{n} yi\,!}\int_0^{\infty} e^{-\theta(n+1)}\theta^{\sum_{i=1}^{n} yi}d\theta$$

Now we take advantage of the property distribution support is always equal 1.

$$\int_0^{\infty} \frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}d\theta = 1$$

$$\int_0^{\infty} \theta^{\alpha-1}e^{-\beta\theta}d\theta = \frac{\Gamma(\alpha)}{\beta^{\alpha}}$$

This way $\alpha = \sum_{i=1}^{n} yi + 1$ and $\beta = n + 1$

Loading [MathJax]/jax/output/HTML-CSS/jax.js nputation follows;

$$\int_0^x \theta^{\sum_{i=1}^n y} e^{-(n+1)\theta} d\theta = \frac{\Gamma(\sum_{i=1}^n yi+1)}{(n+1)^{\sum_{i=1}^n yi+1}}$$

And finally,

$$m(y1,\ldots yn) = \int_0^x \theta^{\sum_{i=1}^n y} e^{-(n+1)\theta} d\theta = \frac{\Gamma(\sum_i = 1^n yi+1)}{\prod_{i=1}^n yi!\,(n+1)^{\sum_{i=1}^n yi+1}} = \frac{\Gamma(25)}{1327104(11^{25})} = 4.315028e-09$$

```r
set.seed(123)

pois=rpois(10,2)
prod=1

for (i in 1:10){
  print(pois[i])
  prod=prod*factorial(pois[i])
}
```

```
## [1] 1
## [1] 3
## [1] 2
## [1] 4
## [1] 4
## [1] 0
## [1] 2
## [1] 4
## [1] 2
## [1] 2
```

```r
#Gamma(25)=!24
factorial(24)/(prod*(11^25))
```

```
## [1] 4.315028e-09
```