4.3 - Support Vector Machines

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Introduction to Machine Learning for Finance

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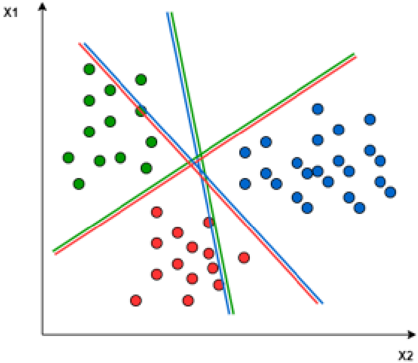
Support Vector Machines

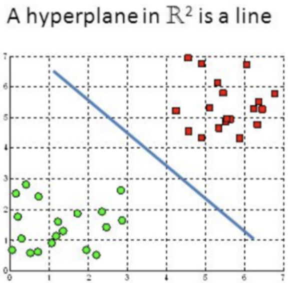
- In this section we consider another popular category of supervised learning models known as **support vector machines**;
- Like **decision trees**, SVMs can be used for either classification or for the prediction of a continuous variable;
- We first consider **linear classification** where a linear function of the feature values is used to separate observations and in particular we will focus on **binary classification** where the separation is into only two categories;

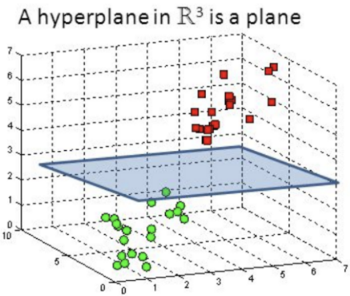
Linear Separation

- **Linearly Separable Data points**: Data points can be said to be linearly separable if a separating boundary/hyperplane can easily be drawn showing distinctively the different class groups.
- Linear separable data points mostly require linear machine learning classifiers such as Logistic regression for example.



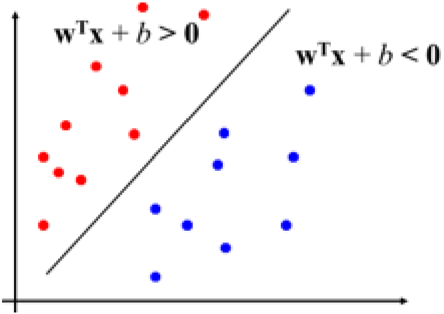
Linear Separation





Linear Separation

- Binary classification can be viewed as the task of separating feature space into two halves;
- A simple situation is that in which we attempt to classify loans into good loans and defaulting loans;



$$f(\mathbf{x}) = \mathrm{sign}(\mathbf{w}^T\mathbf{x} + b)$$

Loans Classification Example

- Consider two features: **credit score** and **income** of the borrower;
- We carry out an approximate scaling by subtracting 620 from the credit score (normalization);
- See Table 5.1 Hull

Credit score	Adjusted credit score	Income ('000s)	Default =0; good loan=1
660	40	30	0
650	30	55	0
650	30	63	0
700	80	35	0
720	100	28	0
650	30	140	1
650	30	100	1
710	90	95	1
740	120	64	1
770	150	63	1

Loans Classification Example

- This is a **balanced data set** in that there are five good loans that defaulted;
- SVM does not work well for a seriously imbalanced data set and, if this is your condition, you need to use procedures to correct for this.

Credit score	Adjusted credit score	Income ('000s)	Default =0; good loan=1
660	40	30	0
650	30	55	0
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700	80	35	0
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650	30	140	1
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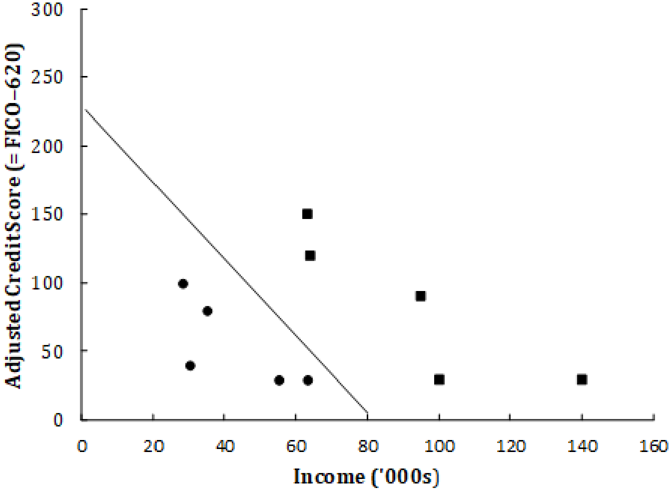
710	90	95	1
740	120	64	1
770	150	63	1

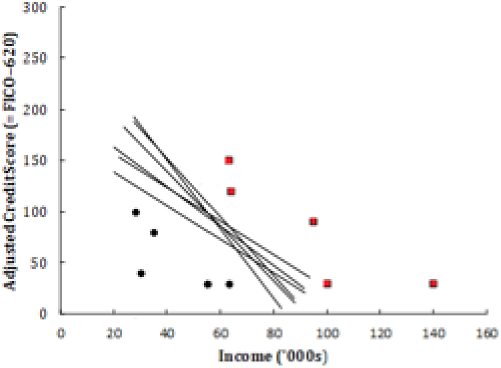
Linear Separation

circles are defaulting loans, squares are good loans



Linear Separation

- Which of the linear separators is optimal?

