3.1 - Linear and Logistic Regression

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Introduction to Machine Learning for Finance

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Back to Linear Regression

Linear Regression

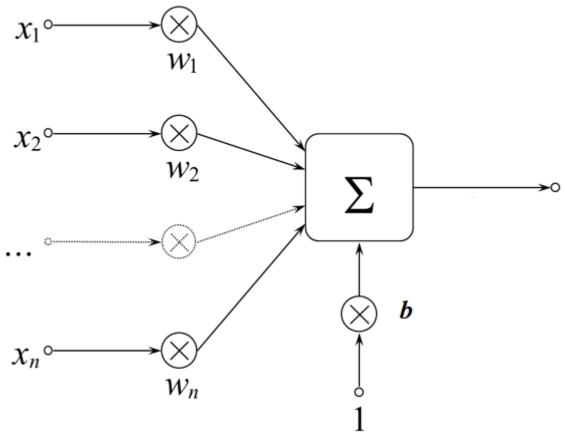
A linear model makes a prediction by simply computing a weighted sum of the input features, plus a constant called the **bias** term (also called the **intercept** term):

$$y = b + w_1 X_1 + w_2 X_2 + \cdots + w_m X_m + \epsilon \tag{1}$$

where:

- y is the predicted value (the value of the target);
- m is the number of features;
- X_i is the i^{th} feature value that are used to predict y;
- b and w_j are the j^{th} model parameters (b being the **bias** term and w_i the **weights**)
- ϵ is the predicton error.

Linear Regression



$$y = \sum_{i=1}^{n} w_i x_i + b$$

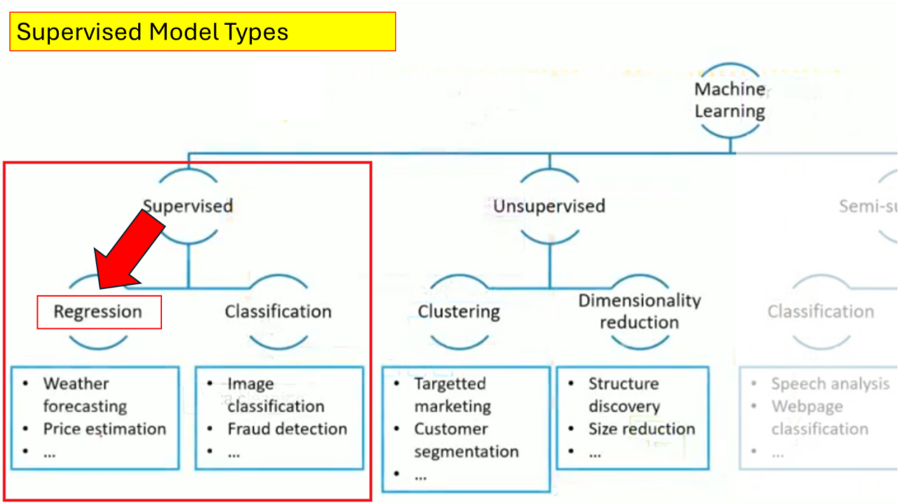
Linear Regression

- The parameters b and w_i are chosen to minimize the mean squared error over the training data set.
- This means that the task in linear regression is to find values for b and w_i that minimize

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(Y - b - w_1 X_{i1} - w_2 X_{i2} - \cdots - w_m X_{im})^2 \tag{2}$$

where n is the size of the training set.

Linear Regression



Iowa House Price Case Study

Linear Regression

The **Iowa House Pricing Dataset** from Kaggle is a popular dataset used for regression problems, particularly in the context of machine learning. It contains comprehensive information on houses in Ames, Iowa, and their sale prices, and it was designed to serve as an alternative to the Boston Housing Dataset. Here's a brief overview: **Dataset Overview**

- **Target Variable**: 'SalePrice' – the final price of the house in USD.
- **Number of Rows (Observations)**: 2,930 (combined training and test datasets).
- **Number of Features (Columns)**: 79 explanatory variables plus the target variable.

