

Exeter Mathematics School Team Mathematical Olympiad
Round 2 : Monday 3rd May 2021

Time allowed One week, commencing 09:00 Monday 3rd

Scoring rules • Questions 1-5 will be marked out of 5, with the final answer given 5 marks (Credit will **not** be given for incomplete solutions).

- Questions 6-9 will be marked out of 10 (Credit **will** be given for incomplete solutions).
- Question 10 will be marked out of 15 (Credit **will** be given for incomplete solutions).

Instructions • Full written solutions for questions 6-10, *not just answers*, are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt (On paper or in L^AT_EX). Rough work *can* be handed in, but should be clearly marked.

- One or two complete solutions (For questions 6-10) will gain more credit than partial attempts at all problems.
- Do not communicate to other teams regarding the ETMO until the round is over.
- The use of the Internet or other sources to assist in the solving of any problems is strictly prohibited
- The use of rulers and compasses is allowed, but calculators and protractors are not only forbidden, but unadvised as they will not give you any meaningful advantage.
- If using paper, staple all the pages neatly together in the top left hand corner, with questions in numerical order.

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Round 2, 2021, Questions 1-5

1. Is $2020^{11} - 1$ a prime number. (*Detailed reasoning must be supplied to receive full marks.*)
2. Malcolm, Will and Ed had an arm-wrestling contest. In each game two men wrestled, while the third rested. After each game, the winner played the next game against the men who had rested. In total, Malcolm played 10 times, Will played 15 times and Ed played 17 times. Who lost the second game?
3. Given the following quartic

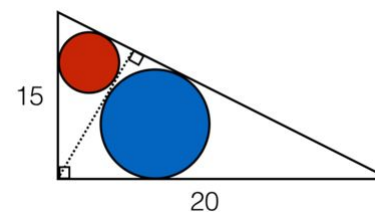
$$x^4 - ax^3 + ax^2 + bx + c = 0$$

What is the minimum value of the sums of the squares of the real and non-real roots.

4. What is the area of the polygon formed by all points (x,y) in the plane satisfying the inequality (*Detailed reasoning must be supplied to receive full marks.*)

$$||x| - 2| + ||y| - 2| \leq 4$$

5. In the right triangle below an altitude is dropped from the right angle two the hypotenuse and circles are inscribed in each of the newly formed triangles. Find the distance between the centers of the red and blue circles



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Round 2, 2021, Questions 6-9

6. Points X and Y lie outside parallelogram $KLMS$ and are such that triangles MLX and SMY are equilateral and lie wholly outside the parallelogram. Prove that KXY is also equilateral
7. Given the function $f(x) = x^2$, is it possible for 4 points on f to form a parallelogram
8. Nick attempts all 10 questions questions on an EMS Teams Olympiad paper in order. Each question is marked on a scale from 0 to 10. He never scores more in a later question than he did in an earlier question. How many possible sequence of marks are there?
9. Prove that if x and y are rational numbers and

$$x^5 = 2x^2y^2 - y^5 \quad (1)$$

then $1 - xy$ is a perfect square

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Round 2, 2021, Question 10

10. Prove that the equation

$$x^n + y^n = z^n$$

where n is an integer greater than 1, has no solutions for integers $x, y, z \in \mathbf{N}$ for $0 < x, y \leq n$

This is the end of the question paper.