

2025-11-29

$$\begin{aligned}
 1 \text{ (a)} \quad P_M &= 4 \left(1 - \frac{1}{\sqrt{M}}\right) Q \left(\sqrt{\frac{3}{M-1}} \frac{E_s}{N_0} \right) \\
 &= 4 \left(1 - \frac{1}{8}\right) Q \left(\sqrt{\frac{E_s}{\pi N_0}} \right) \\
 &= \frac{7}{2} Q \left(\sqrt{\frac{2 \times 10^{-4}}{4 \times 10^{-7}}} \right) \\
 &\approx 3.5 Q(9.78900073) \\
 &\approx 3.5 \times 8.44052 \times 10^{-23} \\
 &\approx 2.954182 \times 10^{-22}
 \end{aligned}$$

(b) SNR is high, ~~2 bit error~~ when symbol error occurs, ~~the~~ almost always has 2 bit error. So

$$\begin{aligned}
 P_e &= \frac{2.954182 P_M}{\log_2 M} \\
 &= \frac{2.954182 \times 10^{-22}}{6} \\
 &\approx 4.923637 \times 10^{-23}
 \end{aligned}$$

$$2 \text{ (a)} \quad \lambda = \frac{3 \times 10^8}{1.8 \times 10^9} = \frac{1}{6} \text{ m}$$

$$\Delta f = \frac{100/3.6}{176} \approx 166.667 \text{ Hz} \leftarrow \text{symbol frequency}$$

$$T_c \approx \frac{9}{16\pi \Delta f} = \frac{9}{16\pi(166.667)} \approx 1.07429 \text{ ms} \leftarrow \text{coherence time}$$

$$B_c \approx \frac{1}{T_c} = 125000 \text{ Hz} \leftarrow \text{coherence bandwidth}$$

$$T_{\text{fading}} \ll T_s = 40 \mu\text{s} \ll T_c = 1.07429 \text{ ms}$$

So ~~the~~ fading is slow relative to the symbol duration

$$B_w = 1/(40 \mu\text{s}) = 25000 \text{ Hz} \ll B_c = 125000 \text{ Hz}$$

So ~~fading is not~~ the fading is flat

$$(c) \text{ we require } SNR_{\text{min}} = \frac{1}{4(10^4)} = 2800$$

$$Pr(\text{dBm}) = -174 + 10 \log_{10}(W) + NR(\text{dB}) + SNR(\text{dB})$$

$$= -174 + 10 \log_{10}(125000) + 8 + 10 \log_{10}(2800)$$

$$\approx -174 + 43.9794 + 8 + 33.9794$$

$$\approx -88.0412 \text{ dBm}$$

$$Path \text{ Loss } (dB) = 20 \log_{10} \left(\frac{4\pi d}{\lambda} \right) = 20 \log_{10} \left(\frac{4\pi(3 \times 10^3)}{\lambda} \right) \approx 107.090 \text{ dB}$$

$$\begin{aligned}
 \text{transmit } (dBm) &= -88.0412 + 107.090 = 19.0488 \text{ dBm} \\
 \text{req. tx power } (dBm) &= 19.0488 \text{ dBm}
 \end{aligned}$$

2. (d) 2025-11-29

2. (d) $\gamma_0 = \text{SNR}_{\text{in}} = 10^{10} = 100$

For BPSK and gray coding, and SNR is high
 $P_e \approx P_{\text{in}}$

$L=1 \Rightarrow P_e \approx \binom{2-1}{1} \left(\frac{1}{400}\right)^1 = \frac{1}{400} \times 10^{-3} > 10^{-8}$

$L=2 \Rightarrow P_e \approx \binom{4-1}{2} \left(\frac{1}{400}\right)^2 = \frac{1.575}{400} \times 10^{-5} > 10^{-8}$

$L=3 \Rightarrow P_e \approx \binom{6-1}{3} \left(\frac{1}{400}\right)^3 = 6.25 \times 10^{-7} > 10^{-8}$

$L=4 \Rightarrow P_e \approx \binom{8-1}{4} \left(\frac{1}{400}\right)^4 = 1.3671875 \times 10^{-9} \leq 10^{-8}$

So $L \geq 4$

3. (a) $\tau_{\text{RMS}}^2 = E[\tau^2] - (E[\tau])^2$

$\tau_{\text{RMS}} = \sqrt{E[\tau^2] - (E[\tau])^2}$

$E[\tau^2] = \frac{(1)^2(0)^2 + (2.6)^2(3)^2 + (2.4)^2(8)^2}{1^2 + 2.6^2 + 0.4^2} = 8.86842 \mu\text{s}^2$

$E[\tau] = \frac{(1)(0) + (2.6)(3) + (2.4)(8)}{1^2 + 2.6^2 + 0.4^2} = 1.55263 \mu\text{s}$

$\tau_{\text{RMS}} = \sqrt{8.86842 - 1.55263^2} = 2.54121 \mu\text{s}$

$\tau_{\text{RMS}} \approx 2.54121 \mu\text{s} \geq 1.5 \mu\text{s}$

Yes, τ_{RMS} is larger than 100% of symbol time

So 25% is significant for this channel

(b) Using BPSK, $\text{SNR} = 15 \text{ dB} = 65 \text{ dB}$

For BPSK, $P_e \approx \frac{1}{2} Q\left(\sqrt{2 \text{SNR}_{\text{in}}}\right) \approx \frac{1}{2} Q\left(\sqrt{10^{6.5}}\right)$

using Gray coding

assuming SNR_{in} is the ratio $\left(\frac{E_s}{N_0/2}\right)$
 $\approx \frac{1}{2} Q\left(\sqrt{3162277.66}\right) \approx \frac{1}{2} (1.775 \times 10^{-14}) \approx 0$

2018-11-27

4. (a)

For ~~PSK~~ SIC,

$$G_p(\text{dB}) = \frac{5 \times 10^6}{10 \times 10^3} = 250$$

$$G_p(\text{dB}) \approx$$

For BPSK using binary coding

$$R_b = R_M \Rightarrow \left(\frac{2 \times 10^6}{N_0} \right) = 2 \left(\sqrt{\frac{2 \times 10^6}{N_0}} \right) = 2 \left(\sqrt{\frac{2 \times 10^6}{N_0}} \right)$$

$$2 \left(\sqrt{\frac{2 \times 10^6}{N_0}} \right) \geq 10^{-6}$$

$$\sqrt{\frac{2 \times 10^6}{N_0}} \geq 4.75342$$

$$\frac{2 \times 10^6}{N_0} \geq 11.2975$$

$$M_j(\text{dB}) \approx 10 \log_{10} 250 - 10 \log_{10} 11.2975 \approx 13.4496 \text{ dB}$$

(b)

$$SIR = \frac{250}{24} = 10.4167$$

$$SIR(\text{dB}) = 10 \log_{10} 10.4167 \approx 10.1773$$

$$\therefore \text{no. } SIR(\text{dB}) < 10 \log_{10} 11.2975$$

$$\frac{P_s}{N_0} = 10 \log_{10} 11.2975$$

$$(\text{dB}) \approx 10.5245 \text{ dB}$$

$$\therefore SIR(\text{dB}) < \frac{P_s}{N_0}(\text{dB})$$

$\therefore N_0$, it does not meet system requirement.