nb-lesson-3-3

February 9, 2025

Run in Google Colab

1 Supervised Models: Decision Trees

1.1 What is a decision tree?

Decision trees can be used for regression (continuous real-valued output, e.g. predicting the price of a house) or classification (categorical output, e.g. predicting email spam vs. no spam), but here we will focus on classification. A decision tree classifier is a binary tree where predictions are made by traversing the tree from root to leaf — at each node, we go left if a feature is less than a threshold, right otherwise. Finally, each leaf is associated with a class, which is the output of the predictor.

1.1.1 Why Decision Trees?

- Decision tress often mimic the human level thinking so its so simple to understand the data and make some good interpretations;
- Decision trees actually make you see the logic for the data to interpret(not like black box algorithms like SVM,NN,etc..);
- There is non requirement that the relationship between the target and the features be linear;
- The tree automatically selects the best features to make the prediction;
- A decision tree is less sensitive to outlying observations than a regression.

For example: if we are classifying bank loan application for a customer, the decision tree may look like this Here we can see the logic how it is making the decision.

It's simple and clear. So what is the decision tree?

1.2 Building a Decision Tree

During model training on feature-target relationships, a tree is grown from a **root** (parent) node (all data containing feature-target relationships), which is then recursively split into child nodes (subset of the entire data) in a binary fashion. Generally, each split is performed on a single feature in the parent node. You will have different kind of splitting depending on the nature of the feature:

- you can have a very simple decision tree in which the decision is a boolean condition (true, false) ...
- ... or numeric data

In the last case we have to define a desired **threshold** value of the feature. For instance, during each split of the parent node, we go to left node (with the corresponding subset of data) if a feature is less than the threshold, and right node otherwise. But how do we decide on the split? The feature to put at the root node is the one with the most **information gain**.

1.2.1 The Impurity Concept

Decision trees use the concept of **impurity** to describe how homogeneous or "pure" a node is. **A node is pure if all its samples belong to the same class**, while **a node with many samples from many different classes is called impure**. The difference between the impurity of a node and that of the child nodes is called **Information Gain**.

The goal of a decision tree is, at each layer, to try to split the data into two (or more) groups, so that data that fall into the same group are most similar to each other (homogeneity), and groups are as different as possible from each other (heterogeneity).

In order to split the nodes at the most informative features, we need to define an objective function that we want to optimize via the tree learning algorithm. Here, our objective function is to maximize the **IG** at each split, which we define as follows:

$$IG(D_p, f) = I(D_p) - \sum_{j=1}^{m} \frac{N_j}{N_p} I(D_j)$$
 (1)

Here,

- f is the feature to perform the split;
- and are the dataset of the parent and *jth* child node;
- *I* is our impurity measure;
- is the total number of training examples at the parent node;
- and is the number of examples in the jth child node.

As we can see, the information gain is simply the difference between the impurity of the parent node and the sum of the child node impurities—the lower the impurities of the child nodes, the larger the information gain. However, for simplicity and to reduce the combinatorial search space, most libraries (including scikit-learn) implement binary decision trees. This means that each parent node is split into two child nodes, D_{left} and D_{right} :

$$IG(D_p, f) = I(D_p) - \frac{N_{left}}{N_p} I(D_{left}) - \frac{N_{right}}{N_p} I(D_{right})$$
(2)

1.2.2 Measures of Information Gain

The three impurity measures or splitting criteria that are commonly used in binary decision trees are **Entropy** ($_{)}$, **Gini impurity** ($_{)}$ and the **Classification Error** ($_{)}$. In the following we will focus only on the first two.

Entropy One measure of information gain is based on *entropy*.

Suppose that there are n possible outcomes and p_i is the probability of outcome i with $\sum_{i=1}^{n} p_i = 1$, entropy can be defined as:

Entropy =
$$-\sum_{i=1}^{n} p_i \ln(p_i)$$

Let's take a very simple example in order to clarify the concept of omogeneity and how entropy can help us to measure it.

Here our target attribute is going to classify star versus diamond. We've got about half stars and half diamonds as a matter of fact we have 49 objects in the box and 24 of them are stars 25 of them are diamonds. You'll also notice that we have color codes here so not all of the diamonds are blue, there are some orange diamonds and some blue diamonds and then there are also some orange stars and some blue stars. So our **partition** might be on color. We're interested in predicting star versus diamond. Our partition then we'll be on the color and so we'll create an orange box and a blue box and then what we'll say here is it's very very much easier to predict whether it will be a star or a diamond in each of these two boxes compared

to the original total data set so there's a lot less omogeneity here so we would expect to have a significant information gain when we partition on color.

Let's see how entropy can give us a measure of how well we have splitted our sample. Now there are 25 orange objects 21 of which are stars and there are 24 blue objects three of which are stars. We'll calculate the full group entropy first:

$$E(\vec{d}) = -\sum_{i} p_i \log(p_i) = -(p_1 \log(p_1) + p_2 \log(p_2))$$

Let's plug those numbers in. So again for diamond there were 24 out of 49 diamonds and there were 25 stars

$$E = -\left(\frac{24}{49}\log\left(\frac{24}{49}\right) + \frac{25}{49}\log\left(\frac{25}{49}\right)\right) \sim 0.9997$$

we get an entropy for the full group of about 0.9997. For the orange box then let's calculate the entropy just within that box. It's the same process but in this case again we have diamonds and we have stars there are four diamonds and 21 stars out of 25 objects in the orange box so we have:

