chapter-4-2

January 23, 2022

Run in Google Colab

TO BE DONE

- Decidere se inserire anche l'esempio del Naive Bayes classifier
- introdurre la misura "errore di classificazione" accanto ad entropia e gini
- Random Forest

1 Supervised Models: Decision Trees

1.1 What is a decision tree?

Decision trees can be used for regression (continuous real-valued output, e.g. predicting the price of a house) or classification (categorical output, e.g. predicting email spam vs. no spam), but here we will focus on classification. A decision tree classifier is a binary tree where predictions are made by traversing the tree from root to leaf — at each node, we go left if a feature is less than a threshold, right otherwise. Finally, each leaf is associated with a class, which is the output of the predictor.

1.1.1 Why Decision Trees?

- Decision tress often mimic the human level thinking so its so simple to understand the data and make some good interpretations;
- Decision trees actually make you see the logic for the data to interpret(not like black box algorithms like SVM,NN,etc..);
- There is non requirement that the relationship between the target and the features be linear;
- The tree automatically selects the best features to make the prediction;
- A decision tree is less sensitive to outlying observations than a regression.

For example: if we are classifying bank loan application for a customer, the decision tree may look like this Here we can see the logic how it is making the decision.

attachment:chapter-4-2_pic_6-2.png

It's simple and clear. So what is the decision tree?

1.2 Building a Decision Tree

During model training on feature-target relationships, a tree is grown from a root (parent) node (all data containing feature-target relationships), which is then recursively split into child nodes (subset of the entire data) in a binary fashion. Each split is performed on a single feature in the parent node, at a desired threshold value of the feature. For instance, during each split of the parent node, we go to left node (with the corresponding subset of data) if a feature is less than the threshold, and right node otherwise. But how do we decide on the split? The feature to put at the root node is the one with the most **information gain**.

In order to split the nodes at the most informative features, we need to define an objective function that we want to optimize via the tree learning algorithm. Here, our objective function is to maximize the **IG** at each split, which we define as follows:

$$IG(D_p, f) = I(D_p) - \sum_{j=1}^{m} \frac{N_j}{N_p} I(D_j)$$
(1)

Here,

- *f* is the feature to perform the split;
- and are the dataset of the parent and *jth* child node;
- *I* is our impurity measure;
- is the total number of training examples at the parent node;
- and is the number of examples in the jth child node.

As we can see, the information gain is simply the difference between the impurity of the parent node and the sum of the child node impurities—the lower the impurities of the child nodes, the larger the information gain. However, for simplicity and to reduce the combinatorial search space, most libraries (including scikit-learn) implement binary decision trees. This means that each parent node is split into two child nodes, D_{left} and D_{right} :

$$IG(D_p, f) = I(D_p) - \frac{N_{left}}{N_p} I(D_{left}) - \frac{N_{right}}{N_p} I(D_{right})$$
(2)

1.2.1 Measures of Information Gain

The three impurity measures or splitting criteria that are commonly used in binary decision trees are **entropy** ($_{)}$, **Gini impurity** ($_{)}$ and the **classification error** ($_{)}$. In the following we will focus only on the first two.

Entropy One measure of information gain is based on *entropy*.

Suppose that there are n possible outcomes and p_i is the probability of outcome i with $\sum_{i=1}^{n} p_i = 1$, entropy can be defined as:

Entropy =
$$-\sum_{i=1}^{n} p_i \ln(p_i)$$

For example let's suppose some data for job application and we find that the 20% of them received an offer. Suppose further that 50% of job applicants have a relevant degree. If both those with a relevant degree and those without a relevant degree had a 20% chance of receiving a job offer, there would be no information gain to knowing wheter an applicant has a relevant degree. Suppose however that: - 30% of those with a relevant degree received a job offer - 10% of those without a relevant degree received a job offer

then there is clearly some information gain to knowing whether an applicant has a relevant degree. Let's calculate the information gain with entropy as a measure.

```
[9]: import math

# initial entropy, the only information we have is that 20% of applicants find a

→ job

p = 0.2

entropy_ini = -p*math.log(p) - (1-p)*math.log(1-p)

print(entropy_ini)
```