Iowa House Price Case Study

Linear Regression

- The objective is to predict the prices of house in Iowa from features
- 800 observations in training set, 600 in validation set, and 508 in test set
- Here the original competition description: **https://www.kaggle.com/c/house-prices-advanced-regression-techniques**

Iowa House Price Case Study
Linear Regression

Types of Features The dataset includes a mix of:

- 1. **Numerical Features**:
  - Continuous (e.g., ‘LotArea‘, ‘GrLivArea‘, ‘SalePrice‘)
  - Discrete (e.g., ‘GarageCars‘, ‘TotRmsAbvGrd‘)
- 2. **Categorical Features**:
  - Nominal (e.g., ‘Neighborhood‘, ‘HouseStyle‘)
  - Ordinal (e.g., ‘ExterQual‘, ‘KitchenQual‘)
- 3. **Temporal Features**:
  - Year-based (e.g., ‘YearBuilt‘, ‘YrSold‘)
- 4. **Location Features**:
  - Specific location details (e.g., ‘Neighborhood‘, ‘MSSubClass‘).

Iowa House Price Case Study

Linear Regression

Categorical Features

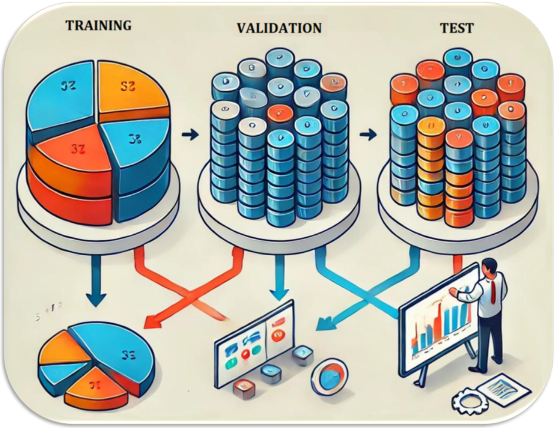
In this problem one of the categorical features is concerned with the basement quality as indicated by the ceiling height. The categories are:
- Excellent (¡ 100 inches)
- Good (90-99 inches)
- Typical (80-89 inches)
- Fair (70-79 inches)
- Poor (¡ 70 inches)
- No Basement

This is an example of a categorical variable where **there is a natural ordering**. We created a new variable that had a values of 5, 4, 3, 2, 1 and 0 for the above six categories respectively.

