Chap 15: Missing Data and Other Opportunities

```
md"# Chap 15: Missing Data and Other Opportunities"
```

```
versioninfo()
```

```
Julia Version 1.11.1
Commit 8f5b7ca12ad (2024-10-16 10:53 UTC)
Build Info:
  Official https://julialang.org/ release
Platform Info:
  OS: Linux (x86_64-linux-gnu)
  CPU: 32 × Intel(R) Xeon(R) CPU E5-2630 v3 @
2.40GHz
  WORD_SIZE: 64
  LLVM: libLLVM-16.0.6 (ORCJIT, haswell)
Threads: 16 default, 0 interactive, 8 GC (on 3
2 virtual cores)
Environment:
  JULIA_PKG_SERVER = https://mirrors.tuna.tsin
ghua.edu.cn/julia
  JULIA_REVISE_WORKER_ONLY = 1
```

```
html"""
<style>
    main {
        margin: 0 auto;
        max-width: max(1600px, 75%);
        padding-left: max(5px, 1%);
        padding-right: max(350px, 10%);
    }
</style>
"""
```

Table of Contents


```julia
begin
  using Pkg, DrWatson
  using PlutoUI
  TableOfContents()
end
```

```
begin
    using Turing
    using Turing
    using DataFrames
    using CSV
    using Random
    using Dagitty
    using Distributions
    using StatisticalRethinking
    #using StatisticalRethinking: link
    using StatisticalRethinkingPlots
    using StatsPlots
    using StatsBase
    using Logging
    using LinearAlgebra
    using LogExpFunctions # for logistic()
end
```

Code 15.1

```
md"## Code 15.1"
```

0.6617857711284418

```
begin
    Random.seed!(2)

    function sim_pancake()
        pancake = [[1, 1], [1, 0], [0, 0]]
        sides = sample(pancake)
        sample([sides, reverse(sides)])
    end

    @time pancakes = vcat([sim_pancake() for _ in 1:100_000]'...)
    up = pancakes[:,1]
    down = pancakes[:,2]

    num_11_10 = sum(up .== 1)
    num_11 = sum((up .== 1) .& (down .== 1))
    num_11 / num_11_10
end
```

```
  0.114458 seconds (1.65 M allocations: 6
4.906 MiB, 54.89% compilation time)
```

pancake = [[1, 1], [1, 0], [0, 0]]

```
pancake = [[1, 1], [1, 0], [0, 0]]
```

sides = [1, 1]

```
sides = sample(pancake)
```

[1, 1]

```
sample([sides, reverse(sides)])
```

[[1, 1], [1, 1]]

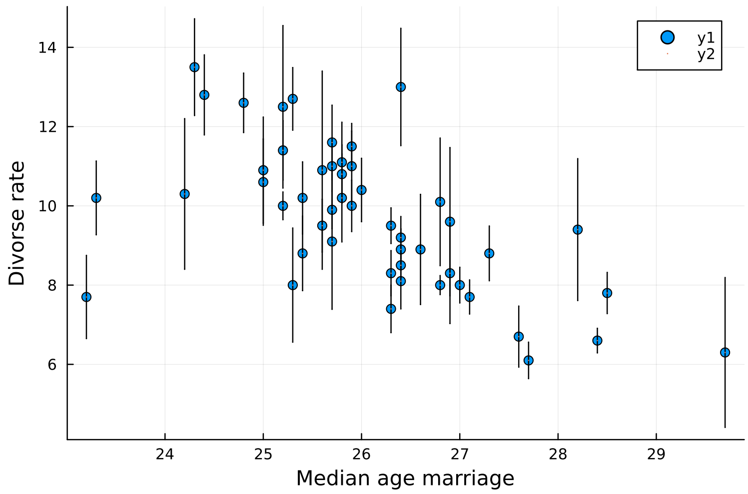
```
[sides, reverse(sides)]
```

15.1 Measurement error

```
md" # 15.1 Measurement error"
```

Code 15.2

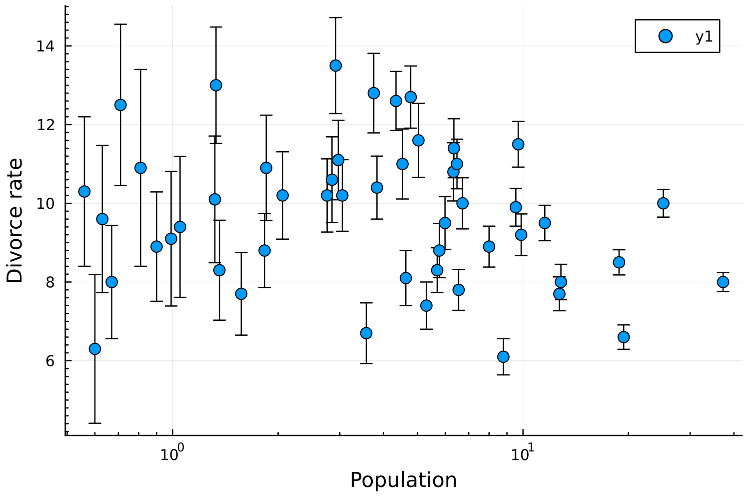
```
md"## Code 15.2"
```




```
begin
    d_divorce =
    DataFrame(CSV.File("data/WaffleDivorce.csv"))

    scatter(d_divorce.MedianAgeMarriage,
    d_divorce.Divorce,
        xlab="Median age marriage", ylab="Divorse
        rate")
    scatter!(d_divorce.MedianAgeMarriage,
    d_divorce.Divorce, yerror=d_divorce."Divorce
    SE", ms=0)
end
```

	Location	Loc	Population	MedianAgeMarriage
1	"Alabama"	"AL"	4.78	25.3
2	"Alaska"	"AK"	0.71	25.2
3	"Arizona"	"AZ"	6.33	25.8

```
first(d_divorce,3)
```




```
begin
    scatter(d_divorce.Population,
    d_divorce.Divorce, xaxis=:log10,
        xlab="Population", ylab="Divorce rate",
        xminorticks=9, yminorticks=10,
        yerror=d_divorce."Divorce SE", ms=5)
    #scatter!(d_divorce.Population,
    d_divorce.Divorce, yerror=d_divorce."Divorce
    SE", ms=0)
end
```

Code 15.3 model `m15_1`

```
md"## Code 15.3 model `m15_1`"
```

	D_true[10]	D_true[11]	D_true[12]	D_true[13]	D
1	-0.784182	0.591328	-0.412152	0.12795	-(
2	-0.418201	1.43575	-0.850778	0.108591	-(
3	-0.638492	0.392377	-0.361222	1.00863	-(
4	-0.509696	1.10383	-0.518881	0.27218	-(
5	-0.691987	0.769628	-0.666241	0.826797	-(
6	-0.521323	0.589482	-0.991947	-0.00510027	-1
7	-0.586819	0.689822	-0.252866	0.751031	-(
8	-0.582998	0.455282	-0.310074	0.56882	-(
9	-0.630576	0.790008	-0.888668	1.30604	-(
10	-0.630576	0.790008	-0.888668	1.30604	-(
more					
1000	-1.02493	0.883494	-0.144567	-0.00200293	-(

```
begin
    d_divorce_ls = (
        D_obs = standardize(ZScoreTransform,
    d_divorce.Divorce),
        D_sd = d_divorce."Divorce SE" ./
    std(d_divorce.Divorce),
        M = standardize(ZScoreTransform,
    d_divorce.Marriage),
        A = standardize(ZScoreTransform,
    d_divorce.MedianAgeMarriage),
        N = nrow(d_divorce),
    )

    @model function m15_1(D_obs, D_sd, M, A, N)
        a ~ Normal(0, 0.2)
        bA ~ Normal(0, 0.5)
        bM ~ Normal(0, 0.5)
        μ = @. a + bA * A + bM * M
        σ ~ Exponential()
        D_true ~ MvNormal(μ, σ)
        @. D_obs ~ Normal(D_true, D_sd)
    end

    Random.seed!(1)
    @time m15_1_ch =
    sample(m15_1(d_divorce_ls...), NUTS(), 1000)
    m15_1_df = DataFrame(m15_1_ch);
end
```

Sampling 100%

Found initial step size
ϵ: 0.2

```
11.457821 seconds (16.59 M allocations:
6.154 GiB, 10.11% gc time, 55.48% compilation
time)
```

Code 15.4

```
md"## Code 15.4"
```

	variable	mean	min	media
1	Symbol("D_true[10]")	-0.622426	-1.17513	-0.6214
2	Symbol("D_true[11]")	0.752743	-0.167793	0.76452
3	Symbol("D_true[12]")	-0.54162	-2.09472	-0.5389
4	Symbol("D_true[13]")	0.191023	-1.80048	0.19718
5	Symbol("D_true[14]")	-0.86873	-1.59464	-0.8784
6	Symbol("D_true[15]")	0.563774	-0.450136	0.55976
7	Symbol("D_true[16]")	0.269308	-0.855484	0.28287
8	Symbol("D_true[17]")	0.505615	-0.78145	0.50451
9	Symbol("D_true[18]")	1.25328	0.14058	1.25724
10	Symbol("D_true[19]")	0.428978	-0.812482	0.44128
	more			
54	:σ	0.579131	0.30084	0.57578

```
describe(m15_1_df)
```