0.5004024235381879

If a candidate has a relevant degree this becomes

```
[6]: p = 0.3
entropy_1 = -p*math.log(p) - (1-p)*math.log(1-p)
print(entropy_1)
```

0.6108643020548935

if a candidate does not have a relevant degree:

```
[7]: p = 0.1
entropy_2 = -p*math.log(p) - (1-p)*math.log(1-p)
print(entropy_2)
```

0.3250829733914482

Because 50% of candidates have a relevant degree, the expected value of entropy after it is determined whether a candidate has a relevant degree is

```
[8]: entropy_exp = .5*entropy_1 + 0.5*entropy_2
print('Expected Entropy = ' + str(entropy_exp) + '\n')
#
# Now we can easily calculate the expected uncertainty reduction as:
#
print('Information Gain = ' + str(entropy_ini - entropy_exp))
```

Expected Entropy = 0.46797363772317085

Information Gain = 0.032428785815017014

When constructing the decision tree, we first search for the feature that has the biggest information gain. This is put at the root of the tree. For each branch emanating from the root we then search for a feature (other than the one at the root) that has the biggest information gain and so on ...

Gini Impurity An alternative to entropy for quantifying information gain is the Gini Impurity, which is basically a concept to quantify how homogeneous or "pure" a node is, with relation to the distribution of the targets in the node. A node is considered pure (G=0) if all training samples in the node belong to the same class, while a node with many training samples from many different classes will have a Gini Impurity close to 1.

$$G = 1 - \sum_{k=1}^{n} p_k^2 = 1 - \sum_{k=1}^{n} \left(\frac{m_k}{m}\right)^2 \tag{3}$$

where p_k is the fraction of samples belonging to class k, n is the number of classes, m are all the training samples in the node and m_k are the training examples in each class.

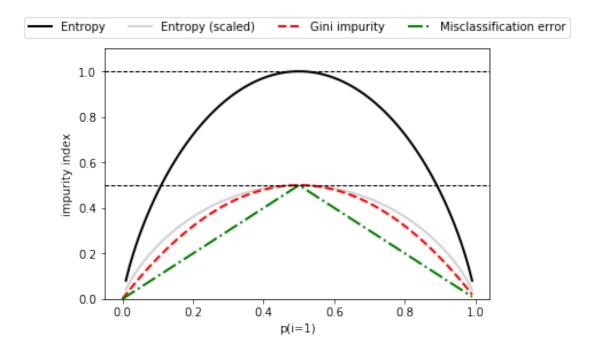
For example if a node contains five samples, with two of class 1, two of class 2, one of class 3 and none of class 4, then

$$G = 1 - \left(\frac{2}{5}\right)^2 - \left(\frac{2}{5}\right)^2 - \left(\frac{1}{5}\right)^2 = 0.64$$

Comparison For a more visual comparison of the three different impurity criteria that we discussed previously, let's plot the impurity indices for the probability range [0,1] for class 1.

```
[10]: import matplotlib.pyplot as plt
      import numpy as np
      def gini(p):
          return (p)*(1 - (p)) + (1 - p)*(1 - (1-p))
      def entropy(p):
          return - p*np.log2(p) - (1 - p)*np.log2((1 - p))
      def error(p):
          return 1 - np.max([p, 1 - p])
      x = np.arange(0.0, 1.0, 0.01)
      ent = [entropy(p) if p != 0 else None for p in x]
      sc_ent = [e*0.5 if e else None for e in ent]
      err = [error(i) for i in x]
      fig = plt.figure()
      ax = plt.subplot(111)
      for i, lab, ls, c, in zip([ent, sc_ent, gini(x), err],
          ['Entropy', 'Entropy (scaled)',
          'Gini impurity',
          'Misclassification error'],
```

[10]: Text(0, 0.5, 'impurity index')