Iowa House Price Case Study

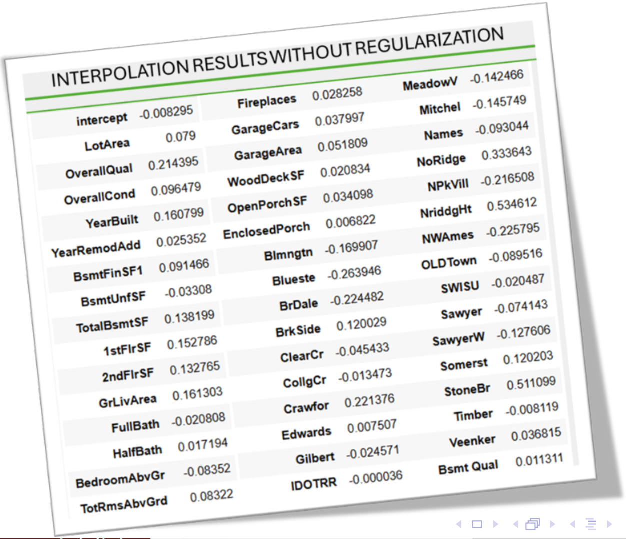
Linear Regression

Dataset splitting: Training, Validation and Test



Iowa House Price Results

Linear Regression



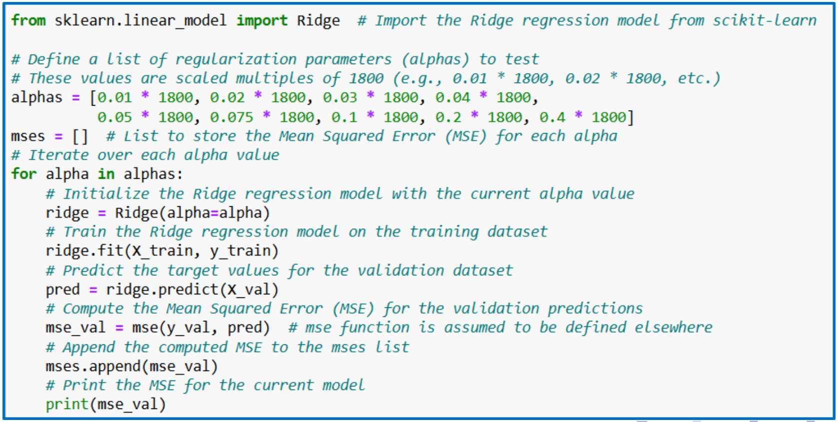
INTERPOLATION RESULTS WITHOUT REGULARIZATION

intercept	-0.008295	Fireplaces	0.028258	MeadowV	-0.142466
LotArea	0.079	GarageCars	0.037997	Mitchel	-0.145749
OverallQual	0.214395	GarageArea	0.051809	Names	-0.093044
OverallCond	0.096479	WoodDeckSF	0.020834	NoRidge	0.333643
YearBuilt	0.160799	OpenPorchSF	0.034098	NPkVill	-0.216508
YearRemodAdd	0.025352	EnclosedPorch	0.006822	NriddgHt	0.534612
BsmtFinSF1	0.091466	Blmngtn	-0.169907	NWAmes	-0.225795
BsmtUnfSF	-0.03308	Blueste	-0.263946	OLDTown	-0.089516
TotalBsmtSF	0.138199	BrDale	-0.224482	SWISU	-0.020487
1stFlrSF	0.152786	BrkSide	0.120029	Sawyer	-0.074143
2ndFlrSF	0.132765	ClearCr	-0.045433	SawyerW	-0.127606
GrLivArea	0.161303	CollgCr	-0.013473	Somerst	0.120203
FullBath	-0.020808	Crawfor	0.221376	StoneBr	0.511099
HalfBath	0.017194	Edwards	0.007507	Timber	-0.008119
BedroomAbvGr	-0.08352	Gilbert	-0.024571	Veenker	0.036815
TotRmsAbvGrd	0.08322	IDOTRR	-0.000036	Bsmt Qual	0.011311

Ridge Regression

Linear Regression

We try using Ridge regression with different values of the hyperparameter λ. The following code shows the effect of this parameter on the prediction error.



```python
from sklearn.linear_model import Ridge  # Import the Ridge regression model from scikit-learn

# Define a list of regularization parameters (alphas) to test
# These values are scaled multiples of 1800 (e.g., 0.01 * 1800, 0.02 * 1800, etc.)
alphas = [0.01 * 1800, 0.02 * 1800, 0.03 * 1800, 0.04 * 1800,
          0.05 * 1800, 0.075 * 1800, 0.1 * 1800, 0.2 * 1800, 0.4 * 1800]
mses = []  # List to store the Mean Squared Error (MSE) for each alpha
# Iterate over each alpha value
for alpha in alphas:
    # Initialize the Ridge regression model with the current alpha value
    ridge = Ridge(alpha=alpha)
    # Train the Ridge regression model on the training dataset
    ridge.fit(X_train, y_train)
    # Predict the target values for the validation dataset
    pred = ridge.predict(X_val)
    # Compute the Mean Squared Error (MSE) for the validation predictions
    mse_val = mse(y_val, pred)  # mse function is assumed to be defined elsewhere
    # Append the computed MSE to the mses list
    mses.append(mse_val)
    # Print the MSE for the current model
    print(mse_val)
```

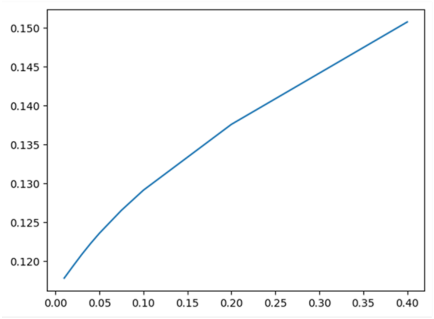
Ridge and Lasso Regression

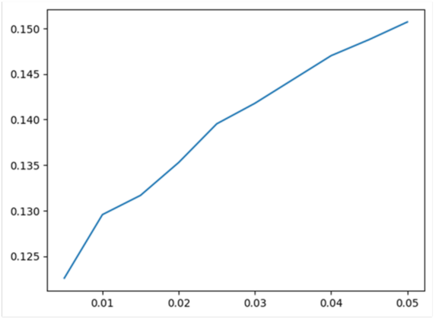
Linear Regression

Error analysis using Ridge and Lasso regression with different values of the hyperparameter λ