Code 15.5 model `m15_2`

```
md"## Code 15.5 model `m15_2`"
```

```
(D_obs = [1.65421, 1.54436, 0.610716, 2.09357, -0.927058
```

```
begin
    dlist2 = (
        D_obs = standardize(ZScoreTransform,
    d_divorce.Divorce),
        D_sd = d_divorce."Divorce SE" ./
    std(d_divorce.Divorce),
        M_obs = standardize(ZScoreTransform,
    d_divorce.Marriage),
        M_sd = d_divorce."Marriage SE" ./
    std(d_divorce.Marriage),
        A = standardize(ZScoreTransform,
    d_divorce.MedianAgeMarriage),
        N = nrow(d_divorce),
    )
end
```

```
[0.083057, 1.01903, 0.0594721, 1.41732, -0.266635, 0.830
```

```
begin

    @model function m15_2(D_obs, D_sd, M_obs,
    M_sd, A, N)
        a ~ Normal(0, 0.2)
        bA ~ Normal(0, 0.5)
        bM ~ Normal(0, 0.5)
        M_true ~ filldist(Normal(), N)

        μ = @. a + bA * A + bM * M_true
        σ ~ Exponential()
        D_true ~ MvNormal(μ, σ)
        @. D_obs ~ Normal(D_true, D_sd)
        @. M_obs ~ Normal(M_true, M_sd)
    end

    Random.seed!(1)
    @time m15_2_ch = sample(m15_2(dlist2...),
    NUTS(), 1000)
    m15_2_df = DataFrame(m15_2_ch);
    D_true = [mean(m15_2_df[!, "D_true[$i]"]) for
    i ∈ 1:dlist2.N]
    M_true = [mean(m15_2_df[!, "M_true[$i]"]) for
    i ∈ 1:dlist2.N]
end
```

```
Sampling 100%

Found initial step size
ϵ: 0.4
```

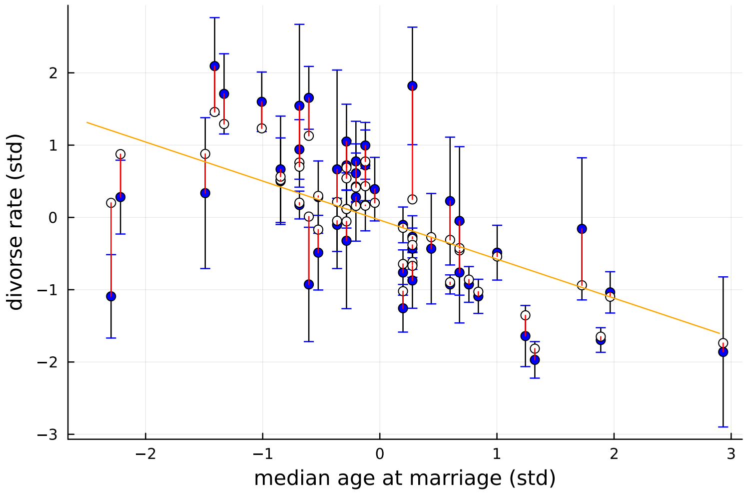
```
37.370198 seconds (53.81 M allocations: 4
2.068 GiB, 22.55% gc time, 20.53% compilation
time)
```

	variable	mean	min	medi
1	Symbol("D_true[10]")	-0.616598	-1.09836	-0.616
2	Symbol("D_true[11]")	0.773391	-0.153289	0.7721
3	Symbol("D_true[12]")	-0.455932	-1.96422	-0.469
4	Symbol("D_true[13]")	0.201203	-1.44406	0.2043
5	Symbol("D_true[14]")	-0.860255	-1.57298	-0.859
6	Symbol("D_true[15]")	0.540992	-0.540644	0.5437
7	Symbol("D_true[16]")	0.297736	-0.943139	0.2935
8	Symbol("D_true[17]")	0.519618	-1.31079	0.5227
9	Symbol("D_true[18]")	1.23177	0.22005	1.2234
10	Symbol("D_true[19]")	0.431547	-0.906202	0.4161
	more			
104	:σ	0.563163	0.242072	0.5583

```
describe(m15_2_df)
```

Figure 15.2

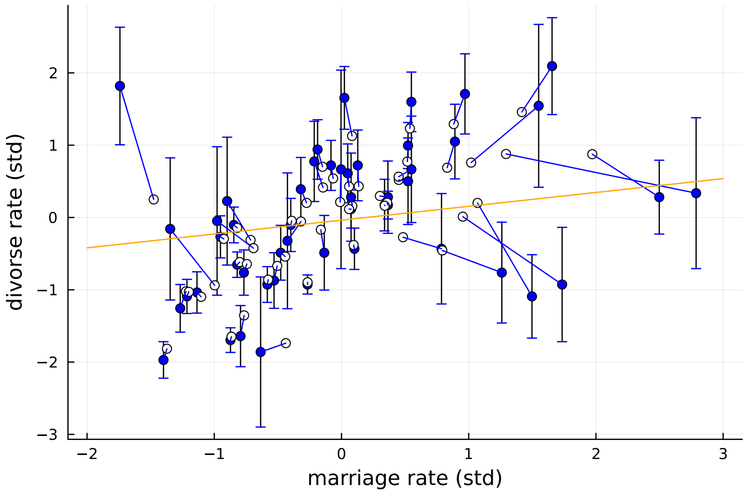
```
md"## Figure 15.2"
```



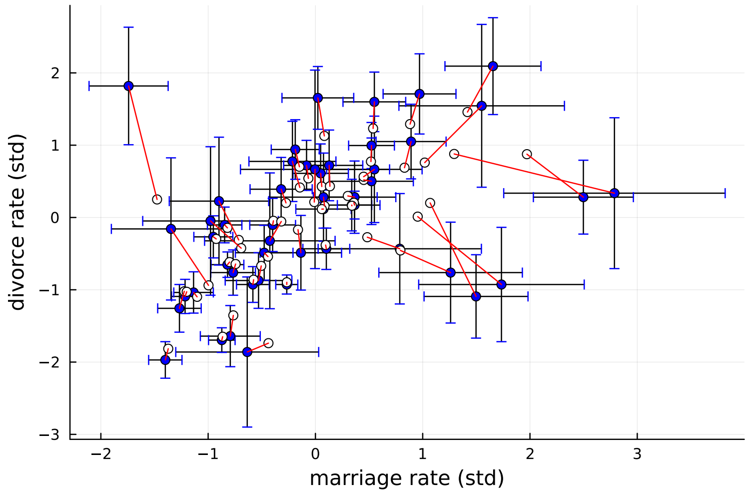

```
begin
    p15_1_1 = scatter(dlist2.A, dlist2.D_obs,
    mc=:blue, yerror=dlist2.D_sd,
        label="observed", xlab="median age at
    marriage (std)", ylab="divorse rate (std)")
    scatter!(dlist2.A, D_true, mc=:white,
    label="true")

    for i ∈ 1:dlist2.N
        plot!([dlist2.A[i], dlist2.A[i]],
    [dlist2.D_obs[i], D_true[i]], c=:red,
    legend=false)
    end
    x = -2.5:0.2:3
    y = -0.0368595 .+  -0.540089 .* x
    plot!(x,y, c=:orange, label="m15_2 estimate")
    p15_1_1
end
```

Code 15.6 Figure 15.3

```
md"## Code 15.6 Figure 15.3"
```




```
begin
    p15_1_2 = scatter(dlist2.M_obs, dlist2.D_obs,
    mc=:blue, yerror=dlist2.D_sd,
        label="observed", xlab="marriage rate
    (std)", ylab="divorse rate (std)",
        legend=true)
    scatter!(M_true, D_true, mc=:white,
    label="true", legend=true)

    for i ∈ 1:dlist2.N
        plot!([dlist2.M_obs[i], M_true[i]],
    [dlist2.D_obs[i], D_true[i]], c=:blue,
    legend=false)
    end
    x2 = -2:0.2:3
    y2 = -0.0368595 .+  0.1915 .* x2
    plot!(x2,y2, c=:orange, label="m15_2 estimate")
    p15_1_2
end
```




```
begin
    p3 = scatter(dlist2.M_obs, dlist2.D_obs,
    mc=:blue, xerror=dlist2.M_sd,
    yerror=dlist2.D_sd,
        label="observed", xlab="marriage rate
    (std)", ylab="divorce rate (std)")
    scatter!(M_true, D_true, mc=:white,
    label="true")

    for i ∈ 1:dlist2.N
        plot!([dlist2.M_obs[i], M_true[i]],
    [dlist2.D_obs[i], D_true[i]], c=:red,
    legend=false)
    end
    p3
end
```

Code 15.7

```
md"## Code 15.7"
```

[-0.860429, 0.151987, 2.67642, 0.24338, -1.82141, -1.683

```
let
    N = 500
    A = rand(Normal(), N)
    M = rand.(Normal.(-A))
    D = rand.(Normal.(A))
    A_obs = rand.(Normal.(A));
end
```

```
Enter cell code...
```