1.3 Application to Credit Decision

```
from IPython.display import Image
       from sklearn.metrics import accuracy_score, recall_score, precision_score,_
       from sklearn.metrics import confusion_matrix, classification_report,
        →roc_auc_score
       from sklearn.metrics import roc_curve, auc, average_precision_score
[118]: #
       # load file lendingclub_traindata.xlsx
       if 'google.colab' in str(get_ipython()):
           from google.colab import files
           uploaded = files.upload()
           path = ''
       else:
           path = './data/'
[119]: train = pd.read_excel(path + 'lendingclub_traindata.xlsx', engine='openpyxl')
       # 1 = qood, 0 = default
       print(train.head())
         home_ownership income
                                   dti fico_low loan_status
                      1 44.304 18.47
      0
                                             690
                                                            0
                      0 38.500 33.73
                                             660
                                                            0
      1
                      1 54.000 19.00
                                             660
                                                            0
                      1 60.000 33.98
      3
                                             695
                                                            0
                      0 39.354 10.85
                                             685
[120]: # remove target column to create feature only dataset
       X = train.drop('loan_status',axis=1)
       # store target column
       y = train['loan_status']
       print(X.shape, y.shape)
      (8695, 4) (8695,)
```

1.3.1 Step-by-Step Analysis

Remember that without any further information, the probability of a good loan is about 82.76%

```
[121]: majority_class = y.mode()[0]  # predict fully paid only
prediction = np.full(shape=y.shape, fill_value=majority_class)
p_good = accuracy_score(y, prediction)
print(p_good)
```

0.8276020701552617

The initial entropy is therefore

```
[122]: entropy_ini = -p_good * math.log(p_good) - (1-p_good)* math.log(1-p_good)
print(entropy_ini)
```

We will consider the same four features and same data as in the previous notebook:

- A categorical variable indicating wheter the applicant rests or owns a home
- The applicant's income
- The applicant's debt to income ratio (dti)
- The applicantion credit score (FICO)

```
[123]: X.columns = ['Owns Home', 'Income', 'dti', 'FICO']
```

First of all let's make a step-by-step calculation before use the magic of scikit-learn library. Let's calculate the percentage of applicants that own their house

Assuming Home Ownership as Root

```
[124]: owners = train[train['home_ownership'] == 1]
    renter = train[train['home_ownership'] == 0]

    owners_perc = owners.shape[0]/X.shape[0]
    print('App Owner = ' + str(owners_perc))
    print('App Rent = ' + str(1-owners_perc))
```

```
App Owner = 0.5913743530764808
App Rent = 0.4086256469235192
```

Then find the percentage of loans that were good for applicants that own their house and for applicants that rent their house

```
[125]: #
# Find the number of loans that were good for owners
#
n1 = owners[owners['loan_status'] == 1]
p1 = n1.shape[0] / owners.shape[0]
print(p1)
```

0.8444185141968106

```
[126]: #
# Find the number of loans that were good for renters
#
n2 = renter[renter['loan_status'] == 1]
p2 = n2.shape[0] / renter.shape[0]
print(p2)
```

0.8032648466084998

The expected entropy if home ownership (but no other feature) becomes known is therefore:

The expected reduction in entropy is therefore a modest:

```
[128]: print(entropy_ini - entropy_exp)
```

0.0014207242593276548

Assuming Applicant's Income as Root The calculation of the expected reduction in entropy from Income requires the specification of a threshold income. Define

- p_1 : probability that income is greater than the threshold
- p_2 : probability that, if income is greater than the threshold, the borrower does not default
- p_3 : probability that if income is less than the threshold the borrower does not default

The expected entropy is

$$\hat{E} = p_1 \left[-p_2 \log(p_2) - (1 - p_2) \log(1 - p_2) \right] + (1 - p_1) \left[-p_3 \log(p_3) - (1 - p_3) \log(1 - p_3) \right]$$

```
[129]: #
       # p1 computation
       threshold = 50
       # applicants with income grater than the threshold
       sample_1 = train[train['income'] > threshold]
       # applicants with income less than the threshold
       sample_2 = train[train['income'] <= threshold]</pre>
       # applicants with income greater than the threshold which dont default
       sample_3 = sample_1[sample_1['loan_status']==1]
       # applicants with income less than the threshold which dont default
       sample_4 = sample_2[sample_2['loan_status']==1]
       p1 = sample_1.shape[0] / train.shape[0]
       p2 = sample_3.shape[0] / sample_1.shape[0]
       p3 = sample_4.shape[0] / sample_2.shape[0]
       print(p1)
       print(p2)
       print(p3)
```