Ridge Regression

Lasso Regression



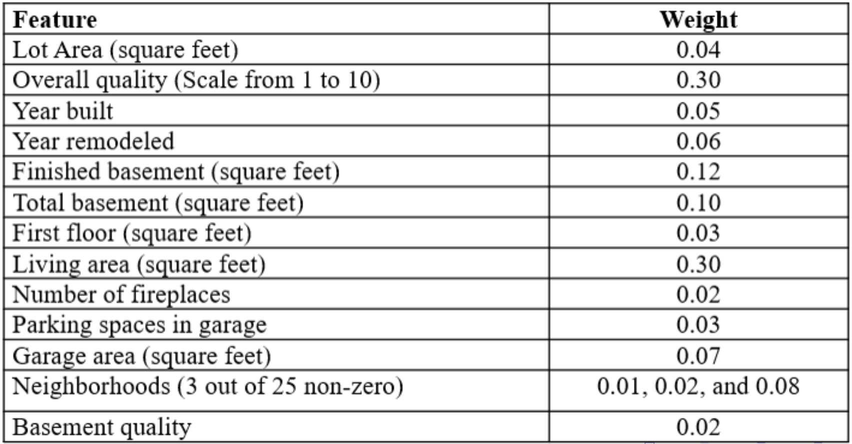


As expected the prediction error increases as λ increases. Values of λ in the range 0 to 0.1 might be reasonably be considered because prediction errors increases only slightly when λ is in this range. However it turns out that the improvement in the model is quite small for these values of λ.

Iowa House Price Results

Linear Regression

Non-zero weights for Lasso when $\lambda = 0.1$ (overall quality and total living area were most important)



Feature	**Weight**
Lot Area (square feet)	0.04
Overall quality (Scale from 1 to 10)	0.30
Year built	0.05
Year remodeled	0.06
Finished basement (square feet)	0.12
Total basement (square feet)	0.10
First floor (square feet)	0.03
Living area (square feet)	0.30
Number of fireplaces	0.02
Parking spaces in garage	0.03
Garage area (square feet)	0.07
Neighborhoods (3 out of 25 non-zero)	0.01, 0.02, and 0.08
Basement quality	0.02

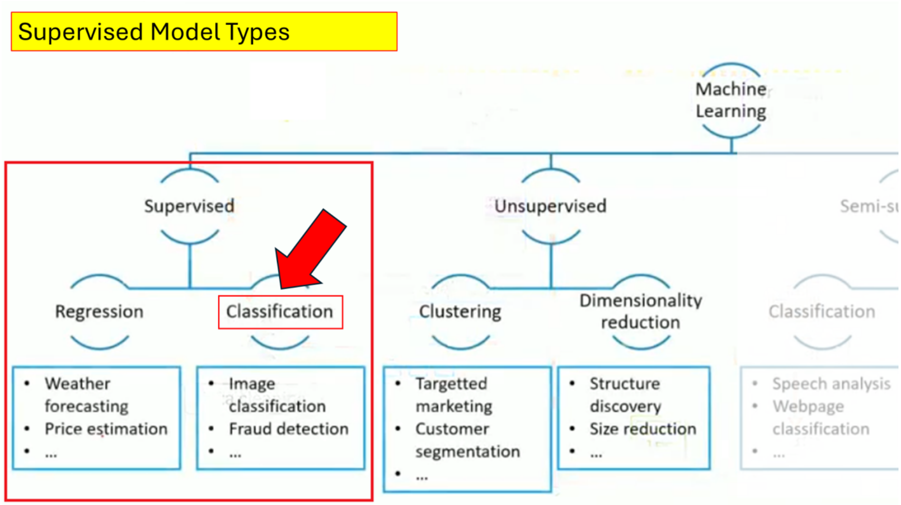
Summary of Iowa House Price Results

Linear Regression

- With no regularization correlation between features leads to some negative weights which we would expect to be positive
- Improvements from Ridge is modest
- Lasso leads to a much bigger improvement in this case
- Elastic net similar to Lasso in this case
- Mean squared error for test set for Lasso with $\lambda = 0.1$ is 14.7

