

Introduction to Monads

Your Everyday Chainable Decorators

2013-10-03



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Brief historical introduction

- Philosophy (a.c.) – The Unit. The One “essence’ that generates all others. Belongs to a category of “generators” like the Dyad and Triad, etc..
- Algebra (1964)- A 3-set construct with mappings between them ($A \rightarrow B \rightarrow C$).
- Category Theory – The Composition of two adjoint *Functors* (*GoF*)
- Computer Science (1980)- Created in Opal and then used in Haskell (~1990) that where functional languages. Nowadays used in OO languages with functional support like Scala and C#

What Are Monads

- $3 = 1 + 2$
- $\text{Box}(3) = \text{Box}(1) + \text{Box}(2)$
- $\text{Box}(3) = \text{Box}(1) (+) \text{Box}(2)$
- $\text{Box}(3) = \bullet (\text{Box}(1), \text{Box}(2), (x, y) \Rightarrow x+y)$
- $\text{Box}(3) = \bullet (\text{Box}(1), \text{Box}(2), \text{Func}(x, y))$
- $\text{Unit}(T \text{ obj}) : \text{Box}\langle T \rangle$
- $\text{Fmap}(\text{Func}\langle T, U, R \rangle g) : \text{Func}\langle \text{Box}\langle T \rangle, \text{Box}\langle U \rangle, \text{Box}\langle R \rangle \rangle$



What are Monads

- Are Types with
 - A method, Unit, to augment other objects.
 - A method, Fmap, to transform a function to another function (Fmap is the equivalent of a Functor)
- Or
 - A method, Fmap, to transform a function to another function
 - A method, Join , to de-augment a double augmented type
- Or
 - A method, Unit, to augment other objects in it.
 - A method, Bind, to operate between two object of the same monad

What are Monads

- $\text{Fmap}(f)(m) \Leftrightarrow \text{Bind}(m, f)$
 - $\text{Join}(m) \Leftrightarrow \text{Bind}(m, I)$
 - $\text{Bind}(m, f) \Leftrightarrow \text{Join}(\text{Fmap}(f)(m))$
-
- where $I(y) = y$
 - f is a function over x
 - m the monad value

Monadic Laws

1. Left Identity

$$\text{Bind}(\text{Unit}(x), f) \Leftrightarrow f(x)$$

2. Right Identity

$$\text{Bind}(m, \text{Unit}) \Leftrightarrow m$$

3. Associativity

$$m = \text{Unit}(x)$$

$$\text{Bind}(\text{Bind}(m, f), g) \Leftrightarrow g(f(x))$$



That is a Monad !?



What is a Decorator

- It's a design pattern
- Provides an object type with new methods
- Normally wraps the original object and even operates over it.



What is a Chainable Decorator

- It's a Decorator with operations that return the same Decorator type allowing to invoke it again

```
decorator.x().y().z()
```

- Each invocation returns a new, immutable object



What are Monads in OO

- Are Chainable Decorator Types with
 - A method, Unit, to wrap (augment) other objects.
 - A method, Bind to operate over other augmented types
- And
 - Additional methods to operate with regardless of the augmented object

Monads and C#

- C# Constructor and/or Extensions features support the implementation of the Unit Monad method.

```
public static Box<T> ToBox<T>(this T obj)
{
    return new Box<T>(obj);
}
```

- This is the expected construction for a decorator factory method
- Typing the extension method allows for polymorphic monad construction

Monads and C#

- Typing the extension method allows for polymorphic monad construction

```
public static Maybe<S> ToMaybe<S>(this Nullable<S> value)
where S : struct
{
    return !value.HasValue
        ? Maybe<S>.Nothing
        : new Maybe<S>(value.Value);
}

public static Maybe<string> ToMaybe(this string value)
{
    return string.IsNullOrEmpty(value)
        ? Maybe<string>.Nothing
        : new Maybe<string>(value);
}
```

Monad and LINQ

- LINQ (Language Integrated Query) is a general, agnostic mechanics to support the Monad concept
- Any type that implements (directly or by extension) the SelectMany method can be used with LINQ.

SelectMany is the Monad Bind operation

```
Box<T> SelectMany<T,U,V> (this Box<T> m , Func<T, M<T>>> k,  
Func<T,U,V> s)
```

- Then you may write:

```
var result = from x in 1.ToBox()  
             from y in 2.ToBox()  
             select (x + y);
```

This is called do-notation (or computation expression in F#)



Fundamental Monads

- Identity – just augments a value. No special operations.
- Maybe – keeps track of the absence of value. A maybe object can be a Nothing or a Just. It allows to compute complex chained expressions that can handle Nothing (null) automatically without exceptions.
- Collection – keeps track of a set of objects. It allows for bulk chained operations like adding , removing, filtering, mapping, etc...
- Writer– allows to write to another object in the “background” while executing a complex chained expression.