- 0.6803910293271995
- 0.8466869506423259
- 0.7869737315581145

```
[130]: entropy_exp = p1*(-p2*math.log(p2) - (1-p2)*math.log(1-p2)) + (1-p1)*(-p3*math.

→log(p3) - (1-p3)*math.log(1-p3))
print(entropy_exp)
```

We can make a function which takes as input a threshold and return the entropy

```
[131]: def ExpectedEntropy(threshold, df):
           s1 = df[df['income'] > threshold]
           # applicants with income less than the threshold
           s2 = df[df['income'] <= threshold]</pre>
           # applicants with income greater than the threshold which dont default
           s3 = s1[s1['loan_status']==1]
           # applicants with income less than the threshold which dont default
           s4 = s2[s2['loan_status']==1]
           p1 = s1.shape[0] / df.shape[0]
           p2 = s3.shape[0] / s1.shape[0]
           p3 = s4.shape[0] / s2.shape[0]
           e = p1*(-p2*math.log(p2) - (1-p2)*math.log(1-p2)) + (1-p1)*(-p3*math.log(p3)_{\square})
        \rightarrow (1-p3)*math.log(1-p3))
           return(e)
       # sanity check
       print(ExpectedEntropy(50, train))
```

0.45702631325619647

We carry out an iterative search to determine the value of the threshold that minimizes this expected entropy for the training set

```
[154]: import matplotlib.pyplot as plt

# 100 linearly spaced numbers

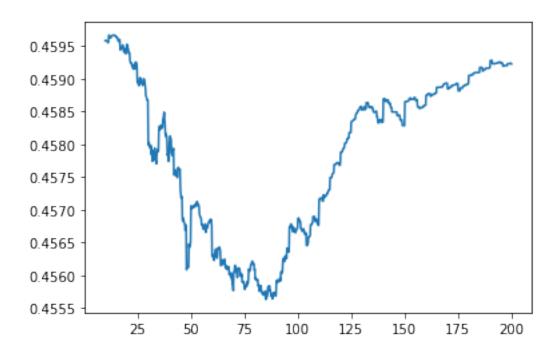
xps = np.linspace(10,200,1000)

yps = []
for xp in xps:
    yps.append(ExpectedEntropy(xp, train))

yps = np.array(yps)

plt.plot(xps, yps)
```

[154]: [<matplotlib.lines.Line2D at 0x2d2ab496848>]



```
[155]: minimum = np.where(yps == np.amin(yps))
    xps_min = xps[minimum][0]
    print(xps_min)
    print(ExpectedEntropy(xps_min, train))

85.12512512512512
    0.4556300235689159
[137]: print(ExpectedEntropy(85.193, train))
```

The result for all the information gain calculations can be found in the textbook of J.C. Hull (Chapter 4) and the FICO score with a threshold of 716 has the greatest information gain. It is therefore put at the node of the three.

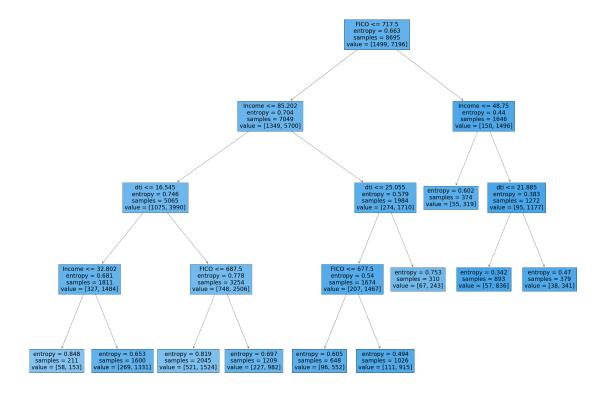
1.3.2 Using the DecisionTreeClassifier

```
[70]: clf = clf = clf.fit(X,y)

fig, ax = plt.subplots(figsize=(40, 30))

plot_tree(clf, filled=True, feature_names=X.columns, proportion=False)

plt.show()
```



```
[71]: r = export_text(clf,feature_names=['Owns Home','Income','dti','FICO'])
[72]: print(r)
     |--- FICO <= 717.50
         |--- Income <= 85.20
             |--- dti <= 16.55
                 |--- Income <= 32.80
                    |--- class: 1
                 |--- Income > 32.80
                | |--- class: 1
             |--- dti > 16.55
                |--- FICO <= 687.50
                | |--- class: 1
                 |--- FICO > 687.50
                | |--- class: 1
         |---| Income > 85.20
             |--- dti <= 25.05
                |--- FICO <= 677.50
                   |--- class: 1
                 |--- FICO > 677.50
                    |--- class: 1
```

```
|--- dti > 25.05
               |--- class: 1
     I --- FICO > 717.50
         |--- Income <= 48.75
            |--- class: 1
         |--- Income > 48.75
             |--- dti <= 21.88
               |--- class: 1
             |--- dti > 21.88
             | |--- class: 1
 []: #
      # load file lendingclub_traindata.xlsx
      if 'google.colab' in str(get_ipython()):
          from google.colab import files
          uploaded = files.upload()
          path = ''
      else:
         path = './data/'
[84]: test = pd.read_excel(path + 'lendingclub_testdata.xlsx', engine='openpyxl')
      # 1 = good, 0 = default
      print(test.head())
        home_ownership income
                                  dti
                                      fico_low loan_status
                         127.0 10.94
                                            675
     0
                                                           0
                        197.0 15.64
                                            710
                                                           0
     1
     2
                          25.5 28.75
                                            670
                                                           0
     3
                          80.0 20.16
                                            660
                     1
                                                           0
     4
                     0
                          57.0 30.60
                                            675
                                                           0
[85]: # remove target column to create feature only dataset
      X_test = test.drop('loan_status',axis=1)
      # store target column
      y_test = test['loan_status']
      y_pred = clf.predict(X_test)
```