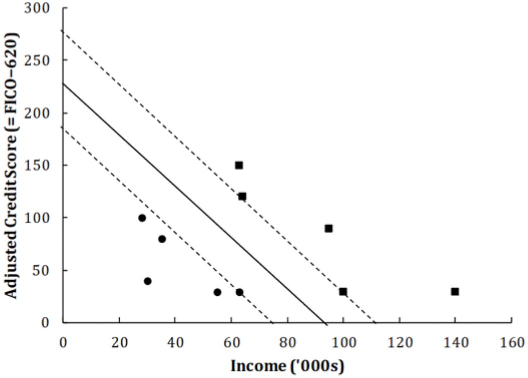


SVM Approach

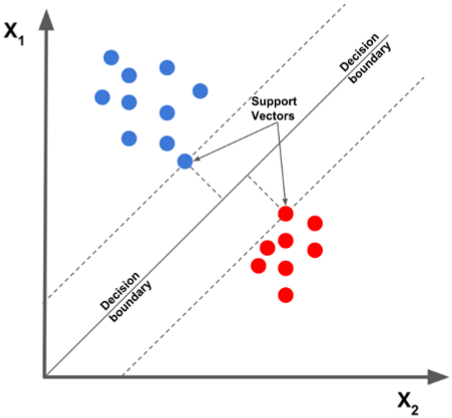
- In the support vector machine (SVM) approach we find a pathway that separates the data into two classes as far as possible
- In the **hard margin** case perfect separation is possible (as in our example)
- The algorithm finds the widest path possible
- Data must be normalized. (We carry out approximate normalization by subtracting 620 from credit score)
- The support vectors are the observations at the edge of the pathway

Example

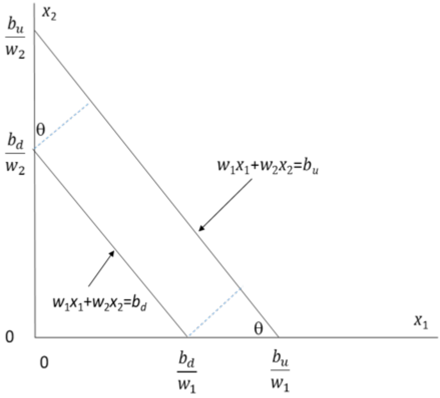
Best pathway for example. Solid line would be used to distinguish good and bad loans

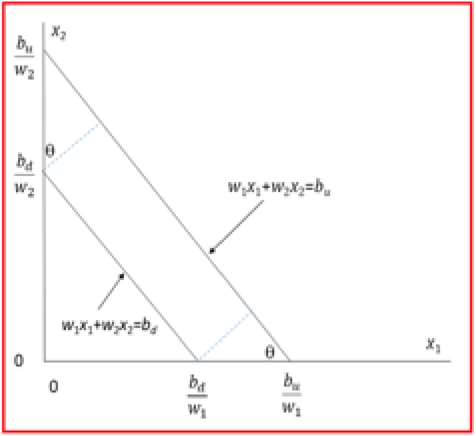


SVM Approach



SVM Approach: Notation





If P is width of pathway

$$\sin\theta = \frac{Pw_1}{b_u - b_d}$$

$$\cos\theta = \frac{Pw_2}{b_u - b_d}$$

$$P = \frac{b_u - b_d}{\sqrt{w_1^2 + w_2^2}}$$

The Math

- We can scale w_1, w_2, b_u, and b_d by the same constant without changing the model.
- We can therefore set $b_u = b + 1$ and $b_d = b - 1$ so that the width of the pathway is

$$P = \frac{2}{\sqrt{w_1^2 + w_2^2}}$$

- In the **hard margin** case the algorithm minimizes $w_1^2 + w_2^2$ subject to **perfect separation** being achieved