Identity Monad

- Simply augments a type to another
- The Box type used in the examples is really and implementation of the Identity Monad
- Using a computation expression is possible to operate with the augment objects.

Identity Monad and C#

```
public class Box<T> {  
    public property Value {get; private set;}  
    protected Box(T x)  
    {  
        this.Value = x;  
    }  
}  
  
public static class BoxExtentions{  
    // Unit  
    public static Box<T> ToBox (this T x ){  
        return new Box(x);  
    }  
    // Bind  
    public static Box<R> SelectMany(this Box<T> box, Func<T, Box<R>> f)  
    {  
        return f(box.Value)  
    }  
    ...  
}
```

Identity Monad em C#

```
public static class BoxExtensions{
    // Unit
    public static Box<T> ToBox (this T x ){
        return new Box(x);
    }
    // Bind
    public static Box<R> SelectMany(this Box<T> box, Func<T, Box<R>> f)
    {
        return f(box.Value)
    }
    // Bind
    public static Box<R> SelectMany(this Box<T> m, Func<T, Box<V>> f,
        Func < T, R, V> s
    {
        return m.SelectMany(x => k(x)
            .SelectMany(y => s(x, y).ToBox())));
    }
}
```




Identity Monad em C#

```
Box<int> three = from x in 1.ToBox()  
                 from y in 2.ToBox()  
                 select (x + y);
```

```
// Or
```

```
Box<int> three = 1.ToBox().SelectMany( x => 2.ToBox(), (x,y) =>x+y );
```



Collection Monad

- Binds together objects in a set
- Allows for bulk operations on every element of the set (foreach)
- Allows for operations on the set like, filtering, ordering, mapping, etc...
- `IEnumerable<T>` is an implementation of the Collection Monad
- Concat is the special operation that adds another element to the collection

Collection Monad

```
public static class CollectionExtensions{  
    // Unit – any object  
    public static Bag<T> ToBag(this T x){  
        return new Bag().Add(x);  
    }  
    // add another element  
    public static Bag<T> Concat(this Bag<T> bag, T x)  
    {  
        return bag.Add(x);  
    }  
}
```



Maybe Monad

- Allows for chaining complex expressions always keeping track of absent values.
 - The absent value is named `Nothing`.
 - The not absent value is named `Just`
- In practice allows to execute always valid operations even in the presence of absent values.
- Special methods are:
 - `Or` - Translates the Maybe to its value , or a default value
 - `Select` - Operates over the inner object and returns a Maybe
- `Nullable<S>` in C# is an implementation of the Maybe Monad with special support in the compiler. But only can be used for struts. No special methods are provided

Maybe Monad

```
public static class MaybeExtensions{
    // Unit - any object
    public static Maybe<T> ToMaybe (this T value ){
        return value == null ? Maybe<T>.Nothing : new Maybe<T>(value);
    }
    // Unit - string
    public static Maybe<string> ToMaybe(this string value)
    {
        return string.IsNullOrEmpty(value)
            ? Maybe<string>.Nothing
            : new Maybe<string>(value);
    }
    // Unit - Nullable
    public static Maybe<S> ToMaybe<S>(this Nullable<S> value)
    where S : struct
    {
        return !value.HasValue
            ? Maybe<S>.Nothing
            : new Maybe<S>(value.Value);
    }
    ...
}
```



Maybe Monad

```
public static class MaybeExtensions{  
    ...  
    // Or - Reduce to value  
    public static T Or (this Maybe<T> x , T defaultValue)  
    {  
        return x.HasValue ? x.Value : defaultValue;  
    }  
    // Select  
    public static Maybe<V> Select<T,V>(this Maybe<T> x, Func<T,V> k)  
    {  
        return !x.HasValue  
        ? Maybe<V>.Nothing  
        : k(m.Value).ToMaybe()  
    }  
    ...  
}
```


Nullable Extensions

```
public static class NullableExtensions{
    ...
    // Or - Reduce to value
    public static S Or (this Nullable<S> x , S defaultValue)
        where S : struct
    {
        return x.HasValue ? x.Value : defaultValue;
    }
    // Select
    public static Nullable<V> Select<T,V>(this Nullable<S> x,
        Func<S,Nullable<V>> k )
        where S : struct
        where V : struct
    {
        return !x.HasValue
            ? (Nullable<V>)null
            : k(m.Value)
    }
    ...
}
```

To Maybe or not To Maybe

```
public InitView(){
    int? id= View.GetId();
    var client;
    if ( id.HasValue)
    {
        client = ServiceSearch(id);
    }
    else
    {
        client = new Client();
    }
    View.Show(client);
}
```

```
public InitView(){
    var client = View.GetId().Select( id =>ServiceSearch(id)).Or(new Client());
    View.Show(client);
}
```

```
public InitView(){
    var client = View.GetId().AlsoNothing( id => id <= 0)
        .Select( id =>ServiceSearch(id)).Or(new Client());
    View.Show(client);
}
```