$$E(\vec{d}, \text{orange}) = -\sum_{i} p_i \log(p_i)$$

$$= -(p_1 \log(p_1) + p_2 \log(p_2))$$

$$= -\left(\frac{4}{25} \log\left(\frac{4}{25}\right) + \frac{21}{25} \log\left(\frac{21}{25}\right)\right)$$

$$\sim 0.6343$$

the entropy within the orange box is 0.6343 which is quite a bit lower. There's a lot more omogeneity. The blue box works very similar. Again it's the same formula so you've got 21 diamonds out of 24 and you've got 3 stars out of 24 so:

$$E(\vec{d}, \text{blue}) = -\sum_{i} p_{i} \log(p_{i})$$

$$= -(p_{1} \log(p_{1}) + p_{2} \log(p_{2}))$$

$$= -\left(\frac{21}{24} \log\left(\frac{21}{24}\right) + \frac{3}{24} \log\left(\frac{3}{24}\right)\right)$$
and 5436

In the blue box the entropy is actually 0.5436 even more omogeneity in the blue box because we only have those 3 diamonds. The combined entropy is going to be the weighted average so there were 25 orange objects out of 49 so we do 25/49 times the entropy of the orange group and then we do 24/49 times the entropy of the blue group add those together we get the weighted average of the entropy

$$\begin{split} E(\vec{d}, \vec{a}) &= \frac{25}{49} E(\text{orange}) + \frac{24}{49} E(\text{blue}) \\ &= \frac{25}{49} (0.6343) + \frac{24}{49} (0.5436) = 0.5899 \end{split}$$

Our information gain then is the original entropy, which is about 0.9997 minus the weighted average of the entropies across the partitions by color which was 0.5899 so our information gained is about 0.4097

$$I(\vec{d}, \vec{a}) = E(\vec{d}) - E(\vec{d}, \vec{a}) = 0.9997 - 0.5899 = 0.4097$$

that's a substantial information gain and again just visually to remind you what that means by partitioning on color if I then know which colored box I'm in it's much easier for me to make an accurate prediction

of what I'm likely to draw out whereas in the original box it was about 50/50 right twenty five stars twenty four diamonds pick one at random I'm almost as likely to get a diamond as I am a star so partitioning on color makes it much easier for us to do that prediction of whether it'll be star or diamond.

Let's do another example. Let's suppose some data for job application and we find that the 20% of them received an offer. Suppose further that 50% of job applicants have a relevant degree. If both those with a relevant degree and those without a relevant degree had a 20% chance of receiving a job offer, there would be no information gain to knowing wheter an applicant has a relevant degree. Suppose however that: - 30% of those with a relevant degree received a job offer - 10% of those without a relevant degree received a job offer

then there is clearly some information gain to knowing whether an applicant has a relevant degree. Let's calculate the information gain with entropy as a measure.

```
In [1]: import math
        # initial entropy, the only information we have is that 20% of applicants find a job
        p = 0.2
        entropy_ini = -p*math.log(p) - (1-p)*math.log(1-p)
        print(entropy ini)
0.5004024235381879
   If a candidate has a relevant degree this becomes
In [2]: p = 0.3
        entropy_1 = -p*math.log(p) - (1-p)*math.log(1-p)
        print(entropy_1)
0.6108643020548935
   if a candidate does not have a relevant degree:
In [3]: p = 0.1
        entropy_2 = -p*math.log(p) - (1-p)*math.log(1-p)
        print(entropy_2)
0.3250829733914482
```

Because 50% of candidates have a relevant degree, the expected value of entropy after it is determined whether a candidate has a relevant degree is

When constructing the decision tree, we first search for the feature that has the biggest information gain. This is put at the root of the tree. For each branch emanating from the root we then search for a feature (other than the one at the root) that has the biggest information gain and so on . . .

Gini Impurity An alternative to entropy for quantifying information gain is the Gini Impurity, which is basically a concept to quantify how homogeneous or "pure" a node is, with relation to the distribution of the targets in the node. A node is considered pure (G=0) if all training samples in the node belong to the same class, while a node with many training samples from many different classes will have a Gini Impurity close to 1

$$G = 1 - \sum_{k=1}^{n} p_k^2 = 1 - \sum_{k=1}^{n} \left(\frac{m_k}{m}\right)^2 \tag{3}$$

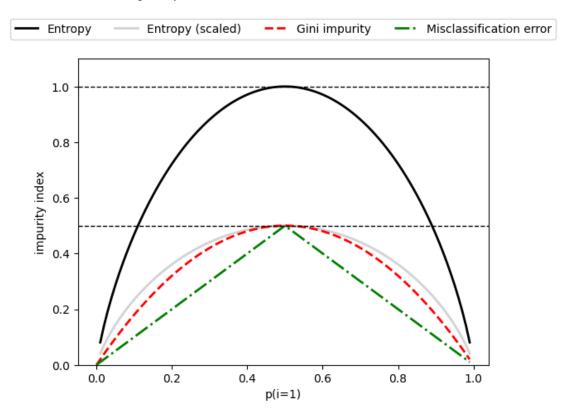
where p_k is the fraction of samples belonging to class k, n is the number of classes, m are all the training samples in the node and m_k are the training examples in each class.

