OUTLINE FOR TOPOLOGICAL COMBINATORICS

The premise of this talk would be to discuss the some of the basics of topology and then eventually state (and prove, maybe[1]) the Borsuk-Ulam Theorem and Brouwer's fixed point theorem. Following which, we present a variety of applications in combinatorics. Most of the material comes from [1].

1. Setup

2. Results in topology

Theorem 2.1 (Brower Fixed Point Theorem)**.** *Let $K \subseteq \mathbb{R}^n$ be convex and compact, and let $f : K \to K$ be continuous. Then, there exists $k \in K$ such that $f(k) = k$.*

Theorem 2.2 (Borsuk-Ulam Theorem)**.** *Suppose $f : \mathbb{S}^n \to \mathbb{R}^n$ was continuous. Then, there is some $x \in \mathbb{S}^n$ such that $f(x) = f(-x)$.*

Theorem 2.3. *For $n \geq 0$, the following statements are equivalent:*

1. *For every continuous mapping $f : \mathbb{S}^n \to \mathbb{R}^n$ there exists a point $\mathbf{x} \in \mathbb{S}^n$ with $f(x) = f(-x)$.*
2. *For every antipodal mapping $f : \mathbb{S}^n \to \mathbb{R}^n$, there exists a point $\mathbf{x} \in \mathbb{S}^n$ satisfying $f(\mathbf{x}) = 0$.*
3. *There is no antipodal mapping $f : \mathbb{S}^n \to \mathbb{S}^{n-1}$.*
4. *There is no continuuos mapping f :*

3. Results in Combinatorics

Theorem 3.1. *Every (open) necklace with d kinds of jewels can be divided between two thieves using no more than d cuts.*

Theorem 3.2. *The game of Hex cannot end in a draw.*

References

[1] Jiří Matoušek, Anders Björner, Günter M Ziegler, et al. *Using the Borsuk-Ulam theorem: lectures on topological methods in combinatorics and geometry.* Springer, 2003.

[1] For the sake of time, we might limit the discussion of Borsuk-Ulam to a proof sketch.