15.2 Missing data

```
md"# 15.2 Missing data"
```

m15_3

- UndefVarError: `logistic` not defined in Main.var
- Suggestion: check for spelling errors or missing imports.
- Hint: a global variable of this name may be made accessible by importing LogExpFunctions in the current active module Main
- Hint: a global variable of this name may be made accessible by importing StatsFuns in the current active module Main

```
md"## `m15_3`

- UndefVarError: `logistic` not defined in Main.var
- Suggestion: check for spelling errors or missing imports.
- Hint: a global variable of this name may be made accessible by importing LogExpFunctions in the current active module Main
- Hint: a global variable of this name may be made accessible by importing StatsFuns in the current active module Main
"
```

```
m15_3 (generic function with 2 methods)
```

```
begin
    @model function m15_3(H, S)
        a ~ Normal()
        bS ~ Normal(0, 0.5)
        p = @. LogExpFunctions.logistic(a + bS*S)
        @. H ~ Binomial(10, p)
    end
end
```

Code 15.8 Vanilla simulation: a=0, b=1

```
md"## Code 15.8 Vanilla simulation: a=0, b=1"
```

```
[6, 4, 4, 5, 5, 4, 2, 7, 3, 8, 7, 5, 5, 6, 4, 6, 9, 9, 7, 4,
```

```
begin
    N0 = 100
    S0 = rand(Normal(), N0)
    a0 = 0
    bS0 = 1
    H0 = rand.([BinomialLogit(10, a0+bS0*l) for l
    in S0]);
end
```

```
(100)
```

```
size(H0)
```

	variable	mean	min	median	max
1	:a	-0.0324142	-0.240182	-0.0326535	0.16273
2	:bS	0.905949	0.680666	0.903763	1.16795

```
begin
    Random.seed!(1)
    @time m15_3_ch0 = sample(m15_3(H0, S0),
    NUTS(100, 0.65, init_ϵ=0.25), 1000)
    m15_3_df0 = DataFrame(m15_3_ch0)
    describe(m15_3_df0)
end
```

Sampling 100%

```
 0.480070 seconds (894.10 k allocations:
100.208 MiB, 10.03% gc time)
```

- Estimates of a and b are close to the truth.

```
md"
- Estimates of a and b are close to the truth."
```

Code 15.9 Simulate a: H* randomly missing (H randomly eaten by the dog)

```
md"## Code 15.9 Simulate a: H* randomly missing (H
randomly eaten by the dog)"
```

```
view(::Vector{Union{Missing, Int64}}, [1, 3, 4, 6, 9, 11,
```

```
begin
    Da = rand(Bernoulli(), N0)
    Hma = Vector{Union{Missing,Int}}(H0)
    Hma[Da .== 1] .= missing;
end
```

```
[missing, 4, missing, missing, 5, missing, 2, 7, missing,
```

```
Hma
```

```
(0.5)
```

```
params(Bernoulli())
```

```
BitVector: [false, true, false, false, true, false, true,
```

```
.!ismissing.(Hma)
```

15.9.1 Complete data fitting `m15_3`

```
md"### 15.9.1 Complete data fitting `m15_3`"
```

	variable	mean	min	median	max
1	:a	-0.0549241	-0.410573	-0.0556711	0.25872
2	:bS	1.04709	0.73206	1.04013	1.39939

```
begin
    Random.seed!(1)
    index_vec = .!ismissing.(Hma)
    @time m15_3_ch_a =
    sample(m15_3(Hma[index_vec], S0[index_vec]),
    NUTS(100, 0.65, init_ϵ=0.25), 1000)
    m15_3_df_a = DataFrame(m15_3_ch_a)
    describe(m15_3_df_a)
end
```

```
Sampling 100%
```

```
0.549874 seconds (1.48 M allocations: 10
0.117 MiB)
```

Code 15.10 Simulate b: Dog only eats Homework of students who study hard (spend less time playing with the dog)

```
md"## Code 15.10 Simulate b: Dog only eats Homework of students who study hard (spend less time playing with the dog)"
```

```
Enter cell code...
```

```
view(::Vector{Union{Missing, Int64}}, [8, 10, 11, 13, 14,
```

```
begin
    Db = S0 .> 0
    Hmb = Vector{Union{Missing,Int}}(H0)
    Hmb[Db .== 1] .= missing;
end
```

15.10.1 Complete data fitting `m15_3`

- Results are reasonably OK.

```
md"### 15.10.1 Complete data fitting `m15_3`
- Results are reasonably OK."
```

	variable	mean	min	median	max
1	:a	-0.120064	-0.580817	-0.117291	0.410149
2	:bS	0.865742	0.267384	0.866562	1.41655

```
begin
    Random.seed!(1)
    index_vecb = .!ismissing.(Hmb)
    @time m15_3_ch_b =
    sample(m15_3(Hmb[index_vecb], S0[index_vecb]),
    NUTS(100, 0.65, init_ϵ=0.25), 1000)
    m15_3_df_b = DataFrame(m15_3_ch_b)
    describe(m15_3_df_b)
end
```

Sampling 100%

```
0.654514 seconds (1.67 M allocations: 112.330 MiB, 6.12% gc time)
```

Code 15.11 Simulate c: X (noisy house) impacts Homework quality and Dog homework-eating behavior

```
md"## Code 15.11 Simulate c: X (noisy house)
impacts Homework quality and Dog homework-eating
behavior"
```

```
view(::Vector{Union{Missing, Int64}}, [5, 12, 29, 31, 51,
```

```
begin
    Random.seed!(501)
    N2 = 1000
    X = rand(Normal(), N2)
    Sc = rand(Normal(), N2)
    Hc = rand.([BinomialLogit(10, l) for l in 2 .+
    Sc .- 2X])
    Dc = X .> 1
    Hmc = Vector{Union{Missing,Int}}(Hc)
    Hmc[Dc .== 1] .= missing;
end
```

Code 15.12 Use true H to fit `m15_3`

- Estimates are off.

```
md"### Code 15.12 Use true H to fit `m15_3`
- Estimates are off."
```

	variable	mean	min	median	max	nm
1	:a	1.19348	1.12922	1.19236	1.26304	0
2	:bS	0.577602	0.485119	0.57752	0.664904	0

```
begin
    Random.seed!(1)
    @time m15_3_ch_c_use_H = sample(m15_3(Hc, Sc),
    NUTS(100, 0.65, init_ϵ=0.25), 1000)
    m15_3_df_c_use_H = DataFrame(m15_3_ch_c_use_H)
    describe(m15_3_df_c_use_H)
end
```

```
Sampling 100%
```

```
  4.468720 seconds (3.80 M allocations: 69
9.392 MiB, 2.73% gc time, 69.74% compilation t
ime)
```

15.12.1 Use Hm but complete-data fitting `m15_3`

- Estimates are off too. Esp. estimate a.
- But estimate b improves a bit.

```
md"### 15.12.1 Use Hm but complete-data fitting
`m15_3`
- Estimates are off too. Esp. estimate a.
- But estimate b improves a bit."
```

	variable	mean	min	median	max	nn
1	:a	1.87375	1.76504	1.87376	1.98606	0
2	:bS	0.822737	0.726672	0.823292	0.924049	0

```
begin
    Random.seed!(1)
    index_vecc = .!ismissing.(Hmc)
    @time m15_3_ch_c =
    sample(m15_3(Hmc[index_vecc], Sc[index_vecc]),
    NUTS(100, 0.65, init_ϵ=0.25), 1000)
    m15_3_df_c = DataFrame(m15_3_ch_c)
    describe(m15_3_df_c)
end
```

Sampling 100%

```
5.519539 seconds (17.96 M allocations:
1.078 GiB, 2.67% gc time, 60.06% compilation t
ime)
```

Code 15.13. Use H and complete-data fitting `m15_3`

- Estimates almost identical to the ones above

```
md"### Code 15.13. Use H and complete-data fitting
`m15_3`

- Estimates almost identical to the ones above"
```

	variable	mean	min	median	max	nn
1	:a	1.87375	1.76504	1.87376	1.98606	0
2	:bS	0.822737	0.726672	0.823292	0.924049	0

```
begin
    Random.seed!(1)
    @time m15_4_ch_c_use_H_complete =
    sample(m15_3(Hc[Dc .== 0], Sc[Dc .== 0]),
    NUTS(100, 0.65, init_ϵ=0.25), 1000)
    m15_4_df_c_use_H_complete =
    DataFrame(m15_4_ch_c_use_H_complete)
    describe(m15_4_df_c_use_H_complete)
end
```

Sampling 100%

```
  1.316840 seconds (950.11 k allocations:
511.360 MiB, 8.74% gc time, 0.34% compilation
time)
```

Code 15.14 Change simulation c: reverse the missingness

```
md"### Code 15.14 Change simulation c: reverse the
missingness
"
```

```
view(::Vector{Union{Missing, Int64}}, [1, 2, 3, 4, 6, 7, 8
```

```
begin
    Dc2 = abs.(X) .< 1;
    Hmc2 = Vector{Union{Missing,Int}}(Hc)
    Hmc2[Dc2 .== 1] .= missing;
end
```

15.14.1 Use Hmc2 but complete-data fitting `m15_3`

- Removing missing data reduces the estimate of b.

```
md"### 15.14.1 Use Hmc2 but complete-data fitting
`m15_3`
- Removing missing data reduces the estimate of b."
```

	variable	mean	min	median	max	nn
1	:a	0.584132	0.478793	0.58252	0.724042	0
2	:bS	0.384676	0.280504	0.382544	0.500593	0

```
begin
    Random.seed!(1)
    index_vec_c2 = .!ismissing.(Hmc2)
    @time m15_3_ch_c_reverse_missing =
    sample(m15_3(Hmc2[index_vec_c2],
    Sc[index_vec_c2]),
        NUTS(100, 0.65, init_ϵ=0.25), 1000)
    m15_3_df_c_reverse_missing =
    DataFrame(m15_3_ch_c_reverse_missing)
    describe(m15_3_df_c_reverse_missing)
end
```