For example if a node contains five samples, with two of class 1, two of class 2, one of class 3 and none of class 4, then

$$G = 1 - \left(\frac{2}{5}\right)^2 - \left(\frac{2}{5}\right)^2 - \left(\frac{1}{5}\right)^2 = 0.64$$

Comparison For a more visual comparison of the three different impurity criteria that we discussed previously, let's plot the impurity indices for the probability range [0, 1] for class 1.

```
In [1]: import matplotlib.pyplot as plt
        import numpy as np
        def gini(p):
            return (p)*(1 - (p)) + (1 - p)*(1 - (1-p))
        def entropy(p):
            return - p*np.log2(p) - (1 - p)*np.log2((1 - p))
        def error(p):
            return 1 - np.max([p, 1 - p])
        x = np.arange(0.0, 1.0, 0.01)
        ent = [entropy(p) if p != 0 else None for p in x]
        sc_ent = [e*0.5 if e else None for e in ent]
        err = [error(i) for i in x]
        fig = plt.figure()
        ax = plt.subplot(111)
        for i, lab, ls, c, in zip([ent, sc_ent, gini(x), err],
            ['Entropy', 'Entropy (scaled)',
            'Gini impurity',
            'Misclassification error'],
            ['-', '-', '--', '-.'],
            ['black', 'lightgray',
            'red', 'green', 'cyan']):
            line = ax.plot(x, i, label=lab,
            linestyle=ls, lw=2, color=c)
        ax.legend(loc='upper center', bbox_to_anchor=(0.5, 1.15), ncol=5, fancybox=True, shadow=False)
        ax.axhline(y=0.5, linewidth=1, color='k', linestyle='--')
        ax.axhline(y=1.0, linewidth=1, color='k', linestyle='--')
        plt.ylim([0, 1.1])
        plt.xlabel('p(i=1)')
        plt.ylabel('impurity index')
```



1.3 Application to Credit Decision (Hull J. C. Chapter 4)

```
In [2]: import math
    import pandas as pd
    import numpy as np
    import seaborn as sns
    import matplotlib.pyplot as plt
    from sklearn.tree import DecisionTreeClassifier, plot_tree, export_graphviz, export_text
    from IPython.display import Image
    from sklearn.metrics import accuracy_score, recall_score, precision_score, f1_score
    from sklearn.metrics import confusion_matrix, classification_report, roc_auc_score
    from sklearn.metrics import roc_curve, auc, average_precision_score
```

We now apply the decision tree approach using the entropy method to the Lending Club Data Introduced in chapter 4.1.

```
In [3]: #
    # load file lendingclub_traindata.xlsx
#
    if 'google.colab' in str(get_ipython()):
        from google.colab import files
        uploaded = files.upload()
        path = ''
    else:
        path = './data/'
```

```
In [4]: train = pd.read_excel(path + 'lendingclub_traindata.xlsx', engine='openpyxl')
        # 1 = good, 0 = default
        print(train.head())
   home_ownership income
                             dti fico_low
                                            loan_status
0
                  44.304
                           18.47
                                       690
                0 38.500
                           33.73
                                       660
                                                      0
1
                                                      0
2
                1 54.000 19.00
                                       660
3
                1 60.000 33.98
                                       695
                                                      0
                0 39.354 10.85
                                                      0
4
                                       685
In [5]: # remove target column to create feature only dataset
        X = train.drop('loan_status',axis=1)
        # store target column
        y = train['loan_status']
        print(X.shape, y.shape)
(8695, 4) (8695,)
```

1.3.1 Step-by-Step Analysis

Remember that without any further information, the probability of a good loan is about 82.76%, infact we have a total of 8695 observations of those 7196 were for good loans and 1499 were for the bad loans that defaulted.

0.45966813572655896

We will consider the same four features and same data as in the previous notebook:

- A categorical variable indicating wheter the applicant rests or owns a home
- The applicant's income
- The applicant's debt to income ratio (dti)
- The applicantion credit score (FICO)

```
In [8]: #X.columns = ['Owns Home', 'Income', 'dti', 'FICO']
```

First of all let's make a step-by-step calculation before use the magic of scikit-learn library. Let's calculate the percentage of applicants that own their house

Assuming Home Ownership as Root

```
In [9]: owners = train[train['home_ownership'] == 1]
        renter = train[train['home_ownership'] == 0]
        owners_perc = owners.shape[0]/X.shape[0]
        print('App Owner = ' + str(owners_perc))
        print('App Rent = ' + str(1-owners_perc))
App Owner = 0.5913743530764808
App Rent = 0.4086256469235192
   Then find the percentage of loans that were good for applicants that own their house and for applicants
that rent their house
In [10]: #
         # Find the number of loans that were good for owners
         n1 = owners[owners['loan_status']==1]
         p1 = n1.shape[0] / owners.shape[0]
         print(p1)
0.8444185141968106
In [11]: #
         # Find the number of loans that were good for renters
         n2 = renter[renter['loan_status']==1]
         p2 = n2.shape[0] / renter.shape[0]
         print(p2)
0.8032648466084998
   The expected entropy if home ownership (but no other feature) becomes known is therefore:
In [12]: entropy_exp = -owners_perc*(p1*math.log(p1) + (1-p1)*math.log(1-p1)) - (1-owners_perc)*(p2*math.log(p1) + (1-p1)*math.log(1-p1))
         print(entropy_exp)
0.4582474114672313
   The expected reduction in entropy is therefore a modest:
In [13]: print(entropy_ini - entropy_exp)
0.0014207242593276548
In [14]: results = pd.DataFrame(columns=('Feature', 'Treshold', 'Expected H', 'IG'))
         results.loc[0] = ['Home Ownership', 0, entropy_exp, entropy_ini - entropy_exp]
```

Assuming Applicant's Income as Root The calculation of the expected reduction in entropy from Income requires the specification of a threshold income. Define

- \bullet p_1 : probability that income is greater than the threshold
- p_2 : probability that, if income is greater than the threshold, the borrower does not default
- p_3 : probability that if income is less than the threshold the borrower does not default