Logistic Regression

Logistic Regression


Supervised Model Types
Machine Learning
Supervised
Unsupervised
Semi-su
Regression
Classification
Clustering
Dimensionality reduction
Classification
• Weather forecasting • Price estimation • ...
• Image classification • Fraud detection • ...
• Targetted marketing • Customer segmentation • ...
• Structure discovery • Size reduction • ...
• Speech analysis • Webpage classification • ...

Logistic Regression

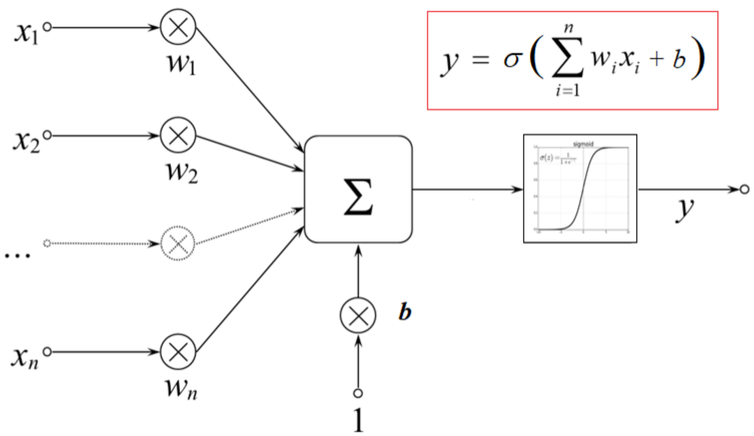
- The objective is to classify observations into a **positive outcome** and **negative outcome** using data on features
- Probability of a positive outcome is assumed to be a sigmoid function:

$$\sigma(y) = \frac{1}{1 + e^{-y}} \tag{3}$$

- where Y is related linearly to the values of the features:

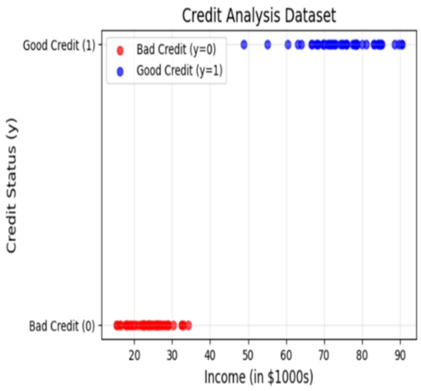
$$y = b + w_1 X_1 + w_2 X_2 + \cdots + w_m X_m \tag{4}$$

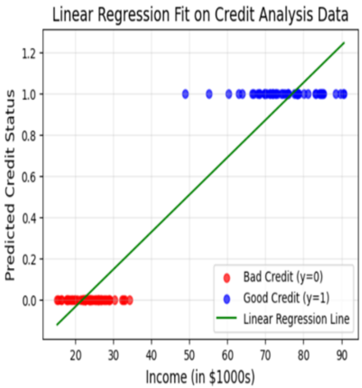
Logistic Regression



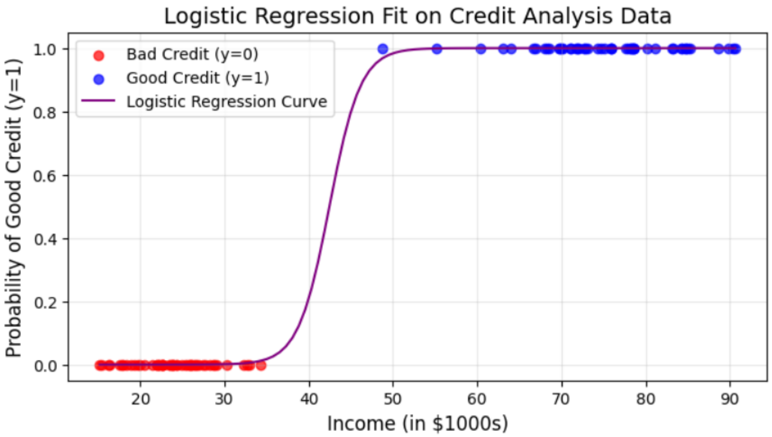
$$y = \sigma\left(\sum_{i=1}^{n} w_i x_i + b\right)$$