The Math

- For the example in table 5.1 we can set x_1 equal to income and x_2 equal to credit score;
- All good loans must be to the north-east of the pathway while all defaulting loans must be to the south west of the pathway;
- This means that, if a loan is good, the income and credit score must satisfy:

$$w_1 x_1 + w_2 x_2 \geq b + 1$$

- While if the loan defaults it must satisfy:

$$w_1 x_1 + w_2 x_2 \leq b - 1$$

Example

- Specification of hard margin problem for our example
- In our example the task is to find b, w_1, and w_2 to minimize subject to

$$
\begin{aligned}
30w_1 + 40w_2 &\leq b - 1 \\
55w_1 + 30w_2 &\leq b - 1 \\
63w_1 + 30w_2 &\leq b - 1 \\
35w_1 + 80w_2 &\leq b - 1 \\
28w_1 + 100w_2 &\leq b - 1 \\
140w_1 + 30w_2 &\geq b + 1 \\
100w_1 + 30w_2 &\geq b + 1 \\
95w_1 + 90w_2 &\geq b + 1 \\
64w_1 + 120w_2 &\geq b + 1 \\
63w_1 + 150w_2 &\geq b + 1
\end{aligned}
$$

The general hard margin problem

- The objective function is

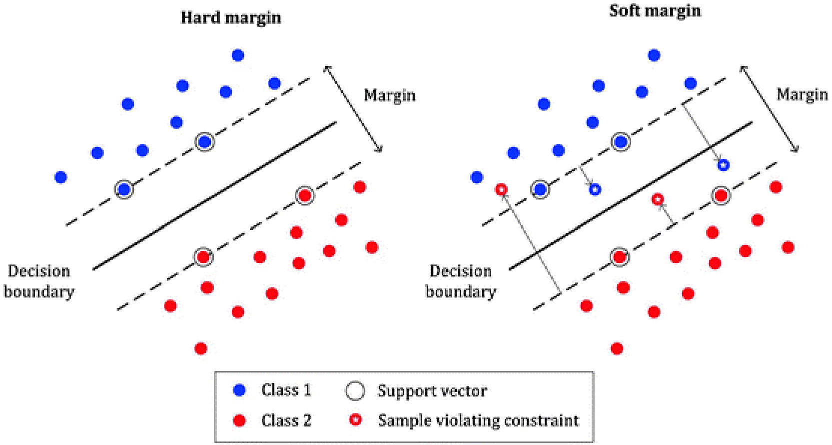
$$\sqrt{w_1^2 + w_2^2 + \cdots + w_n^2}$$

- We minimize this for values of w_i and b subject to the condition that there are no violations, i.e.:

$$\sum_i w_i x_i - b > 1 \quad \text{if loan good}$$

$$\sum_i w_i x_i - b < -1 \quad \text{if loan bad}$$

Hard Margin Vs Soft Margin



The soft margin problem

- We measure the violation of an observation as the extent to which the hard margin condition is violated
- we minimize

$$C \cdot \text{sum of violations} + \sqrt{\sum_i w_i^2}$$

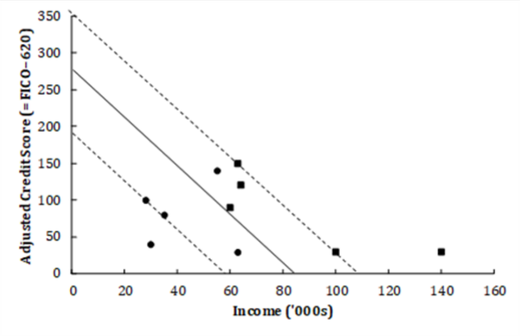
- Changing C changes the trade-off between the width of the path and the violations
- As C becomes smaller the pathway becomes wider with more violations

Changed example:

Credit score	Adjusted credit score	Income ('000s)	Default =0; good loan=1
660	40	30	0
650	**140**	55	0
650	30	63	0
700	80	35	0
720	100	28	0
650	30	140	1
650	30	100	1
710	90	**60**	1
740	120	64	1
770	150	63	1

Example

$C = 0.001$ results

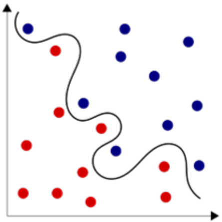


Impact of C for Example

C	w_1	w_2	b	Loans mis-classified	Width of pathway
0.01	0.054	0.022	5.05	10%	34.4
0.001	0.040	0.012	3.33	10%	48.2
0.0005	0.026	0.010	2.46	10%	70.6
0.0003	0.019	0.006	1.79	20%	102.2
0.0002	0.018	0.003	1.69	30%	106.6