The expected entropy is

```
\hat{E} = p_1 \left[ -p_2 \log(p_2) - (1 - p_2) \log(1 - p_2) \right] + (1 - p_1) \left[ -p_3 \log(p_3) - (1 - p_3) \log(1 - p_3) \right]
In [15]: #
                             # p1 computation
                             threshold = 50
                             # applicants with income grater than the threshold
                             sample_1 = train[train['income'] > threshold]
                              # applicants with income less than the threshold
                             sample_2 = train[train['income'] <= threshold]</pre>
                             # applicants with income greater than the threshold which dont default
                             sample_3 = sample_1[sample_1['loan_status']==1]
                             # applicants with income less than the threshold which dont default
                             sample_4 = sample_2[sample_2['loan_status']==1]
                             p1 = sample_1.shape[0] / train.shape[0]
                             p2 = sample_3.shape[0] / sample_1.shape[0]
                             p3 = sample_4.shape[0] / sample_2.shape[0]
                            print(p1)
                             print(p2)
                            print(p3)
0.6803910293271995
0.8466869506423259
0.7869737315581145
In [16]: entropy_exp = p1*(-p2*math.log(p2) - (1-p2)*math.log(1-p2)) + (1-p1)*(-p3*math.log(p3) - (1-p3)*math.log(p3) - (1-p3)*mat
                             print(entropy_exp)
0.45702631325619647
```

We can make a function which takes as input a threshold and return the entropy

```
In [17]: def ExpectedEntropy1(threshold, df):
    s1 = df[df['income'] > threshold]
    # applicants with income less than the threshold
    s2 = df[df['income'] <= threshold]
    # applicants with income greater than the threshold which dont default
    s3 = s1[s1['loan_status']==1]
    # applicants with income less than the threshold which dont default
    s4 = s2[s2['loan_status']==1]

p1 = s1.shape[0] / df.shape[0]
    p2 = s3.shape[0] / s1.shape[0]</pre>
```

```
p3 = s4.shape[0] / s2.shape[0]

e = p1*(-p2*math.log(p2) - (1-p2)*math.log(1-p2)) + (1-p1)*(-p3*math.log(p3) - (1-p3)*math
    return(e)

# sanity check
print(ExpectedEntropy1(50, train))
```

0.45702631325619647

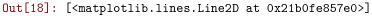
We carry out an iterative search to determine the value of the threshold that minimizes this expected entropy for the training set

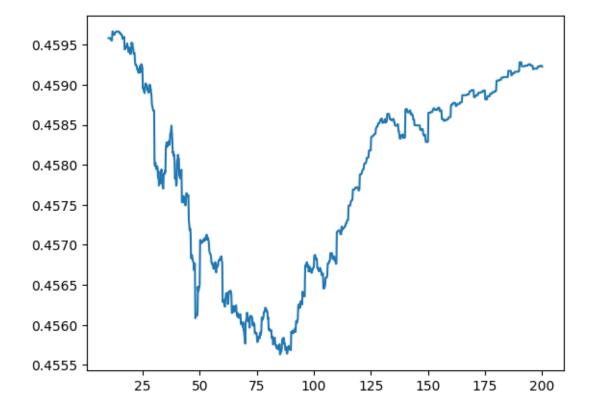
```
In [18]: import matplotlib.pyplot as plt

# 100 linearly spaced numbers
    xps = np.linspace(10,200,1000)
    yps = []
    for xp in xps:
        yps.append(ExpectedEntropy1(xp, train))

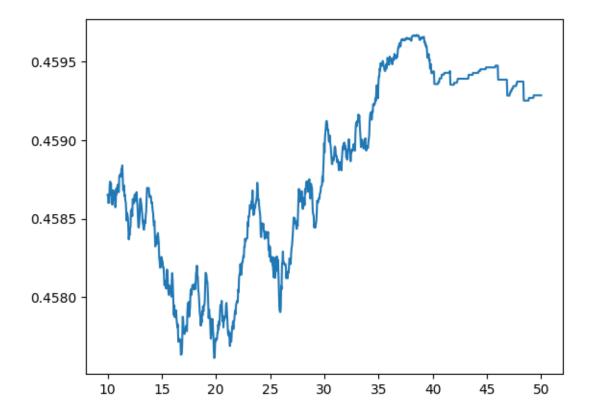
    yps = np.array(yps)

    plt.plot(xps, yps)
```