Note that for the default prediciton (i.e., with threshold=0.5), it totally missed all the bad loans.

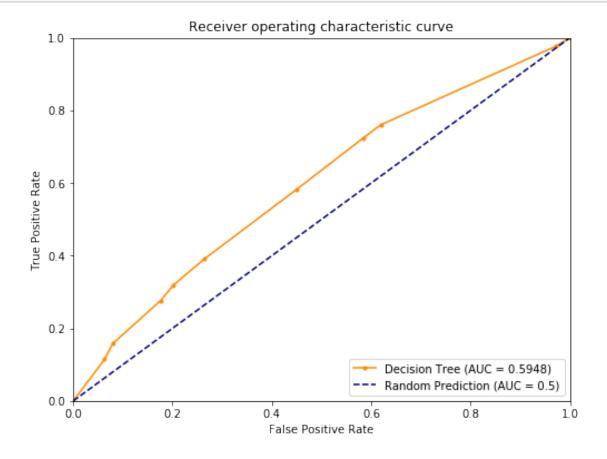
```
ax.xaxis.tick_top()
ax.yaxis.set_ticklabels(['Good','Defaulted'],verticalalignment='center')
ax.xaxis.set_ticklabels(['Good','Defaulted'])
plt.show()
```



```
[87]: THRESHOLD = [.5, .75, .80, .85]
      results = pd.DataFrame(columns=["THRESHOLD", "accuracy", "recall", "tnr", "fpr", |
       →"precision", "f1_score"]) # df to store results
      results['THRESHOLD'] = THRESHOLD
                                     # threshold column
      n_test = len(y_test)
      Q = clf.predict_proba(X_test)[:,1]
      j = 0
      for i in THRESHOLD:
                                     # iterate over each threshold
                                                                                     # fit_
       \rightarrow data to model
          preds = np.where(Q>i, 1, 0)
                                                                                 # if prob_
       \Rightarrow threshold, predict 1
          cm = (confusion_matrix(y_test, preds,labels=[1, 0], sample_weight=None)/
       \rightarrown_test)*100
           # confusion matrix (in percentage)
```

```
print('Confusion matrix for threshold =',i)
    print(cm)
    print(' ')
    TP = cm[0][0]
                                                                                 Ш
                             # True Positives
    FN = cm[0][1]
                             # False Positives
    FP = cm[1][0]
                                                                                 ш
                             # True Negatives
    TN = cm[1][1]
                             # False Negatives
    results.iloc[j,1] = accuracy_score(y_test, preds)
    results.iloc[j,2] = recall_score(y_test, preds)
    results.iloc[j,3] = TN/(FP+TN)
                             # True negative rate
    results.iloc[j,4] = FP/(FP+TN)
                             # False positive rate
    results.iloc[j,5] = precision_score(y_test, preds)
    results.iloc[j,6] = f1_score(y_test, preds)
    j += 1
print('ALL METRICS')
print(results.T.to_string(header=False))
Confusion matrix for threshold = 0.5
[82.11629479 0.
                         11
[17.88370521 0.
Confusion matrix for threshold = 0.75
[[62.42393509 19.6923597 ]
 [11.07167005 6.81203516]]
Confusion matrix for threshold = 0.8
[[59.46585531 22.65043949]
[10.44624746 7.43745774]]
Confusion matrix for threshold = 0.85
[[32.15010142 49.96619337]
[ 4.73292765 13.15077755]]
ALL METRICS
THRESHOLD
                         0.75
                                             0.85
                0.5
                                    0.8
accuracy 0.821163
                     0.69236 0.669033 0.453009
```

```
recall
                       1 0.760189 0.724166 0.391519
     tnr
                       0 0.380907 0.415879 0.73535
                       1 0.619093 0.584121
     fpr
                                              0.26465
     precision 0.821163 0.849356
                                     0.85058 0.871677
     f1_score
                0.901801 0.802303 0.782299 0.540341
[88]: # Compute the ROC curve and AUC
      fpr, tpr, _ = roc_curve(y_test, Q)
      roc_auc = auc(fpr,tpr)
[89]: plt.figure(figsize=(8,6))
                                     # format the plot size
      lw = 1.5
      plt.plot(fpr, tpr, color='darkorange', marker='.',
               lw=lw, label='Decision Tree (AUC = %0.4f)' % roc_auc)
      plt.plot([0, 1], [0, 1], color='navy', lw=lw, linestyle='--',
               label='Random Prediction (AUC = 0.5)' )
      plt.xlim([0.0, 1.0])
      plt.ylim([0.0, 1.0])
      plt.xlabel('False Positive Rate')
      plt.ylabel('True Positive Rate')
      plt.title('Receiver operating characteristic curve')
      plt.legend(loc="lower right")
      plt.show()
```