Logistic Regression





Logistic Regression



Logistic Regression

- The output of the sigmoid function is then interpreted as the probability of a particular example belonging to class 1, $\sigma(z) = P(y = 1|\mathbf{x}; \mathbf{w})$, given its features, x, parameterized by the weights, w.
- The predicted probability can then simply be converted into a binary outcome via a **threshold function**:

$$\hat{y} = \begin{cases} 1 & \text{if } \sigma(z) \geq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

Cost Function

Logistic Regression

- For example, for a simple binary classification problem we can use the logistic loss function

$$L(y, \hat{y}) = -[y \log \hat{y} + (1 - y) \log (1 - \hat{y})] \tag{5}$$

- y is the true label (0 or 1).
- $\hat{y}$ is the predicted probability of class 1.
- The function penalizes incorrect predictions more severely.

This cannot be maximized analytically but we can use a gradient ascent algorithm

Cost Function

Logistic Regression

$$L(y, \hat{y}) = -[y \log \hat{y} + (1 - y) \log (1 - \hat{y})] \tag{6}$$

When $y = 1$:

$$L(1, \hat{y}) = -\log \hat{y}$$

- If $\hat{y}$ is close to 1, the loss is small.
- If $\hat{y}$ is close to 0, the loss is large.

When $y = 0$:

$$L(0, \hat{y}) = -\log (1 - \hat{y})$$

- If $\hat{y}$ is close to 0, the loss is small.
- If $\hat{y}$ is close to 1, the loss is large.

Why Accuracy Can Be Misleading

Logistic Regression

Why Accuracy can be Misleading in Credit Risk Models

- In **credit risk modeling**, we often predict whether a loan will **default** (bad loan) or not (good loan).
- The dataset is often **highly unbalanced**: good loans are much more frequent than bad loans.
- A simple **accuracy metric** can be misleading in such cases.

Why Accuracy Can Be Misleading

Logistic Regression

Definition of Accuracy:

$$\text{Accuracy} = \frac{\text{Number of correct predictions}}{\text{Total number of predictions}}. \tag{7}$$

Example:

- Suppose 95% of loans are **good**, and only 5% are **bad**.
- A naive model that **always predicts "good"** achieves **95% accuracy**.
- However, this model is completely **useless** for detecting bad loans.

Conclusion: Accuracy does not capture the impact of misclassification.

Cost of Misclassification

- In credit risk, **False Positive (FP)** are more dangerous than **False Negatives (FN)**.
- **False Positives:** Predicting a bad loan as good $\rightarrow$ **high financial risk**.
- **False Negatives:** Predicting a good loan as bad $\rightarrow$ missed opportunity, but not catastrophic.
- **Accuracy does not distinguish between FN and FP**, but we need to!

Confusion Matrix

	Predicted: Good	**Predicted: Bad**
Actual: Good	**True Positives (TP)**	**False Negatives (FN)**
Actual: Bad	**False Positives (FP)**	**True Negatives (TN)**

- The confusion matrix shows where the model makes **errors**.
- Helps to analyze **False Positives vs. False Negatives**.