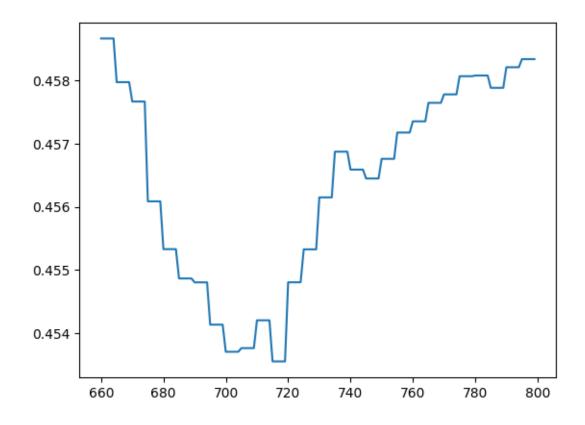




```
In [19]: minimum = np.where(yps == np.amin(yps))
                       xps_min = xps[minimum][0]
                       print(xps_min)
                       print(ExpectedEntropy1(xps_min, train))
85.12512512512512
0.4556300235689159
In [20]: entropy_exp = ExpectedEntropy1(85.193, train)
                       print(entropy_exp)
                       print(entropy_ini - entropy_exp)
0.4556300235689159
0.004038112157643048
In [21]: results.loc[1] = ['Income',xps_min, entropy_exp, entropy_ini - entropy_exp]
Assuming Applicant's dti (debt to income ratio) as Root
In [22]: def ExpectedEntropy2(threshold, df):
                                  s1 = df[df['dti'] > threshold]
                                  # applicants with income less than the threshold
                                  s2 = df[df['dti'] <= threshold]</pre>
                                  # applicants with dti greater than the threshold which dont default
                                  s3 = s1[s1['loan_status']==1]
                                  # applicants with dti less than the threshold which dont default
                                  s4 = s2[s2['loan_status']==1]
                                 p1 = s1.shape[0] / df.shape[0]
                                 p2 = s3.shape[0] / s1.shape[0]
                                 p3 = s4.shape[0] / s2.shape[0]
                                  e = p1*(-p2*math.log(p2) - (1-p2)*math.log(1-p2)) + (1-p1)*(-p3*math.log(p3) - (1-p3)*math.log(p3) - (1-p3)*
                                  return(e)
In [23]: # 100 linearly spaced numbers
                       xps = np.linspace(10,50,1000)
                       yps = []
                       for xp in xps:
                                  yps.append(ExpectedEntropy2(xp, train))
                       yps = np.array(yps)
                       plt.plot(xps, yps)
Out[23]: [<matplotlib.lines.Line2D at 0x21b10041b10>]
```



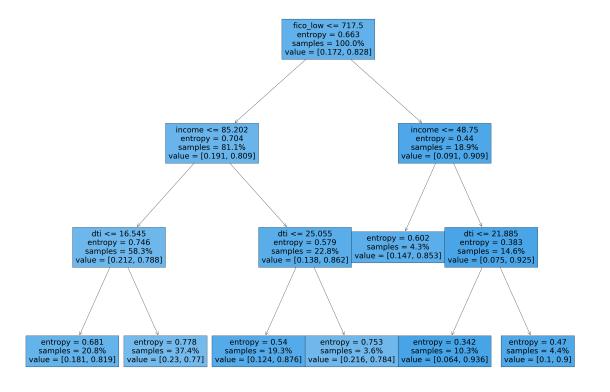
```
In [24]: minimum = np.where(yps == np.amin(yps))
         xps_min = xps[minimum][0]
         entropy_exp = ExpectedEntropy2(xps_min, train)
         print(xps_min)
         print(entropy_exp)
         print(entropy_ini - entropy_exp)
19.84984984984985
0.45761406059416887
0.0020540751323900874
In [25]: results.loc[2] = ['dti',xps_min, entropy_exp, entropy_ini - entropy_exp]
Assuming Applicant's FICO as Root
In [26]: def ExpectedEntropy3(threshold, df):
             s1 = df[df['fico_low'] > threshold]
             # applicants with income less than the threshold
             s2 = df[df['fico_low'] <= threshold]</pre>
             # applicants with fico greater than the threshold which dont default
             s3 = s1[s1['loan_status']==1]
             # applicants with fico less than the threshold which dont default
             s4 = s2[s2['loan_status']==1]
```



```
print(xps_min)
         print(entropy_exp)
         print(entropy_ini - entropy_exp)
715
0.45355502172339535
0.006113114003163611
In [29]: results.loc[3] = ['FICO',xps_min, entropy_exp, entropy_ini - entropy_exp]
In [30]: results
Out[30]:
                   Feature
                              Treshold Expected H
                                                           ΙG
            Home Ownership
                              0.000000
                                          0.458247
                                                    0.001421
         1
                    Income
                             85.125125
                                          0.455630
                                                    0.004038
         2
                             19.849850
                                          0.457614
                                                    0.002054
                       dti
                      FICO 715.000000
                                          0.453555 0.006113
```

The FICO score with a threshold of 715 has the greatest information gain. It is therefore put at the node of the three.

1.3.2 Using the DecisionTreeClassifier



```
In [32]: r = export_text(clf,feature_names=['Owns Home','Income','dti','FICO'])
In [33]: print(r)
|--- FICO <= 717.50
  --- Income <= 85.20
   | |--- dti <= 16.55
   | | |--- class: 1
       |--- dti > 16.55
      | |--- class: 1
   |--- Income > 85.20
   |--- dti <= 25.05
      | |--- class: 1
       |--- dti > 25.05
   | | |--- class: 1
|--- FICO > 717.50
  --- Income <= 48.75
   | |--- class: 1
   |--- Income > 48.75
   | |--- dti <= 21.88
       |--- class: 1
       |--- dti > 21.88
       |--- class: 1
  Let's make a test prediction . . .
In [34]: data = {
            'home_ownership' :[1],
            'income'
                              :[80],
            'dti'
                              :[20],
            'fico_low'
                              : [600]
        check = pd.DataFrame(data)
        check
Out[34]:
           home_ownership income dti fico_low
                                            600
In [35]: Q = clf.predict_proba(check)[:,1]
        print(Q)
[0.77012907]
  Now check the performance of the method.
In [36]: #
        \# load file lendingclub_testdata.xlsx
        if 'google.colab' in str(get_ipython()):
            from google.colab import files
```

```
uploaded = files.upload()
             path = ''
         else:
            path = './data/'
In [37]: test = pd.read_excel(path + 'lendingclub_testdata.xlsx', engine='openpyxl')
         # 1 = good, 0 = default
         print(test.head())
   home_ownership income
                           dti fico_low loan_status
0
                   127.0 10.94
                                       675
                                                      0
                1
                    197.0 15.64
                                       710
                                                      0
1
                1
2
                                       670
                                                      0
                1
                    25.5 28.75
3
                     80.0 20.16
                                       660
                                                      0
                1
                     57.0 30.60
                                       675
                                                      0
4
                0
In [38]: # remove target column to create feature only dataset
        X_test = test.drop('loan_status',axis=1)
         # store target column
         y_test = test['loan_status']
         y_pred = clf.predict(X_test)
```