1.4 Continuous Target Variables

So far we have considered the use of decision trees for classification. We now describe how they can be used to predict the value of a continuous variable. Suppose that the feature at the root node is *X* and the threshold value for *X* is *Q*. We choose *X* and *Q* to minimize the expected mean squared error (mse) in the prediction of the target for the training set. In other words, we minimize

$$Prob(X > Q) \times (\text{mse if } X > Q) + Prob(X \le Q) \times (\text{mse if } X \le Q)$$

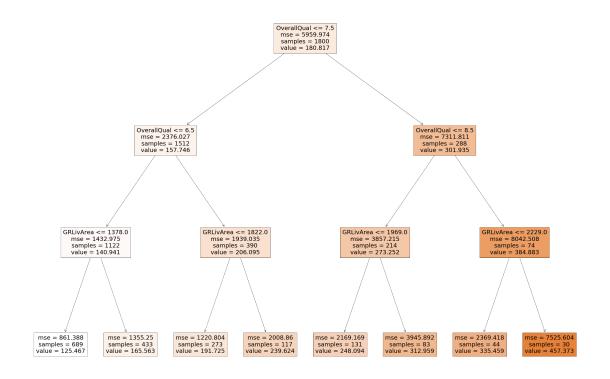
The feature at the next node and its threshold are chosen similarly. The value predicted at a tree leaf is the average of the values for the observations corresponding to the leaf.