Alternative Performance Metrics

Precision: Out of all predicted bad loans, how many are truly bad?

$$\text{Precision} = \frac{TP}{TP + FP} \tag{8}$$

Recall (Sensitivity): Out of all actual bad loans, how many did we detect?

$$\text{Recall} = \frac{TP}{TP + FN} \tag{9}$$

F1-score: Balances Precision and Recall.

$$F1 = 2 \cdot \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}} \tag{10}$$

Specificity: Measures correct identification of good loans.

$$\text{Specificity} = \frac{TN}{TN + FP} \tag{11}$$

Key Takeaways

- **Accuracy alone is misleading** in unbalanced datasets.
- **Confusion matrix** provides deeper insights into model performance.
- **Precision, Recall, F1-score, and Specificity** are better suited for evaluating credit risk models.
- A strong credit risk model should minimize **False Positive**, as these represent loans that default but were wrongly classified as good.

Subsection 1

Credit Risk Example

Lending Club Case Study

- Data consists of loans made and whether they proved to be good or defaulted.
- We use only four features
  - Home ownership (rent vs. own)
  - Income
  - Debt to income
  - Credit score
- Training set has 8,695 observations (7,196 good loans and 1,499 defaulting loans). Test set has 5,196 observations (4,858 good loans and 1,058 defaulting loans)

Interpretation of FICO Scores

- The **FICO** credit score is a three-digit number used by lenders to assess an individual's creditworthiness.
- It ranges from **300 to 850**, with higher scores indicating lower credit risk.
- Developed by **Fair Isaac Corporation (FICO)**.
- Used in the U.S. for lending decisions such as **mortgages, car loans, and credit cards**.

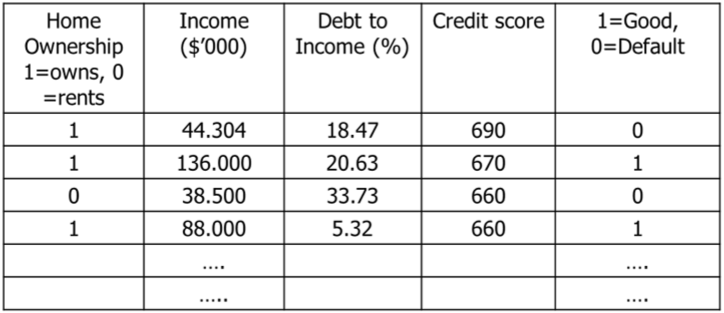
FICO Score Range Breakdown

- **300-579 (Poor)**: High interest rates, difficult loan approvals.
- **580-669 (Fair)**: Some lenders approve loans, but with high interest.
- **670-739 (Good)**: Average U.S. credit score, decent interest rates.
- **740-799 (Very Good)**: Better loan terms and lower interest rates.
- **800-850 (Exceptional)**: Best rates and highest credit limits.

Use in LendingClub

- In the **LendingClub dataset**, FICO scores are given as a **range** (e.g., 700-724).
- **fico_low**: The lower bound of the range.
- **fico_high**: The upper bound of the range.
- This range helps investors assess **borrower risk** when making lending decisions.

The Data



Home Ownership 1=owns, 0 =rents	Income ($'000)	Debt to Income (%)	Credit score	1=Good, 0=Default
1	44.304	18.47	690	0
1	136.000	20.63	670	1
0	38.500	33.73	660	0
1	88.000	5.32	660	1
	....			....
	.....			....

Results for Lending Club Training Set

- X_1= Home Ownership
- X_2= Income
- X_3= Debt to income ratio
- X_4 = Credit score

$$Y = -6.5645 + 0.1395 \cdot X_1 + 0.0041 \cdot X_2 - 0.0011 \cdot X_3 + 0.0113 \cdot X_4$$

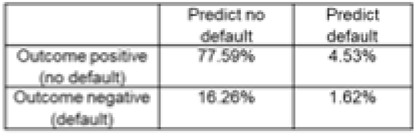
Decision Criterion

- The data set is imbalanced with more good loans than defaulting loans
- There are procedures for creating a balanced data set
- With a balanced data set we could classify an observation as positive if $Q > 0.5$ and negative otherwise
- However this does not consider the cost of misclassifying a bad loan and the lost profit from misclassifying a good loan
- A better approach is to investigate different thresholds, Z
  - If $Q > Z$ we accept a loan
  - If $Q \leq Z$ we reject the loan

Test Results

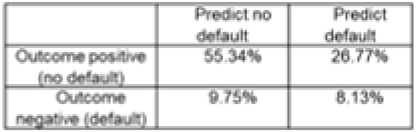
See Hull, Tables 3.10, 3.11, and 3.12

Z = 0.75:



