# Outline for Topoligical Combinatorics

The premise of this talk would be to discuss the some of the basics of topology and then eventually state (and prove, maybe[1]) the Borsuk-Ulam Theorem, Sperner's Lemma, and Brouwer's fixed point theorem. Following which, we present a variety of applications in combinatorics. Most of the material comes from [1].

Topological Results:

**Theorem** (Brower Fixed Point Theorem)

Let $K \subseteq \mathbb{R}^n$ be convex and compact, and let $f : K \to K$ be continuous. Then, there exists $k \in K$ such that $f(k) = k$.

**Theorem** (Borsuk-Ulam Theorem)

Suppose $f : \mathbb{S}^n \to \mathbb{R}^n$ was continuous. Then, there is some $x \in \mathbb{S}^n$ such that $f(x) = f(-x)$.

Combinatorial Results:

**Theorem**

Every (open) necklace with $d$ kinds of jewels can be divided between two thieves using no more than $d$ cuts.

**Theorem** (Kneser Conjecture)

For all $k > 0$ and $n \geq 2k - 1$, the chromatic number of the Kneser graph $KG_{n,k}$ is $\chi(KG_{n,k}) = n - 2k + 2$.

The game of Hex is played between two players, red and green. Each of them takes turns coloring one hexagon in a grid. After all hexagons are filled, red wins if there is a red path connecting the top and bottom, and green wins of there is a green path connecting the left and right.

**Theorem**

The game of Hex cannot end in a draw.

[1] For the sake of time, we might limit the discussion of Borsuk-Ulam to a proof sketch.

## References

[1] Jiří Matoušek, Anders Björner, Günter M Ziegler, et al. *Using the Borsuk-Ulam theorem: lectures on topological methods in combinatorics and geometry*. Springer, 2003.