Non Linear Separability

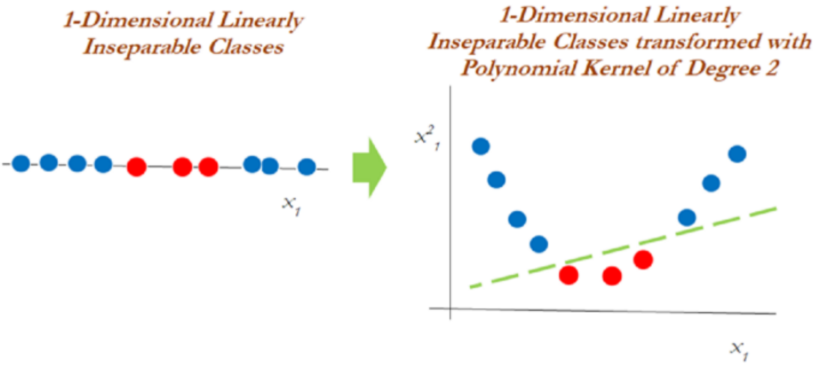
- **Non-Linearly Separable data points**: This is the exact opposite of Linearly separable data points.
- View the image below, notice that no matter how one tries to draw a straight line, some data points will one way or the other get misclassified.



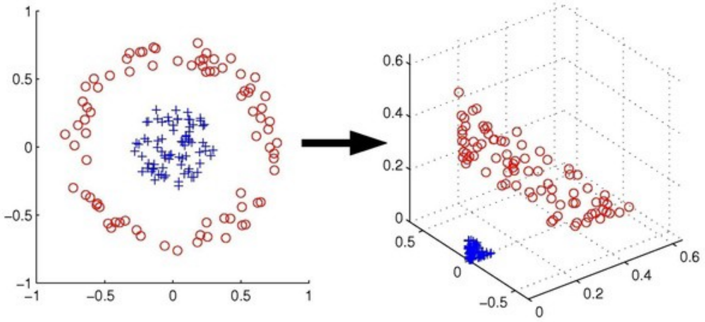
Non-linear classification

- The objective is to create new features so that the boundary becomes linear
- Suppose there is a single feature (age?) and we find the low and high values of the feature tend to give one outcome while intermediate values give another outcome
- We could form a new feature as $(\nu - m)^2$ where ν is the feature value and m is its mean

The Kernel Trick



The Kernel Trick



Forming new features

- We can add powers of each feature as a new feature.
- Alternatively, we can choose particular landmarks and create new features using the Gaussian Radial Basis Function (a similarity function). If values of features at a landmark are $l_1, l_2, \ldots, l_m$, the new feature values are calculated as

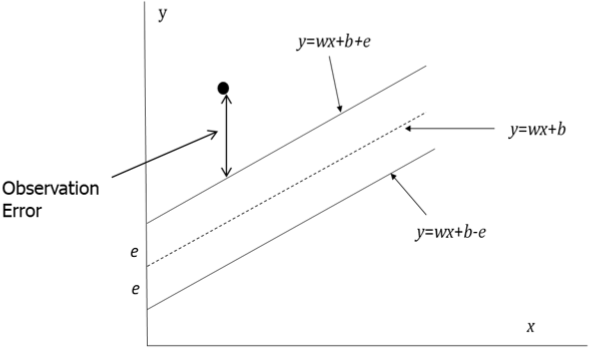
$$\exp\left(-\gamma \sum_{j=1}^{m} (x_j - l_j)^2\right)$$

- As the parameter γ increases the span of influence of a landmark decreases and the boundary becomes less smooth

SVM Regression: using SVM to predict a continuous variable

- We search for a pathway with a certain width that includes as many target values as possible
- If a target value lies within the pathway there is assumed to be no error
- If it lies outside the pathway the error is the difference between the actual value and the value predicted by the outer edge of the pathway

The single feature case



General Case

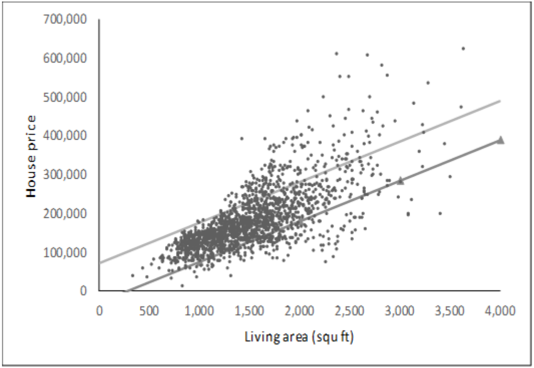
- We minimize

$$C \sum_{i=1}^{n} z_i + \sum_{j=1}^{m} w_j^2$$

  where C is a hyperparameter
- z_i is the error (zero if observation lies within the pathway)
- The first term is concerned with reducing errors for observations outside the pathway
- The second term provides some regularization. It avoids large positive and negative ws

Example

Predicting Iowa House Prices from Living Area when e=50,000 and C=0.01 (Hull Figure 5.7)



Example

Predicting Iowa House Prices from Living Area when e=100,000 and C=0.1 (Hull Figure 5.8)