Note that for the default prediciton (i.e., with threshold=0.5), it totally missed all the bad loans.

```
In [39]: n_test = len(y_test)
    cm = (confusion_matrix(y_test,y_pred,labels=[1, 0], sample_weight=None)/n_test)*100

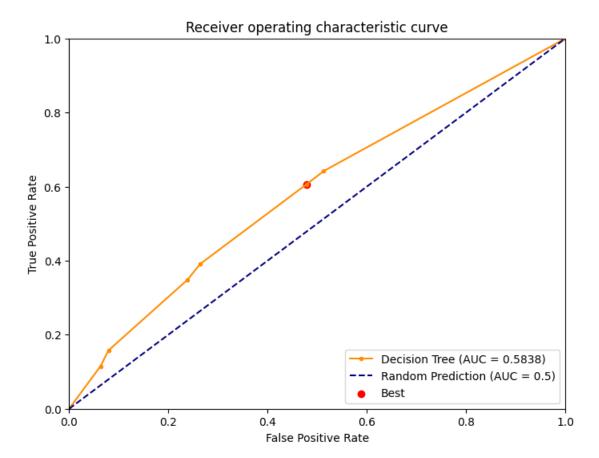
plt.figure(figsize=(6, 4))  # format the plot size
    ax = plt.subplot()
    sns.heatmap(cm, annot=True, ax = ax, fmt='.4g', cmap="Blues")
    ax.set_xlabel('\nPredicted labels'); ax.set_ylabel('True labels\n')
    ax.xaxis.tick_top()
    ax.yaxis.set_ticklabels(['Good','Defaulted'],verticalalignment='center')
    ax.xaxis.set_ticklabels(['Good','Defaulted'])
    plt.show()
```



```
In [40]: THRESHOLD = [.5, .75, .80, .85]
         results = pd.DataFrame(columns=["THRESHOLD", "accuracy", "recall", "tnr", "fpr", "precision",
         results['THRESHOLD'] = THRESHOLD
         n_test = len(y_test)
         Q = clf.predict_proba(X_test)[:,1]
         j = 0
         for i in THRESHOLD:
                                  # iterate over each threshold
             preds = np.where(Q > i, 1, 0)
                                                                                  # if prob > threshold,
             cm = (confusion_matrix(y_test, preds,labels=[1, 0], sample_weight=None)/n_test)*100
             # confusion matrix (in percentage)
             print('Confusion matrix for threshold =',i)
            print(cm)
            print(' ')
             TP = cm[0][0]
             FN = cm[0][1]
             FP = cm[1][0]
             TN = cm[1][1]
             results.iloc[j,1] = accuracy_score(y_test, preds)
             results.iloc[j,2] = recall_score(y_test, preds)
             results.iloc[j,3] = TN/(FP+TN)
             results.iloc[j,4] = FP/(FP+TN)
             results.iloc[j,5] = precision_score(y_test, preds)
```

```
results.iloc[j,6] = f1_score(y_test, preds)
            i += 1
        print('ALL METRICS')
        print(results.T.to_string(header=False))
Confusion matrix for threshold = 0.5
Γ[82.11629479 0.
                        1
 Γ17.88370521 0.
                        11
Confusion matrix for threshold = 0.75
[[82.11629479 0.
                        7
 [17.88370521 0.
                        ]]
Confusion matrix for threshold = 0.8
[[49.76335362 32.35294118]
 [ 8.5530764
              9.3306288 ]]
Confusion matrix for threshold = 0.85
[[32.15010142 49.96619337]
[ 4.73292765 13.15077755]]
ALL METRICS
THRESHOLD
                        0.75
                                   0.8
                                            0.85
               0.5
accuracy 0.821163 0.821163 0.59094 0.453009
               1.0
                        1.0 0.606011 0.391519
               0.0
                         0.0 0.521739 0.73535
tnr
                         1.0 0.478261
fpr
               1.0
                                         0.26465
precision 0.821163 0.821163 0.853333 0.871677
          0.901801 0.901801 0.708714 0.540341
f1_score
In [41]: # Compute the ROC curve and AUC
        fpr, tpr, threshold = roc_curve(y_test, Q)
        roc_auc = auc(fpr,tpr)
         # calculate the g-mean for each threshold
         gmeans = np.sqrt(tpr * (1-fpr))
         # locate the index of the largest g-mean
        ix = np.argmax(gmeans)
        print(threshold[ix])
0.819436775262286
In [42]: plt.figure(figsize=(8,6))
                                      # format the plot size
         lw = 1.5
        plt.plot(fpr, tpr, color='darkorange', marker='.',lw=lw, label='Decision Tree (AUC = %0.4f)' %
        plt.plot([0, 1], [0, 1], color='navy', lw=lw, linestyle='--', label='Random Prediction (AUC = '
        plt.scatter(fpr[ix], tpr[ix], marker='o', color='red', label='Best')
```

```
plt.xlim([0.0, 1.0])
plt.ylim([0.0, 1.0])
plt.xlabel('False Positive Rate')
plt.ylabel('True Positive Rate')
plt.title('Receiver operating characteristic curve')
plt.legend(loc="lower right")
plt.show()
```