Sampling 100%

```
  1.077021 seconds (4.28 M allocations: 32
5.958 MiB, 4.76% gc time)
```

Code 15.15 Simulate d: Homework affects dog. Bad homework more likely gets eaten.

```
md"## Code 15.15 Simulate d: Homework affects dog.
Bad homework more likely gets eaten."
```

```
view(::Vector{Union{Missing, Int64}}, [2, 3, 5, 6, 8, 9, 1
```

```
begin
    Sd = rand(Normal(), N0)
    Hd = rand.([BinomialLogit(10, l) for l in Sd])
    Dd = Hd .< 5
    Hmd = Vector{Union{Missing,Int}}(Hd)
    Hmd[Dd .== 1] .= missing;
end
```

15.15.1 Complete-data fitting `m15_3`

```
md"### 15.15.1 Complete-data fitting `m15_3`"
```

	variable	mean	min	median	max	nn
1	:a	0.398245	0.026878	0.396347	0.848112	0
2	:bS	0.782425	0.36436	0.780993	1.19886	0

```
begin
    Random.seed!(1)
    index_vec_d = .!ismissing.(Hmd)
    @time m15_3_ch_d =
    sample(m15_3(Hmd[index_vec_d],
    Sd[index_vec_d]),
        NUTS(100, 0.65, init_ϵ=0.25), 1000)
    m15_3_df_d = DataFrame(m15_3_ch_d)
    describe(m15_3_df_d)
end
```

Sampling 100%

```
  0.590207 seconds (1.68 M allocations: 11
3.420 MiB)
```

Code 15.16 Milk calories ~ Mass + Brain size. Load data and standardize

```
md"## Code 15.16 Milk calories ~ Mass + Brain
size.  Load data and standardize"
```

```
(K = [-0.940041, -0.816126, -1.12591, -1.002, -0.258511,
```

```
begin
    d_milk = DataFrame(CSV.File("data/milk.csv",
    missingstring="NA"))

    # get rid of dots in column names
    rename!(n -> replace(n, "." => "_"), d_milk)

    d_milk.neocortex_prop = d_milk.neocortex_perc
    ./ 100
    d_milk.logmass = log.(d_milk.mass)

    t = Vector{Union{Missing, Float64}}(missing,
    nrow(d_milk))
    present_mask = completecases(d_milk,
    :neocortex_prop)
    t[present_mask] .=
    standardize(ZScoreTransform,
        Vector{Float64}
        (d_milk.neocortex_prop[present_mask]))

    dat_list = (
        K = standardize(ZScoreTransform,
d_milk.kcal_per_g),
        B = t,
        M = standardize(ZScoreTransform,
d_milk.logmass),
    );
end
```

	clade	species
1	"Strepsirrhine"	"Eulemur fulvus"
2	"Strepsirrhine"	"E macaco"
3	"Strepsirrhine"	"E mongoz"
4	"Strepsirrhine"	"E rubriventer"
5	"Strepsirrhine"	"Lemur catta"
6	"New World Monkey"	"Alouatta seniculus"
7	"New World Monkey"	"A palliata"
8	"New World Monkey"	"Cebus apella"
9	"New World Monkey"	"Saimiri boliviensis"
10	"New World Monkey"	"S sciureus"
11	"New World Monkey"	"Cebuella pygmaea"
12	"New World Monkey"	"Callimico goeldii"
13	"New World Monkey"	"Callithrix jacchus"
14	"New World Monkey"	"Leontopithecus rosalia"
15	"Old World Monkey"	"Chlorocebus pygerythrus"

```
d_milk
```

17

```
sum(present_mask)
```

```
[-2.0802, missing, missing, missing, missing, -0.508641,
```

```
t
```

Code 15.17 `m15_5` Model imputation and fitting

```
md"### Code 15.17 `m15_5` Model imputation and
fitting"
```

	B_impute[10]	B_impute[11]	B_impute[12]	B_impu
1	-1.55351	0.0844085	1.08743	-2.165
2	-2.5169	0.278236	1.1723	-2.229
3	-2.34466	-0.189826	0.420911	-1.094
4	-2.12903	-0.405498	0.328087	-0.924
5	1.06764	-0.880436	1.31149	-0.216
6	-1.31796	3.33127	-0.49526	1.4558
7	-0.224652	2.50136	0.35028	0.3448
8	-0.922883	-2.37955	-0.0131398	-1.029
9	0.0159517	1.75816	0.0486348	-0.2671
10	-1.00052	-1.31987	-1.01234	0.1258
more				
1000	0.1668	-0.840739	0.57879	-1.270

```julia
begin
    @model function m15_5(K, B, M)
        σ ~ Exponential()
        σ_B ~ Exponential()
        a ~ Normal(0, 0.5)
        ν ~ Normal(0, 0.5)
        bB ~ Normal(0, 0.5)
        bM ~ Normal(0, 0.5)

        N_missing = sum(ismissing.(B))
        B_impute ~ filldist(Normal(ν, σ_B), N_missing)

        i_missing = 1
        for i in eachindex(B)
            if ismissing(B[i])
                #B_impute[i_missing] ~ Normal(ν, σ_B) # this line is bug!
                b = B_impute[i_missing]
                i_missing += 1
            else
                B[i] ~ Normal(ν, σ_B)
                b = B[i]
            end
            μ = a + bB * b + bM * M[i]
            K[i] ~ Normal(μ, σ)
        end
    end

    Random.seed!(1)
    @time m15_5_ch = sample(m15_5(dat_list...), NUTS(), 1000);
    m15_5_df = DataFrame(m15_5_ch);
end
```

Sampling 100%

Found initial step size
ϵ: 0.05

```
8.173490 seconds (9.36 M allocations: 959.330 MiB, 2.12% gc time, 76.74% compilation time)
```

	variable	mean	min	
1	Symbol("B_impute[10]")	-0.421178	-3.19289	-(
2	Symbol("B_impute[11]")	-0.297335	-3.66384	-(
3	Symbol("B_impute[12]")	0.158509	-3.03178	0
4	Symbol("B_impute[1]")	-0.574773	-4.84528	-(
5	Symbol("B_impute[2]")	-0.666931	-3.83844	-(
6	Symbol("B_impute[3]")	-0.706487	-4.51215	-(
7	Symbol("B_impute[4]")	-0.275485	-3.07226	-(
8	Symbol("B_impute[5]")	0.522288	-2.87903	0
9	Symbol("B_impute[6]")	-0.14819	-3.99323	-(
10	Symbol("B_impute[7]")	0.148524	-4.43891	0
11	Symbol("B_impute[8]")	0.28102	-2.47574	0
12	Symbol("B_impute[9]")	0.486673	-2.94571	0
13	:a	0.0213586	-0.688616	0
14	:bB	0.492542	-0.376103	0
15	:bM	-0.544161	-1.12879	-(

```
describe(m15_5_df)
```

Code 15.19 `m15_6` Model fitting using only the non-missing values

```
md"### Code 15.19 `m15_6` Model fitting using only the non-missing values"
```

	a	bB	bM	ν	
1	0.37319	0.658656	-0.607679	-0.53481	0.`
2	0.16674	0.828162	-0.931949	0.0587369	0.`
3	0.0823521	0.541567	-0.638686	-0.0399501	0.`
4	-0.0467399	0.697646	-0.713644	-0.115263	0.`
5	0.0919513	0.836835	-0.955096	0.00590266	0.`
6	0.0864378	0.462202	-0.395563	-0.0157	0.8
7	0.31745	0.397549	-0.567001	0.14617	0.9
8	-0.147122	0.705595	-0.791945	-0.073959	0.`
9	0.372846	0.846506	-0.644569	-0.206733	0.8
10	0.252837	0.0665011	-0.434392	0.296508	1.:
more					
1000	0.157658	0.848862	-0.982046	0.260027	0.`

```
begin
    dat_list_obs = (
    K = dat_list.K[present_mask],
    B = Vector{Float64}(dat_list.B[present_mask]),
    M = dat_list.M[present_mask]
)

@model function m15_6(K, B, M)
    σ ~ Exponential()
    σ_B ~ Exponential()
    a ~ Normal(0, 0.5)
    ν ~ Normal(0, 0.5)
    bB ~ Normal(0, 0.5)
    bM ~ Normal(0, 0.5)

    @. B ~ Normal(ν, σ_B)
    μ = @. a + bB * B + bM * M
    @. K ~ Normal(μ, σ)
end

Random.seed!(1)
@time m15_6_ch = sample(m15_6(dat_list_obs...),
NUTS(), 1000)
m15_6_df = DataFrame(m15_6_ch);
end
```

```
Sampling 100%

Found initial step size
ϵ: 0.4
```