To Maybe or not to Maybe

```
public bool IsReadOnly()  
{  
    var isReadOnlyString = Request["isReadOnly"];  
    if (string.IsNullOrEmpty(isReadOnlyString))  
        return false;  
    bool isReadOnly;  
  
    return bool.TryParse(isReadOnlyString, out isReadOnly) ? isReadOnly : false;  
}
```

```
public bool IsReadOnly()  
{  
    return Request["isReadOnly"].ToMaybe().Convert<bool>().Or(false);  
}
```

To Maybe or not to Maybe

```
public Money Tax( Money base, Fraction taxInterest)
{
    if (base != null && taxInterest != null)
    {
        return money * taxInterest;
    }
    else
    {
        return ??????
    }
}
```


To Maybe or not to Maybe

```
public Maybe<Money> Tax( Maybe<Money> base,  
    Maybe<Fraction> taxInterest)  
{    // do -notation  
    return from x in base  
        from y in taxInterest  
        select ( x * y);  
}
```

```
public Maybe<Money> Tax( Maybe<Money> base,  
    Maybe<Fraction> taxInterest)  
{    // explicit linq  
    return base.SelectMany ( base => taxInterest , (base,  
taxInterest) => base * taxInterest );  
}
```



Writer Monad

- Allows to write to a “background” object while executing chained operations
- Originally used for file writing I/O operations, logging or debugging (as functions cannot have secondary effects)
- Today is the basis for a robust implementations of the Builder pattern (specially when fluent interface is used) and simple Domains Specific Languages (DSL)
- LINQ to Relational Data uses this monad to write to Expression objects (with the help of the compiler) that are then run through an Interpreter (the LINQ Provider)



Writer Monad

```
public class Client {  
  
    public string Name {get;set;}  
    public string Address {get;set;}  
  
    public override int GetHashCode()  
    {  
        return Name.GetHashCode() * 17  
            + Address.GetHashCode();  
    }  
}
```



Writer Monad

```
public class Hash {  
    private int hash;  
    public Hash (int hash){  
        this.hash = hash;  
    }  
    // get value  
    public override int GetHashCode()  
    {  
        return hash;  
    }  
    ... // equals  
}
```


Writer Monad

```
public static class HashExtentions{
    private static readonly int prime = 17

    // Unit - any object
    public static Hash ToHash (this object value ){
        return new Hash(value.GetHashCode())
    }

    // Add another object to the hash
    public static Hash Concat(this Hash value, object other )
    {
        return other == null
            ? value
            : new Hash( // the composite hash rule
                value.GetHashCode() * prime + other.GetHashCode()
            );
    }

    // Add a collection of objects
    public static Hash Concat(this Hash value, IEnumerable<object> others)
    {
        var result = value;
        foreach ( T element in others)
        {
            result = result.Concat(element);
        }
        return result;
    }
}
```



Writer Monad

```
public class Client {  
  
    public string Name {get;set;}  
    public string Address {get;set;}  
  
    public override int GetHashCode()  
    {  
        return Name.ToHash()  
            .Concat(Address)  
            .GetHashCode();  
    }  
}
```




Writer Monad

```
public class Client {  
  
    public string Name {get;set;}  
    public string Address {get;set;}  
  
    public override int GetHashCode()  
    {  
        return from x in Name.ToHash()  
               from y in Address.ToHash()  
               select (x * 17 + y)  
    }  
}
```

Other Monads

- Reader (aka Environment) – The counterpart of the Writer Monad. Allows for accessing values in the chain from outside the original augmented value

```
string userName = user.ToReader()  
    .Select ( (ctx, user) => ctx.IsUserAuthenticated(user)  
        ? user.Name  
        : "Guest" )  
    .RunIn(appcontext);
```

- State – Allows for tracking state in a chained operation.

```
var state = microwave.ToState(State.Closed)  
    .Open()  
    .Put(meal)  
    .SetTimer(2, Time.Minutes);  
    // changes the door state from Closed to Open  
    // changes the timer state from NotSet to Set  
if (state.IsDoorClosed() && state.IsTimerSet()) {  
    microwave.Start();  
}
```

- Continuation* – Allows for postponing operations. Useful in the presence of distributed/multithreaded operations.

```
list.Do(list => list.Sum()).ContinueWith( sum => label.Text = sum)
```




Use Monads

- They are a special type of Decorator that
 - augment code and types capabilities
 - simplify code by encapsulating rules
 - reduce code and increase readability as they are chainable and strong typed
- Use Maybe Monad
 - to remove `null` checks and `NullReferenceException`
 - to simplify chained calculations that compute correctly even in the absence of some values
 - Reduces decisions around `null` (use of `if` and ternary conditions) increasing test coverage
- Use Write
 - to construct fluent powerful Builders
 - connect with LINQ and Expression
- Use Collection
 - to operate in bulk (for each, filter, map)
 - to do aggregations (sum, avg, reduce)
- Mix them up
 - A collection of maybe
 - Filtering a collection with a writer



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