We will illustrate this procedure for the house price data considered in the previous notebook (see also J. C. Hull Chapter 3). To keep the example manageable we consider only two features: Overall Quality (scale 1 to 10) and Living Area (Square Feet).

```
[94]:
# load file IOWA_Training_Data.xlsx
#
if 'google.colab' in str(get_ipython()):
    from google.colab import files
    uploaded = files.upload()
    path = ''
else:
    path = './data/'
```

```
[96]: train = pd.read_excel(path + 'IOWA_Training_Data.xlsx', engine='openpyxl')
# 1 = good, 0 = default
print(train.head())
```

```
OverallQual
                      GrLivArea Sale Price
     0
                   7
                           1710
                                       208.5
                   6
                           1262
                                       181.5
     1
     2
                   7
                           1786
                                       223.5
     3
                   7
                           1717
                                       140.0
     4
                   8
                           2198
                                       250.0
[97]: # remove target column to create feature only dataset
      X_train = train.drop('Sale Price',axis=1)
      # store target column
      y_train = train['Sale Price']
      print(X_train.shape, y_train.shape)
     (1800, 2) (1800,)
[98]: X_train.columns = ['OverallQual', 'GRLivArea']
[99]: | pred = DecisionTreeRegressor(criterion='mse', max_depth=3, random_state=0)
      pred = pred.fit(X_train,y_train)
      fig, ax = plt.subplots(figsize=(40, 30))
      plot_tree(pred, filled=True, feature_names=X_train.columns, proportion=False)
      plt.show()
```



```
[100]: r = export_text(pred,feature_names=['OverallQual','GRLivArea'])
[101]: print(r)
      |--- OverallQual <= 7.50
          |--- OverallQual <= 6.50
             |--- GRLivArea <= 1378.00
              | |--- value: [125.47]
              |--- GRLivArea > 1378.00
              | |--- value: [165.56]
          |--- OverallQual > 6.50
            |--- GRLivArea <= 1822.00
             | |--- value: [191.72]
             |--- GRLivArea > 1822.00
             | |--- value: [239.62]
      |--- OverallQual > 7.50
          |--- OverallQual <= 8.50
              |--- GRLivArea <= 1969.00
             | |--- value: [248.09]
              |--- GRLivArea > 1969.00
             | |--- value: [312.96]
          |--- OverallQual > 8.50
             |--- GRLivArea <= 2229.00
             | |--- value: [335.46]
              |--- GRLivArea > 2229.00
             | |--- value: [457.37]
 []: #
      # load file IOWA_Validation_Data.xlsx
      if 'google.colab' in str(get_ipython()):
          from google.colab import files
          uploaded = files.upload()
          path = ''
      else:
          path = './data/'
[104]: validation = pd.read_excel(path + 'IOWA_Validation_Data.xlsx', engine =
       # 1 = good, 0 = default
      print(validation.head())
         OverallQual GrLivArea Sale Price
      0
                          1045
                                     127.0
                  6
                          1378
                                     128.9
      1
                          1944
                                     103.5
      2
                  6
      3
                  5
                          1306
                                     130.0
```

```
1464
                                      129.0
      4
[105]: # remove target column to create feature only dataset
       X_validation = validation.drop('Sale Price',axis=1)
       # store target column
       y_validation = validation['Sale Price']
       print(X_validation.shape, y_validation.shape)
      (600, 2) (600,)
[107]: #
       \# load file IOWA_Test_Data.xlsx
       if 'google.colab' in str(get_ipython()):
           from google.colab import files
           uploaded = files.upload()
           path = ''
       else:
           path = './data/'
[108]: test = pd.read_excel(path + 'IOWA_Test_Data.xlsx', engine='openpyxl')
       # 1 = good, 0 = default
       print(test.head())
         OverallQual GrLivArea Sale Price
      0
                   5
                           1053
                                     142.10
                   5
                           1144
                                     120.00
      1
      2
                   6
                           1721
                                     174.85
      3
                   5
                                      116.00
                            922
                   5
                           1411
                                     130.00
[109]: # remove target column to create feature only dataset
       X_test = test.drop('Sale Price',axis=1)
       # store target column
       y_test = test['Sale Price']
       print(X_test.shape, y_test.shape)
      (508, 2) (508,)
[110]: y_pred_train=pred.predict(X_train)
       mse = mean_squared_error(y_pred_train,y_train)
       rmse=math.sqrt(mse)
       print("rmse for training set")
       print(rmse)
      rmse for training set
```

```
[111]: y_pred_validation=pred.predict(X_validation)
    mse = mean_squared_error(y_pred_validation,y_validation)
    rmse=math.sqrt(mse)
    print("rmse for validation set")
    print(rmse)

rmse for validation set
    40.462035147026455

[112]: y_pred_test=pred.predict(X_test)
    mse = mean_squared_error(y_pred_test,y_test)
    rmse=math.sqrt(mse)
    print("rmse for test set")
    print(rmse)

rmse for test set
    39.048800554631995

1.5 Random Forest
```