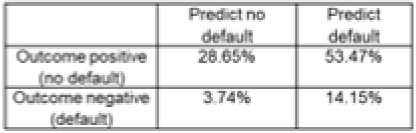
	Predict no default	Predict default
Outcome positive (no default)	77.59%	4.53%
Outcome negative (default)	16.26%	1.62%

Z=0.80:



	Predict no default	Predict default
Outcome positive (no default)	55.34%	26.77%
Outcome negative (default)	9.75%	8.13%

Z=0.85:



	Predict no default	Predict default
Outcome positive (no default)	28.65%	53.47%
Outcome negative (default)	3.74%	14.15%

The Confusion matrix and common ratios

	Predict positive outcome	Predict negative outcome
Outcome positive	TP	FN
Outcome negative	FP	TN

$$\text{Accuracy} = \frac{\text{TP} + \text{TN}}{\text{TP} + \text{FN} + \text{FP} + \text{TN}}$$

$$\text{True Positive Rate (TPR also called sensitivity or recall)} = \frac{\text{TP}}{\text{TP} + \text{FN}}$$

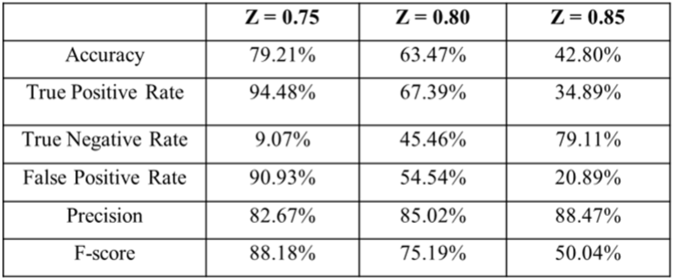
$$\text{The True Negative rate(also called specificity)} = \frac{\text{TN}}{\text{TN} + \text{FP}}$$

$$\text{The False Positive Rate} = \frac{\text{FP}}{\text{TN} + \text{FP}}$$

$$\text{Precision, P} = \frac{\text{TP}}{\text{TP} + \text{FP}}$$

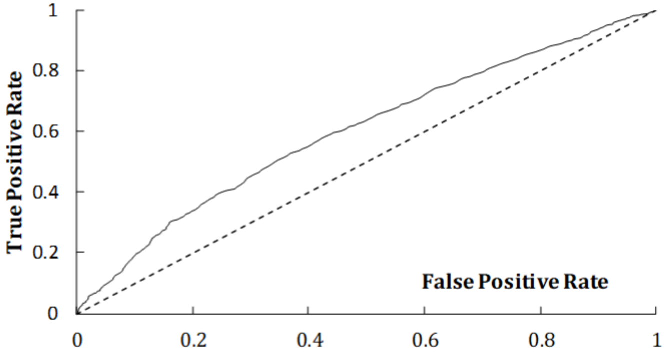
$$\text{F score} = 2 \times \frac{\text{P} \times \text{TPR}}{\text{P} + \text{TPR}}$$

Test Set Ratios for different Z values



	Z = 0.75	**Z = 0.80**	**Z = 0.85**
Accuracy	79.21%	63.47%	42.80%
True Positive Rate	94.48%	67.39%	34.89%
True Negative Rate	9.07%	45.46%	79.11%
False Positive Rate	90.93%	54.54%	20.89%
Precision	82.67%	85.02%	88.47%
F-score	88.18%	75.19%	50.04%

As we change the Z criterion we get an ROC



Area Under Curve (AUC)

- The area under the curve is a popular way of summarizing the predictive ability of a model to estimate a binary variable
- When $AUC = 1$ the model is perfect.
- When $AUC = 0.5$ the model has no predictive ability
- When $AUC < 0.5$ the model is worse than random
- In this case $AUC = 0.6020$

Choosing Z

- The value of Z can be based on
- The expected profit from a loan that is good, P
- The expected loss from a loan that defaults, L
- We need to maximize: $P \times \text{TP} - L \times \text{FP}$