1.4 Exercise: Continuous Target Variables

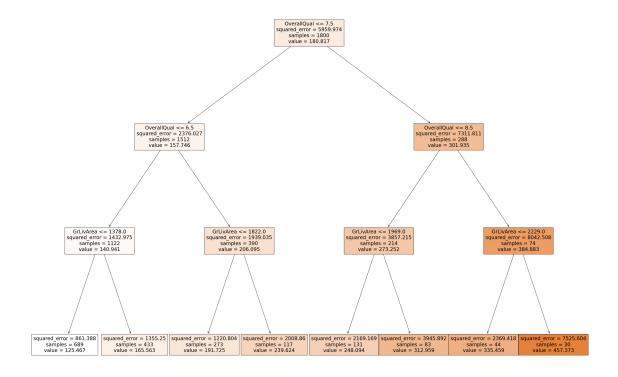
So far we have considered the use of decision trees for classification. We now describe how they can be used to predict the value of a continuous variable. Suppose that the feature at the root node is X and the threshold value for X is Q. We choose X and Q to minimize the expected mean squared error (mse) in the prediction of the target for the training set. In other words, we minimize

$$Prob(X > Q) \times (\text{mse if } X > Q) + Prob(X \leq Q) \times (\text{mse if } X \leq Q)$$

The feature at the next node and its threshold are chosen similarly. The value predicted at a tree leaf is the average of the values for the observations corresponding to the leaf.

We will illustrate this procedure for the house price data considered in the previous notebook (see also J. C. Hull Chapter 3). To keep the example manageable we consider only two features: Overall Quality (scale 1 to 10) and Living Area (Square Feet).

```
In [43]: import pandas as pd
         import numpy as np
         import seaborn as sns
         import matplotlib.pyplot as plt
         from sklearn.tree import DecisionTreeRegressor, plot_tree, export_graphviz, export_text
         from IPython.display import Image
         from sklearn.metrics import accuracy_score, recall_score, precision_score, f1_score
         from sklearn.metrics import confusion_matrix, classification_report, mean_squared_error
         from sklearn.metrics import roc_curve, auc, average_precision_score
         import math
In [44]: #
         # load file IOWA_Training_Data.xlsx
         if 'google.colab' in str(get_ipython()):
             from google.colab import files
             uploaded = files.upload()
            path = ''
         else:
            path = './data/'
In [45]: train = pd.read_excel(path + 'IOWA_Training_Data.xlsx', engine='openpyxl')
         # 1 = good, 0 = default
         print(train.head())
   OverallQual GrLivArea Sale Price
0
            7
                    1710
                                208.5
1
             6
                     1262
                                181.5
2
            7
                     1786
                                223.5
            7
3
                     1717
                               140.0
                     2198
                                250.0
In [46]: # remove target column to create feature only dataset
         X_train = train.drop('Sale Price',axis=1)
         # store target column
         v_train = train['Sale Price']
         print(X_train.shape, y_train.shape)
(1800, 2) (1800,)
In [47]: X_train.columns = ['OverallQual', 'GrLivArea']
         X_train.head()
Out[47]:
           OverallQual GrLivArea
         0
                     7
                              1710
         1
                      6
                              1262
         2
                      7
                              1786
         3
                      7
                              1717
                              2198
In [48]: pred = DecisionTreeRegressor(criterion='squared_error',max_depth=3,random_state=0)
         pred = pred.fit(X_train,y_train)
         fig, ax = plt.subplots(figsize=(40, 30))
        plot_tree(pred, filled=True, feature_names=X_train.columns, proportion=False)
        plt.show()
```



```
In [49]: r = export_text(pred,feature_names=['OverallQual','GrLivArea'])
In [50]: print(r)
|--- OverallQual <= 7.50
    |--- OverallQual <= 6.50
        |--- GrLivArea <= 1378.00
          |--- value: [125.47]
        |--- GrLivArea > 1378.00
          |--- value: [165.56]
    \mid --- OverallQual > 6.50
        |--- GrLivArea <= 1822.00
          |--- value: [191.72]
        |--- GrLivArea > 1822.00
          |--- value: [239.62]
|--- OverallQual > 7.50
    |--- OverallQual <= 8.50
        |--- GrLivArea <= 1969.00
          |--- value: [248.09]
        |--- GrLivArea > 1969.00
        | |--- value: [312.96]
    |--- OverallQual > 8.50
        |--- GrLivArea <= 2229.00
          |--- value: [335.46]
       |--- GrLivArea > 2229.00
```

```
| | | |--- value: [457.37]
In [51]: #
         {\it \#\ load\ file\ IOWA\_Validation\_Data.xlsx}
         if 'google.colab' in str(get_ipython()):
             from google.colab import files
             uploaded = files.upload()
             path = ''
         else:
             path = './data/'
In [52]: validation = pd.read_excel(path + 'IOWA_Validation_Data.xlsx', engine = 'openpyxl')
         # 1 = good, 0 = default
         print(validation.head())
   OverallQual GrLivArea Sale Price
0
             6
                     1045
                                127.0
                                128.9
             6
                     1378
1
2
             6
                     1944
                                103.5
3
             5
                     1306
                                130.0
4
                     1464
                                129.0
In [53]: # remove target column to create feature only dataset
         X_validation = validation.drop('Sale Price',axis=1)
         # store target column
         y_validation = validation['Sale Price']
         print(X_validation.shape, y_validation.shape)
         X_validation.head()
(600, 2) (600,)
Out[53]:
            OverallQual GrLivArea
         0
                      6
                              1045
         1
                      6
                               1378
         2
                      6
                              1944
         3
                      5
                               1306
         4
                      5
                               1464
In [54]: #
         # load file IOWA_Test_Data.xlsx
         if 'google.colab' in str(get_ipython()):
             from google.colab import files
             uploaded = files.upload()
             path = ''
         else:
             path = './data/'
In [55]: test = pd.read_excel(path + 'IOWA_Test_Data.xlsx', engine='openpyxl')
         # 1 = good, 0 = default
         print(test.head())
```

```
OverallQual GrLivArea Sale Price
0
             5
                     1053
                                142.10
             5
                     1144
1
                               120.00
2
             6
                     1721
                                174.85
3
             5
                      922
                                116.00
4
             5
                     1411
                                130.00
In [56]: # remove target column to create feature only dataset
         X_test = test.drop('Sale Price',axis=1)
         # store target column
         y_test = test['Sale Price']
         print(X_test.shape, y_test.shape)
(508, 2) (508,)
In [57]: y_pred_train=pred.predict(X_train)
         mse = mean_squared_error(y_pred_train,y_train)
         rmse=math.sqrt(mse)
         print("rmse for training set")
         print(rmse)
rmse for training set
38.660405648678584
In [58]: y_pred_validation=pred.predict(X_validation)
         mse = mean_squared_error(y_pred_validation,y_validation)
         rmse=math.sqrt(mse)
         print("rmse for validation set")
         print(rmse)
rmse for validation set
40.462035147026455
In [59]: y_pred_test=pred.predict(X_test)
         mse = mean_squared_error(y_pred_test,y_test)
         rmse=math.sqrt(mse)
         print("rmse for test set")
         print(rmse)
rmse for test set
39.048800554631995
```