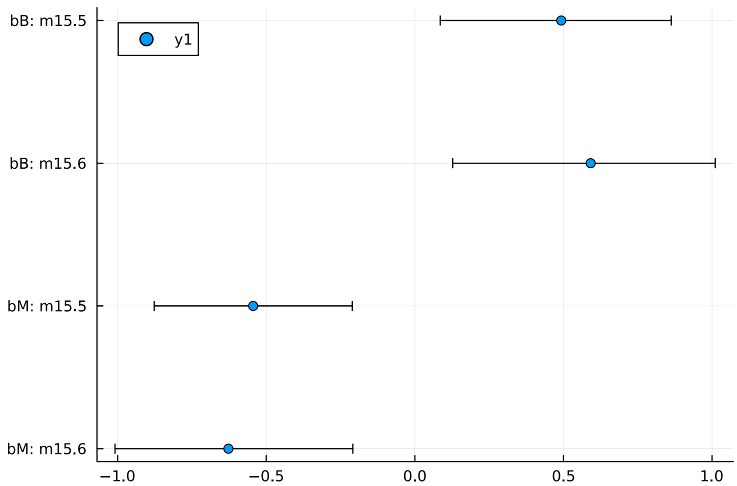
```
  6.575922 seconds (6.97 M allocations: 531.774 MiB, 1.85% gc time, 82.97% compilation time)
```

	variable	mean	min	median	ma:
1	:a	0.090448	-0.546451	0.0942606	0.652
2	:bB	0.591536	-0.578625	0.599326	1.351
3	:bM	-0.627407	-1.30115	-0.634604	0.323
4	:ν	-9.05504e-5	-0.921007	-0.00634828	0.719
5	:σ	0.880438	0.486684	0.849687	1.922
6	:σ_B	1.02796	0.620253	1.00107	2.118

```
describe(m15_6_df)
```

Code 15.20 Compare parameter estimates and CI between `m15_5` and `m15_6`

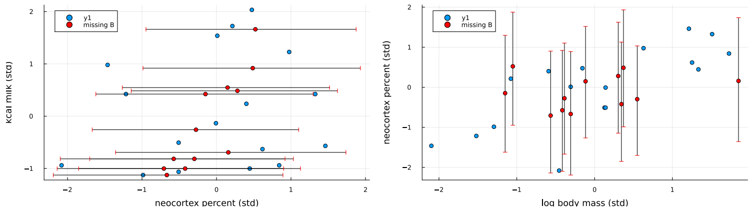
```
md"### Code 15.20 Compare parameter estimates and CI between `m15_5` and `m15_6`"
```




```
coeftab_plot(m15_5_df, m15_6_df; pars=(:bB, :bM), names=("m15.5", "m15.6"))
```

Code 15.21 Fig 15.5 Plot the imputed values and its confidence

```
md"### Code 15.21 Fig 15.5 Plot the imputed values and its confidence"
```




```
let
    N_missing = sum(ismissing.(dat_list.B))
    miss_mask = ismissing.(dat_list.B)

    B_impute_μ = [
        mean(m15_5_df[!,"B_impute[$i]"])
        for i ∈ 1:N_missing
    ]

    B_impute_pi = [
        PI(m15_5_df[!,"B_impute[$i]"])
        for i ∈ 1:N_missing
    ]

    err = (
        B_impute_μ .- first.(B_impute_pi),
        last.(B_impute_pi) .- B_impute_μ
    )

    p1 = scatter(dat_list.B, dat_list.K,
    xlab="neocortex percent (std)", ylab="kcal
    milk (std)")
    Ki = dat_list.K[miss_mask]
    scatter!(B_impute_μ, Ki, xerr=err, mc=:red,
    label="missing B")
    #scatter!(B_impute_μ, Ki, xerr=err, ms=0)

    p2 = scatter(dat_list.M, dat_list.B,
    ylab="neocortex percent (std)", xlab="log body
    mass (std)")
    Mi = dat_list.M[miss_mask]
    scatter!(Mi, B_impute_μ, yerr=err, mc=:red,
    label="missing B")
    #scatter!(Mi, B_impute_μ, yerr=err, ms=0)

    plot(p1, p2, size=(1400, 400),
    margin=5*Plots.mm)
end
```

Code 15.22 `m15_7_1` : add a bivariate normal between two predictors.

```
md"### Code 15.22 `m15_7_1`: add a bivariate
normal between two predictors."
```

```
m15_7_1 (generic function with 2 methods)
```

```
@model function m15_7_1(K, MB, M_missingB)
    σ ~ Exponential()
    σ_BM ~ Exponential()
    a ~ Normal(0, 0.5)
    μB ~ Normal(0, 0.5)
    μM ~ Normal(0, 0.5)
    bB ~ Normal(0, 0.5)
    bM ~ Normal(0, 0.5)
    Rho_BM ~ LKJ(2, 2)

    Σ = (σ_BM .* σ_BM') .* Rho_BM

    # process complete cases
    for i ∈ eachindex(MB)
        MB[i] ~ MvNormal([μM, μB], Σ)
    end

    # impute and process incomplete cases
    N_missing = length(M_missingB)
    #B_impute = Array{Float64}(undef, N_missing)
    # Note =, not ~. Note Float64, not Real.
    Vector{..} also works.
    B_impute ~ filldist(Normal(0, 3),
    N_missing)  # this would cause all estimates
    to be from the prior.
    #B_impute ~ filldist(Normal(μB, σ_BM),
    N_missing)  # this would fail to sample.
    MB_missingB = [
        [m, b]
        for (m, b) ∈ zip(M_missingB, B_impute)
    ]

    for i ∈ eachindex(MB_missingB)
        MB_missingB[i] ~ MvNormal([μM, μB], Σ)
    end

    # from both sets, build mean vector for K
    μ = [
        a + bB * b + bM * m
        for (m, b) ∈ Iterators.flatten((MB,
MB_missingB))
    ]

    @. K ~ Normal(μ, σ)
end
```

```
[-0.940041, -1.06396, -0.50634, 1.53825, 1.72412, 0.9806
```

```
begin
    # prepare data for sampling

    # to improve stability and performance, need
    to separate full samples and samples need to
    be imputed
    pres_mask =  @. !ismissing(dat_list.B)
    _miss_mask = ismissing.(dat_list.B)
    MB = [
        [m, b]
        for (m, b) ∈ zip(dat_list.M[pres_mask],
    Vector{Float64}(dat_list.B[pres_mask]))
    ]
    M_missingB = dat_list.M[_miss_mask]

    # very important to reorder K values to match
    order of samples
    KK = vcat(dat_list.K[pres_mask],
    dat_list.K[_miss_mask])
end
```

```
[-0.415002, -0.307158, -0.565025, -0.387477, -1.05084, -
```

```
M_missingB
```

	B_impute[10]	B_impute[11]	B_impute[12]	B_impu
1	-0.470577	0.757419	3.46814	2.28901
2	0.813646	0.395205	-4.13456	-2.8953
3	5.28155	0.282224	-2.28238	-0.7551
4	-6.22276	-0.358102	2.47944	0.77917
5	-4.3162	-1.77658	-3.10947	2.86125
6	2.58361	-0.557708	0.901645	1.31044
7	-3.03621	0.398519	-0.223246	-1.3126
8	2.87532	-0.180627	0.333789	0.65251
9	-3.77811	0.162254	-0.498088	-0.6210
10	1.05255	-0.121459	1.35053	1.81235
more				
1000	2.47606	-5.37574	1.83822	1.74409

```
begin
    Random.seed!(1)
    @time m15_7_1_ch = sample(m15_7_1(KK, MB,
    M_missingB), NUTS(), 1000)
    m15_7_1_df = DataFrame(m15_7_1_ch);
end
```

Sampling 100%

Found initial step size
ϵ: 0.2

```
29.675125 seconds (117.78 M allocations:
17.016 GiB, 9.27% gc time, 31.96% compilation
time)
```

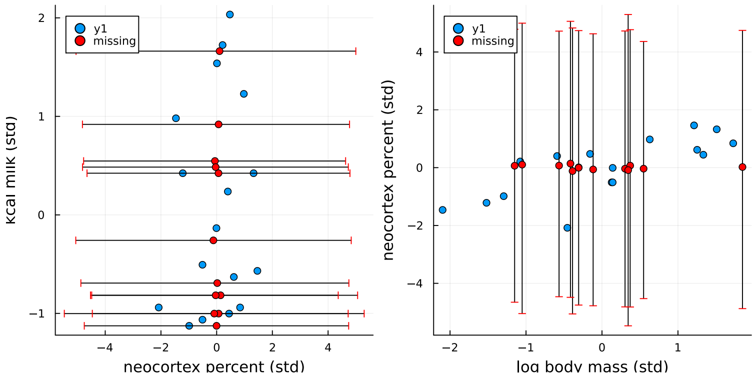
	variable	mean	min
1	Symbol("B_impute[10]")	-0.086222	-10.78
2	Symbol("B_impute[11]")	-0.0347327	-10.04
3	Symbol("B_impute[12]")	0.0215038	-9.089
4	Symbol("B_impute[1]")	0.140579	-9.849
5	Symbol("B_impute[2]")	-0.0038949	-9.790
6	Symbol("B_impute[3]")	0.0713118	-9.462
7	Symbol("B_impute[4]")	-0.117313	-10.41
8	Symbol("B_impute[5]")	0.104244	-10.99
9	Symbol("B_impute[6]")	0.0656626	-9.575
10	Symbol("B_impute[7]")	-0.0615376	-9.230
11	Symbol("B_impute[8]")	-0.0358849	-8.321
12	Symbol("B_impute[9]")	0.0693097	-8.798
13	Symbol("MB_missingB[10][1]")	0.293279	-3.397
14	Symbol("MB_missingB[10][2]")	-0.2171	-3.389
15	Symbol("MB_missingB[11][1]")	0.255512	-4.224

```
describe(m15_7_1_df)
```

Plot `m15_7_1` estimates

- Sampling not working very well. Estimates of missing B hovers around 0.

```
md"### Plot `m15_7_1` estimates
- Sampling not working very well. Estimates of
missing B hovers around 0."
```




```
let
    N_missing = sum(ismissing.(dat_list.B))
    miss_mask = ismissing.(dat_list.B)

    B_impute_μ = [
        #mean(m15_7_2_df[!,"MB_missingB[$i][2]"])
        mean(m15_7_1_df[!,"B_impute[$i]"])
        for i ∈ 1:N_missing
    ]

    B_impute_pi = [
        #PI(m15_7_2_df[!,"MB_missingB[$i][2]"])
        PI(m15_7_1_df[!,"B_impute[$i]"])
        for i ∈ 1:N_missing
    ]

    err = (
        B_impute_μ .- first.(B_impute_pi),
        last.(B_impute_pi) .- B_impute_μ
    )

    p1 = scatter(dat_list.B, dat_list.K,
    xlab="neocortex percent (std)", ylab="kcal
    milk (std)")
    Ki = dat_list.K[miss_mask]
    scatter!(B_impute_μ, Ki, mc=:red,
    label="missing", xerr=err)


    #scatter!(B_impute_μ, Ki, xerr=err, ms=0)

    p2 = scatter(dat_list.M, dat_list.B,
    ylab="neocortex percent (std)", xlab="log body
    mass (std)")
    Mi = dat_list.M[miss_mask]
    scatter!(Mi, B_impute_μ, mc=:red,
    label="missing", yerr=err)
    #scatter!(Mi, B_impute_μ, yerr=err, ms=0)

    plot(p1, p2, size=(800, 400))
end
```

model `m15_7_2` : B_impute is undef Float64.

- Still buggy. The observed M is regarded as missing as well in this model.
- A better option might be employing two conditional distributions. M|B, and B|M.

```
md"### model `m15_7_2`: B_impute is undef Float64.
- Still buggy. The observed M is regarded as
missing as well in this model.
- A better option might be employing two
conditional distributions. M|B, and B|M."
```

```
m15_7_2 (generic function with 2 methods)
```

```
@model function m15_7_2(K, MB, M_missingB)
    σ ~ Exponential()
    σ_BM ~ Exponential()
    a ~ Normal(0, 0.5)
    μB ~ Normal(0, 0.5)
    μM ~ Normal(0, 0.5)
    bB ~ Normal(0, 0.5)
    bM ~ Normal(0, 0.5)
    Rho_BM ~ LKJ(2, 2)

    Σ = (σ_BM .* σ_BM') .* Rho_BM

    # process complete cases
    for i ∈ eachindex(MB)
        MB[i] ~ MvNormal([μM, μB], Σ)
    end

    # impute and process incomplete cases
    N_missing = length(M_missingB)
    B_impute = Array{Float64}(undef, N_missing)  #
    Note =, not ~. Note Float64, not Real.
    Vector{..} also works.
    #B_impute ~ filldist(Normal(), N_missing)   #
    this would cause all estimates to be from the
    prior.
    #B_impute ~ filldist(Normal(μB, σ_BM),
    N_missing)  # this would fail to sample.
    MB_missingB = [
        [m, b]
        for (m, b) ∈ zip(M_missingB, B_impute)
    ]

    for i ∈ eachindex(MB_missingB)
        MB_missingB[i] ~ MvNormal([μM, μB], Σ)
        MB_missingB[i][1] = M_missingB[i] # this
        would pull the estimated B values closer
        to the main trend, but not as much as the
        book.

        #MB_missingB[i] ~ MvNormal([M_missingB[i],
        μB], Σ) # this didn't improve.

        #MB_missingB[i][1] ~ Normal(μM, σ_BM) #
        this is wrong. same random variable
        defined twice.

    end

    # from both sets, build mean vector for K
    μ = [
        a + bB * b + bM * m
        for (m, b) ∈ Iterators.flatten((MB,
MB_missingB))
    ]

    @. K ~ Normal(μ, σ)
end
```

	MB_missingB[10][1]	MB_missingB[10][2]	MB_missingB[1][1]
1	0.334615	-0.379059	-0.70782
2	-1.32381	-0.846944	0.649774
3	-0.0387845	0.206419	-0.438227
4	-0.527676	0.282668	-0.948367
5	-0.325789	0.951219	1.60756
6	-2.5738	-1.19229	-2.82478
7	-0.450555	0.537884	-0.0785012
8	-1.64276	-1.82066	-0.297684
9	1.36486	1.17069	0.540476
10	-0.7454	-0.262098	0.741061
more			
1000	-1.093	-0.534637	-0.166601

```
begin
    Random.seed!(1)

    @time m15_7_2_ch = sample(m15_7_2(KK, MB, M_missingB), NUTS(), 1000)
    m15_7_2_df = DataFrame(m15_7_2_ch);
end
```

```
Sampling 100%

Found initial step size
ϵ: 0.05

19.886576 seconds (66.76 M allocations: 8.762 GiB, 8.28% gc time, 44.88% compilation time)
```

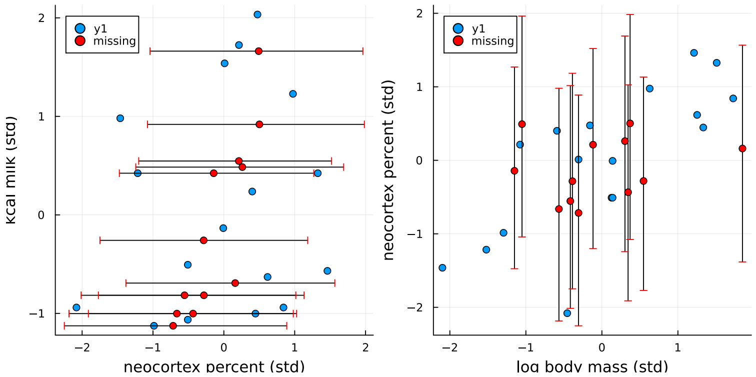
	variable	mean	min
1	Symbol("MB_missingB[10][1]")	-0.238076	-3.594
2	Symbol("MB_missingB[10][2]")	-0.434491	-3.632
3	Symbol("MB_missingB[11][1]")	-0.141753	-3.342
4	Symbol("MB_missingB[11][2]")	-0.28145	-3.191
5	Symbol("MB_missingB[12][1]")	0.123383	-3.026
6	Symbol("MB_missingB[12][2]")	0.160404	-3.138
7	Symbol("MB_missingB[1][1]")	-0.328377	-4.049
8	Symbol("MB_missingB[1][2]")	-0.554389	-3.410
9	Symbol("MB_missingB[2][1]")	-0.407122	-3.894
10	Symbol("MB_missingB[2][2]")	-0.716034	-3.708
11	Symbol("MB_missingB[3][1]")	-0.381969	-3.711
12	Symbol("MB_missingB[3][2]")	-0.662566	-3.535
13	Symbol("MB_missingB[4][1]")	-0.132596	-3.963
14	Symbol("MB_missingB[4][2]")	-0.284366	-3.675
15	Symbol("MB_missingB[5][1]")	0.353378	-3.419

```
describe(m15_7_2_df)
```

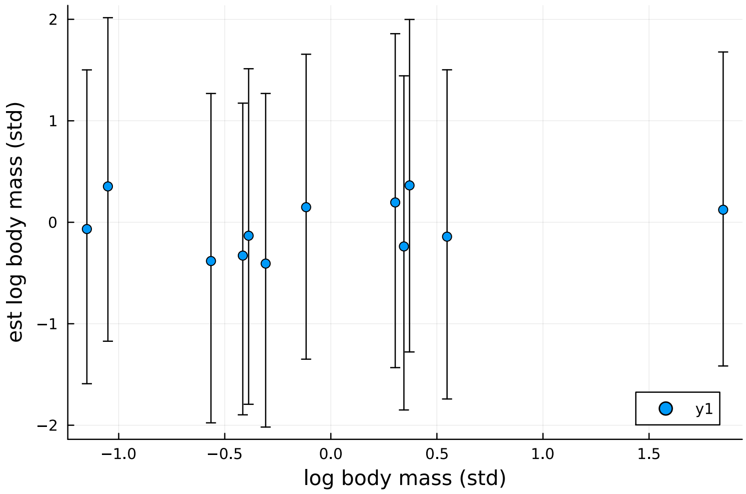
Plot `m15_7_2` estimates

- The Julia model didn't use the observed values for the M variable and instead sampled M as well.
- That results in imputation not working very well. Both estimated M and estimated B hover around 0.

```
md"### Plot `m15_7_2` estimates
- The Julia model didn't use the observed values
for the M variable and instead sampled M as well.
- That results in imputation not working very
well. Both estimated M and estimated B hover
around 0."
```




```
let
    N_missing = sum(ismissing.(dat_list.B))
    miss_mask = ismissing.(dat_list.B)

    B_impute_μ = [
        mean(m15_7_2_df[!,"MB_missingB[$i][2]"])
        #mean(m15_7_df[!,"B_impute[$i]"])
        for i ∈ 1:N_missing
    ]

    B_impute_pi = [
        PI(m15_7_2_df[!,"MB_missingB[$i][2]"])
        #PI(m15_7_df[!,"B_impute[$i]"])
        for i ∈ 1:N_missing
    ]

    err = (
        B_impute_μ .- first.(B_impute_pi),
        last.(B_impute_pi) .- B_impute_μ
    )

    p1 = scatter(dat_list.B, dat_list.K,
    xlab="neocortex percent (std)", ylab="kcal
    milk (std)")
    Ki = dat_list.K[miss_mask]
    scatter!(B_impute_μ, Ki, xerr=err, mc=:red,
    label="missing")


    #scatter!(B_impute_μ, Ki, xerr=err, ms=0)

    p2 = scatter(dat_list.M, dat_list.B,
    ylab="neocortex percent (std)", xlab="log body
    mass (std)")
    Mi = dat_list.M[miss_mask]
    scatter!(Mi, B_impute_μ, yerr=err, mc=:red,
    label="missing")
    #scatter!(Mi, B_impute_μ, yerr=err, ms=0)

    plot(p1, p2, size=(800, 400))
end
```




```
let
    N_missing = sum(ismissing.(dat_list.B))
    miss_mask = ismissing.(dat_list.B)

    M_impute_μ = [
        mean(m15_7_2_df[!,"MB_missingB[$i][1]"])
        for i ∈ 1:N_missing
    ]

    M_impute_pi = [
        PI(m15_7_2_df[!,"MB_missingB[$i][1]"])
        for i ∈ 1:N_missing
    ]


    err = (
        M_impute_μ .- first.(M_impute_pi),
        last.(M_impute_pi) .- M_impute_μ
    )

    Mi = dat_list.M[miss_mask]
    p2 = scatter(Mi, M_impute_μ, yerr=err,
    ylab="est log body mass (std)", xlab="log body
    mass (std)")
    #scatter!(Mi, M_impute_μ, yerr=err, ms=0)

    p2
end
```

ToDo: split the bivariate normal into two univariate conditional normal

```
md"### ToDo: split the bivariate normal into two
univariate conditional normal"
```

Code 15.23 Obtain index of data with missing B (Brain/Neocortex size)

```
md"## Code 15.23 Obtain index of data with missing B (Brain/Neocortex size)"
```

```
BitVector: [false, true, true, true, true, false, false,
```

```
ismissing.(dat_list.B)
```

Code 15.24 Load the Gods dataset

```
md"## Code 15.24 Load the Gods dataset"
```

	variable	mean	min	me
1	:polity	nothing	"Big Island Hawaii"	not
2	:year	-1339.35	-9600	-60
3	:population	4.86246	1.40832	4.7
4	:moralizing_gods	0.949405	0	1.0
5	:writing	0.459491	0	0.0

```
begin
    d_gods =
    DataFrame(CSV.File("data/Moralizing_gods.csv",
    missingstring="NA"))
    describe(d_gods)
end
```

	polity	year	population	moralizing_g
1	"Big Island Hawaii"	1000	3.72964	missing
2	"Big Island Hawaii"	1100	3.72964	missing
3	"Big Island Hawaii"	1200	3.59834	missing
4	"Big Island Hawaii"	1300	4.02624	missing

```
first(d_gods, 4)
```

Code 15.25 Count rows with different moralizing_gods

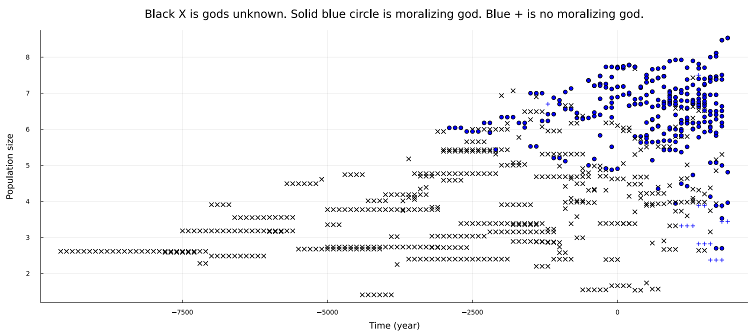
```
md"## Code 15.25 Count rows with different moralizing_gods"
```

```
Dict(0 ⇒ 17, missing ⇒ 528, 1 ⇒ 319)
```

```
countmap(d_gods.moralizing_gods)
```

Code 15.26 Fig 15.7 Plot pop vs time

```
md"## Code 15.26 Fig 15.7 Plot pop vs time"
```




```
let
    symbol = map(g -> ismissing(g) ? :x : (g == 1 ? :circle : :+), d_gods.moralizing_gods)
    color = map(g -> ismissing(g) ? :black : :blue, d_gods.moralizing_gods)
    scatter(d_gods.year, d_gods.population, m=symbol, mc=color,
        xlab="Time (year)", ylab="Population size", title="Black X is gods unknown. Solid blue circle is moralizing god. Blue + is no moralizing god.", legend=false,
        size=(1400,600), margin=5*Plots.mm)
end
```

Code 15.27

- Check how god missingness is associated with writing/literacy.

```
md"## Code 15.27
- Check how god missingness is associated with writing/literacy."
```

```
Dict((0, 0) ⇒ 16, (1, 1) ⇒ 310, (missing, 1) ⇒ 86, (mis
```

```
countmap(zip(d_gods.moralizing_gods, d_gods.writing))
```

Code 15.28 Check how moralizing_gods varies over years for Hawaii

```
md"## Code 15.28 Check how moralizing_gods varies over years for Hawaii"
```

	year	writing	moralizing_gods
1	1000	0	missing
2	1100	0	missing
3	1200	0	missing
4	1300	0	missing
5	1400	0	missing
6	1500	0	missing
7	1600	0	missing
8	1700	0	missing
9	1800	0	1

```
d_gods[d_gods.polity .== "Big Island Hawaii", ["year", "writing", "moralizing_gods"]]
```

15.3 Categorical errors and discrete absences

```
md"# 15.3 Categorical errors and discrete absences"
```

Code 15.29 Simulate data

- parameter `r` (the rate of missing) has a large impact on accuracy of the β estimate (less so on the α estimate). The higher it is, the less accurate the β estimate is.

```
md"## Code 15.29 Simulate data
- parameter `r` (the rate of missing) has a large impact on accuracy of the β estimate (less so on the α estimate). The higher it is, the less accurate the β estimate is.

"
```

`simulate_missing_cat_data`

- cat_probability/k: probability that there is a cat in a house.
- `missing_rate`/r: The probability if the presence of cat in a house is unknown. The higher the `missing_rate` is, the less accurate the β estimate is.

```
"""
- cat_probability/k: probability that there is a cat in a house.
- `missing_rate`/r: The probability if the presence of cat in a house is unknown. The higher the `missing_rate` is, the less accurate the β estimate is.

"""
function simulate_missing_cat_data(N_houses::Int; α=5, β=-2, cat_probability=0.5, missing_rate=0.2)
    Random.seed!(9)


    cat = rand(Bernoulli(cat_probability), N_houses)
    # music_notes is the number of notes that the songbird in the house will sing.
    music_notes = rand.([Poisson(exp(α + β * c)) for c ∈ cat])   # wrongly omitted exp() before.

    R_C = rand(Bernoulli(missing_rate), N_houses)

    cat_obs = Vector{Int}(cat)
    cat_obs[R_C] .= -9 # -9 means unknown/missing .

    dat = (
        notes = music_notes,
        cat = cat_obs,
        RC = R_C,
        N = N_houses,
    )
end
```

Code 15.30 `m15_8`

```
md"## Code 15.30 `m15_8`"
```

```
m15_8 (generic function with 2 methods)
```

```julia
@model function m15_8(notes, cat, RC, N)
    α ~ Normal(0, 2)
    β ~ Normal(0, 2) #Uniform(-10, 10) does not help.
    k ~ Beta(2, 2)
    λ = @. exp(α + β * cat) # was logistic() in the original code.

    for i ∈ eachindex(cat)
        if !RC[i]  # Cat is not missing. RC[i]==0.
            cat[i] ~ Bernoulli(k)
            notes[i] ~ Poisson(λ[i])
            #Turing.@addlogprob! poislogpdf(λ[i], notes[i])   #equivalent to above ~.
        else
            # OR replace the following with https://discourse.julialang.org/t/how-to-logsumexp-jl/103894
            Turing.@addlogprob! log(k*poispdf(exp(α+β), notes[i]) + (1-k)*poispdf(exp(α), notes[i]))

            #Turing.@addlogprob! log(k) + poislogpdf(exp(α+β), notes[i]) #wrong
            #Turing.@addlogprob! log(1-k) + poislogpdf(exp(α), notes[i])
        end
    end
end
```

`m15_8_1`

Difference vs `m15_8`: use logsumexp(log(a), log(b)) instead of log(a+b)

```
"""
Difference vs `m15_8`: use logsumexp(log(a), log(b)) instead of log(a+b)
"""
@model function m15_8_1(notes, cat, RC, N)
    α ~ Normal(0, 2)
    β ~ Normal(0, 2) #Uniform(-10, 10) does not help.
    k ~ Beta(2, 2)
    λ = @. exp(α + β * cat) # was logistic() in the original code.

    for i ∈ eachindex(cat)
        if !RC[i]  # Cat is not missing. RC[i]==0.
            cat[i] ~ Bernoulli(k)
            notes[i] ~ Poisson(λ[i])
            #Turing.@addlogprob! poislogpdf(λ[i], notes[i])   #equivalent to above ~.
        else
            # OR replace the following with https://discourse.julialang.org/t/how-to-logsumexp-jl/103894
            #Turing.@addlogprob! log(k*poispdf(exp(α+β), notes[i]) + (1-k)*poispdf(exp(α), notes[i]))
            Turing.@addlogprob! logsumexp(log(k) + poislogpdf(exp(α+β), notes[i]), log(1-k) + poislogpdf(exp(α), notes[i]))
        end
    end
end
```

	variable	mean	min	median	max	nn
1	:k	0.497986	0.44571	0.498075	0.549806	0
2	:α	4.99897	4.9879	4.99889	5.01093	0
3	:β	-1.97624	-2.00892	-1.97627	-1.94196	0

```
begin
    dat1 = simulate_missing_cat_data(1000, α=5,
    β=-2, cat_probability=0.5, missing_rate=0.2)
    @time m15_8_df1 =
    DataFrame(sample(m15_8(dat1...), NUTS(), 2000))
    describe(m15_8_df1)
end
```

```
Sampling 100%

Found initial step size
ϵ: 0.0015625

 7.440322 seconds (6.40 M allocations: 88
3.491 MiB, 2.92% gc time, 51.82% compilation t
ime)

(notes = [16, 13, 10, 22, 18, 141, 145, 29, 147,    more ,1
```

```
dat1
```

	variable	mean	min	median	max	nn
1	:k	0.497871	0.441962	0.497624	0.552211	0
2	:α	4.99902	4.98768	4.99901	5.01396	0
3	:β	-1.9764	-2.01855	-1.97631	-1.93809	0

```
let
    dat_tmp = simulate_missing_cat_data(1000, α=5,
    β=-2, cat_probability=0.5, missing_rate=0.2)
    @time m15_8_1_df_tmp =
    DataFrame(sample(m15_8_1(dat_tmp...), NUTS(),
    2000))
    describe(m15_8_1_df_tmp)
end
```

```
Sampling 100%

Found initial step size
ϵ: 0.0015625

 7.931540 seconds (6.56 M allocations: 93
3.997 MiB, 3.06% gc time, 49.13% compilation t
ime)
```

	variable	mean	min	median	max	nn
1	:k	0.497956	0.44724	0.498179	0.560882	0
2	:α	4.99899	4.98747	4.99896	5.01088	0
3	:β	-1.97613	-2.00797	-1.97628	-1.94324	0

```
begin
    dat2 = simulate_missing_cat_data(1000, α=5,
    β=-2, cat_probability=0.5, missing_rate=0.01)
    @time m15_8_df2 =
    DataFrame(sample(m15_8(dat2...), NUTS(), 2000))
    describe(m15_8_df2)
end
```

```
Sampling 100%

Found initial step size
ϵ: 0.0015625

3.828394 seconds (3.34 M allocations: 89
1.546 MiB, 4.13% gc time)
```

	variable	mean	min	median	max	nn
1	:k	0.498062	0.44529	0.497948	0.549062	0
2	:α	4.99909	4.9872	4.99896	5.01028	0
3	:β	-1.97655	-2.01338	-1.97657	-1.93935	0

```
let
    dat3 = simulate_missing_cat_data(1000, α=5,
    β=-2, cat_probability=0.5, missing_rate=0.001)
    @time m15_8_df3 =
    DataFrame(sample(m15_8(dat3...), NUTS(), 2000))
    describe(m15_8_df3)
end
```

```
Sampling 100%

Found initial step size
ϵ: 0.0015625

3.415802 seconds (3.16 M allocations: 82
0.227 MiB, 3.40% gc time)
```

	variable	mean	min	median	max	nn
1	:k	0.496482	0.438809	0.496547	0.550727	0
2	:α	3.32288	3.30037	3.32287	3.34949	0
3	:β	1.6364	1.60497	1.63654	1.66679	0

```
let
    dat_tmp = simulate_missing_cat_data(1000, α=5,
    β=-2, cat_probability=0.5, missing_rate=0.95)
    @time m15_8_df_tmp =
    DataFrame(sample(m15_8(dat_tmp...), NUTS(),
    2000))
    describe(m15_8_df_tmp)
end
```

```
Sampling 100%

Found initial step size
ϵ: 0.003125
```

```
8.536856 seconds (4.23 M allocations: 1.224 GiB, 2.07% gc time)
```

	variable	mean	min	median	max	nn
1	:k	0.496173	0.446996	0.495608	0.547061	0
2	:α	3.32272	3.29307	3.32286	3.34824	0
3	:β	1.63657	1.60228	1.63628	1.67	0

```
let
    dat_tmp = simulate_missing_cat_data(1000, α=5,
    β=-2, cat_probability=0.5, missing_rate=0.95)
    @time m15_8_1_df_tmp =
    DataFrame(sample(m15_8_1(dat_tmp...), NUTS(),
    2000))
    describe(m15_8_1_df_tmp)
end
```

```
Sampling 100%

Found initial step size
ϵ: 0.003125
```

```
10.869776 seconds (4.31 M allocations: 1.253 GiB, 2.11% gc time)
```

cat prob=0.8

```
md"### cat prob=0.8"
```

	variable	mean	min	median	max	nn
1	:k	0.227394	0.176452	0.227297	0.285104	0
2	:α	3.13295	3.10584	3.13299	3.15594	0
3	:β	1.73059	1.69734	1.73065	1.76578	0

```
let
    dat_tmp = simulate_missing_cat_data(1000, α=5, β=-2, cat_probability=0.8, missing_rate=0.95)
    @time m15_8_df_tmp = DataFrame(sample(m15_8(dat_tmp...), NUTS(), 2000))
    describe(m15_8_df_tmp)
end
```

```
Sampling 100%
Found initial step size
ϵ: 0.003125
```

```
6.109495 seconds (3.31 M allocations: 885.675 MiB, 2.18% gc time)
```

	variable	mean	min	median	max	nn
1	:k	0.35218	0.295682	0.351507	0.404161	0
2	:α	3.45952	3.43594	3.4597	3.48342	0
3	:β	0.862368	0.831262	0.862348	0.891627	0

```
let
    dat_tmp = simulate_missing_cat_data(1000, α=5, β=-2, cat_probability=0.8, missing_rate=0.75)
    @time m15_8_df_tmp = DataFrame(sample(m15_8(dat_tmp...), NUTS(), 2000))
    describe(m15_8_df_tmp)
end
```

```
Sampling 100%
Found initial step size
ϵ: 0.003125
```

```
5.725503 seconds (3.35 M allocations: 899.454 MiB, 2.97% gc time)
```

	variable	mean	min	median	max	nn
1	:k	0.790216	0.735548	0.790597	0.830519	0
2	:α	5.00003	4.97665	4.99995	5.02018	0
3	:β	-1.98676	-2.02405	-1.98696	-1.95199	0

```
let
    dat_tmp = simulate_missing_cat_data(1000, α=5,
    β=-2, cat_probability=0.8, missing_rate=0.5)
    @time m15_8_df_tmp =
    DataFrame(sample(m15_8(dat_tmp...), NUTS(),
    2000))
    describe(m15_8_df_tmp)
end
```

```
Sampling 100%

Found initial step size
ϵ: 0.003125

4.626353 seconds (3.12 M allocations: 80
4.544 MiB, 3.48% gc time)
```

	variable	mean	min	median	max	nn
1	:k	0.789995	0.74857	0.790038	0.833331	0
2	:α	5.00019	4.98285	5.00037	5.01959	0
3	:β	-1.98718	-2.03088	-1.98703	-1.95718	0

```
let
    dat_tmp = simulate_missing_cat_data(1000, α=5,
    β=-2, cat_probability=0.8, missing_rate=0.25)
    @time m15_8_df_tmp =
    DataFrame(sample(m15_8(dat_tmp...), NUTS(),
    2000))
    describe(m15_8_df_tmp)
end
```

```
Sampling 100%

Found initial step size
ϵ: 0.003125

4.076087 seconds (3.15 M allocations: 81
7.418 MiB, 3.37% gc time)
```

	variable	mean	min	median	max	nn
1	:k	0.790049	0.75132	0.790476	0.827859	0
2	:α	5.00029	4.98122	5.00022	5.0206	0
3	:β	-1.98733	-2.01634	-1.98772	-1.95452	0

```
let
    dat_tmp = simulate_missing_cat_data(1000, α=5,
    β=-2, cat_probability=0.8, missing_rate=0.05)
    @time m15_8_df_tmp =
    DataFrame(sample(m15_8(dat_tmp...), NUTS(),
    2000))
    describe(m15_8_df_tmp)
end
```

```
Sampling 100%

Found initial step size
ϵ: 0.003125

3.620584 seconds (3.26 M allocations: 86
2.116 MiB, 3.09% gc time)
```

	variable	mean	min	median	max	nn
1	:k	0.789279	0.7493	0.789522	0.830389	0
2	:α	5.00024	4.98038	5.00009	5.0216	0
3	:β	-1.98737	-2.01635	-1.98742	-1.95488	0

```
let
    dat_tmp = simulate_missing_cat_data(1000, α=5,
    β=-2, cat_probability=0.8, missing_rate=0.01)
    @time m15_8_df_tmp =
    DataFrame(sample(m15_8(dat_tmp...), NUTS(),
    2000))
    describe(m15_8_df_tmp)
end
```

```
Sampling 100%

Found initial step size
ϵ: 0.003125

3.375091 seconds (3.15 M allocations: 81
7.728 MiB, 3.63% gc time)
```