1.5 Random Forest

The random forest method is a versatile and powerful machine learning algorithm used primarily for classification and regression tasks. It is an ensemble method, which means it combines the predictions of multiple individual models (in this case decision trees) to achieve better performance and robustness than a single model.

At its core, the random forest works by building a large number of decision trees during training. Each tree is trained on a random subset of the data, and at each decision point (or split) in a tree, it selects a random subset of features to determine the best split. These two randomization steps—sampling the data

and selecting a subset of features—help the model avoid overfitting, which is a common issue with individual decision trees.

The training process involves the following steps: first, the algorithm creates multiple bootstrap samples from the original dataset by sampling with replacement. Each bootstrap sample will likely exclude some of the original data points (called out-of-bag samples), which can be used later to estimate model performance. For each bootstrap sample, the algorithm grows a decision tree to its maximum depth without pruning, but at each split, it considers only a random subset of features to find the best split. This randomness reduces correlation between the trees and leads to more diverse models in the ensemble.

Once the forest of decision trees is trained, the random forest makes predictions by aggregating the outputs of the individual trees. For classification tasks, this aggregation typically involves a majority vote, where the class predicted most often by the trees is chosen as the final prediction. For regression tasks, the predictions of the trees are averaged to produce the final output.

One of the key advantages of the random forest method is its ability to handle a large number of features, including irrelevant ones, without significant overfitting. It also provides a measure of feature importance, which can help identify the most relevant variables in the dataset.

Here is an example of how to use the random forest algorithm in Python for both classification and regression:

1.5.1 Classification Example

```
In [61]: from sklearn.datasets import load_iris
         from sklearn.ensemble import RandomForestClassifier
         from sklearn.model_selection import train_test_split
         from sklearn.metrics import accuracy_score
         # Load the Iris dataset
         data = load_iris()
         X, y = data.data, data.target
         # Split the data into training and test sets
         X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3, random_state=42)
         # Initialize and train the Random Forest classifier
         rf classifier = RandomForestClassifier(n estimators=100, random state=42)
         rf_classifier.fit(X_train, y_train)
         # Make predictions on the test set
         y_pred = rf_classifier.predict(X_test)
         # Evaluate the model
         accuracy = accuracy_score(y_test, y_pred)
         print(f"Classification Accuracy: {accuracy:.2f}")
Classification Accuracy: 1.00
```

1.5.2 Regression Example

```
X, y = data.data, data.target

# Split the data into training and test sets
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3, random_state=42)

# Initialize and train the Random Forest regressor
rf_regressor = RandomForestRegressor(n_estimators=100, random_state=42)
rf_regressor.fit(X_train, y_train)

# Make predictions on the test set
y_pred = rf_regressor.predict(X_test)

# Evaluate the model
mse = mean_squared_error(y_test, y_pred)
print(f"Mean Squared Error: {mse:.2f}")
Mean Squared Error: 0.26
```

In the classification example, we use the Iris dataset, which contains information about different species of flowers, to build a model that predicts the species of a flower based on its measurements. In the regression example, we use the Boston housing dataset to predict housing prices based on various features. These examples demonstrate how to quickly implement the random forest algorithm using the scikit-learn library, one of the most popular machine learning libraries in Python.

The reason that the random forest model works is that a large number of relatively uncorrelated models (trees) operating as a committee will outperform any of the individual constituent models. The low correlation between models is the key. Just like how investments with low correlations (like stocks and bonds) come together to form a portfolio that is greater than the sum of its parts, uncorrelated models can produce ensemble predictions that are more accurate than any of the individual predictions. The reason for this wonderful effect is that the trees protect each other from their individual errors (as long as they don't constantly all err in the same direction). While some trees may be wrong, many other trees will be right, so as a group the trees are able to move in the correct direction.

1.6 References and Credits

In []: