secondary 2 mathematics@2024-12-07 with solutions

expand

1. Expand $-20(7-w)w^2(6w+4)$.
2. Expand $-10(5-7y)(6y-3)y^2$.
3. Expand $2(-6y-2)(2-3y)(6-y)y$.

factor

1. Factor $x^2 - y^2$.
2. Factor $x^2 + y^2$.
3. Factor $x^3 + y^3$.
4. Factor $x^3 - y^3$.
5. Factor $x^2 - 2xy + y^2$.
6. Factor $x^2 + 2xy + y^2$.
7. Factor $u^2 + 5u - 14$.
8. Factor $-476v^2 - 2380v + 6664$.
9. Factor $-952v^{10000} - 4760v^{9999} + 13328v^{9998}$.
10. Factor $2w^4 + 10w^2 - 28$.
11. Factor $4z^6 + 20z^3 - 56$.
12. Factor $8a^{10000} + 40a^{5000} - 112$.
13. Factor $16b^{16000} + 80b^{11000} - 224$.
14. Factor $y^2 + 8yz + 12z^2$.
15. Factor $y^2 + 8yz^3 + 12z^6$.
16. Factor $y^4 + 8y^3z^4 + 12y^2z^7$.
17. Factor $32j^{16000}k^{2000} + 160j^{11000}k^{3000} - 448k^{4000}$.

identities

1. Find P, Q in $Px^2 - 7x + Q \equiv (x-5)(x-2)$.
2. Find R, S, T in $2(4x+1)(Rx-S) \equiv 48x^2 + Tx - 8$.
3. Find A, B in $Ax + B$. When $x = 3$, $Ax + B = 16$. When $x = 5$, $Ax + B = 30$.
4. Find A, B, C in $Ax^2 + Bx + C$. When $x = 5$ or $x = -4$, $Ax^2 + Bx + C = 0$. When $x = 0$, $Ax^2 + Bx + C = 100$.
5. Find X, Y, Z in $Xa^2 + Ya + Z$. When $a = -3$ or $a = 5$, $Xa^2 + Ya + Z = 0$. When $x = 0$, $Xa^2 + Ya + Z = -30$.

expand (solutions)

1. Expand $-20(7-w)w^2(6w+4)$.

$$-20(7-w)w^2(6w+4)$$
$$= -20w^2\left(42w+28-6w^2-4w\right)$$
$$= -20w^2\left(-6w^2+38w+28\right)$$
$$= 120w^4-760w^3-560w^2.$$

2. Expand $-10(5-7y)(6y-3)y^2$.

$$-10(5-7y)(6y-3)y^2$$
$$= -10y^2\left(30y-15-42y^2+21y\right)$$
$$= -10y^2\left(-42y^2+51y-15\right)$$
$$= 420y^4-510y^3+150y^2.$$

3. Expand $2(-6y-2)(2-3y)(6-y)y$.

$$2(-6y-2)(2-3y)(6-y)y$$
$$= 2y\left(-12y+18y^2-4+6y\right)(6-y)$$
$$= 2y\left(18y^2-6y-4\right)(6-y)$$
$$= 2y\left(108y^2-36y-24-18y^3+6y^2+4y\right)$$
$$= 2y\left(-18y^3+114y^2-32y-24\right)$$
$$= -36y^4+228y^3-64y^2-48y.$$

factor (solutions)

1. Factor x^2-y^2.

$$x^2-y^2=(x-y)(x+y).$$

2. Factor x^2+y^2.

$$x^2+y^2.$$

3. Factor x^3+y^3.

$$x^3+y^3=(x+y)\left(x^2-xy+y^2\right).$$

4. Factor x^3-y^3.

$$x^3-y^3=(x-y)\left(x^2+xy+y^2\right).$$

5. Factor $x^2-2xy+y^2$.

$$x^2-2xy+y^2=(x-y)^2.$$

6. Factor $x^2+2xy+y^2$.

$$x^2+2xy+y^2=(x+y)^2.$$

7. Factor $u^2 + 5u - 14$.

$$u^2 + 5u - 14 = (u + 7)(u - 2).$$

8. Factor $-476v^2 - 2380v + 6664$.

$$-476v^2 - 2380v + 6664$$
$$= -476\left(v^2 + 5v - 14\right)$$
$$= -476(v + 7)(v - 2).$$

9. Factor $-952v^{10000} - 4760v^{9999} + 13328v^{9998}$.

$$-952v^{10000} - 4760v^{9999} + 13328v^{9998}$$
$$= -952v^{9998}\left(v^2 + 5v - 14\right)$$
$$= -952v^{9998}(v + 7)(v - 2).$$

10. Factor $2w^4 + 10w^2 - 28$.

$$2w^4 + 10w^2 - 28$$
$$= 2\left(w^4 + 5w^2 - 14\right)$$
$$= 2\left(\left(w^2\right)^2 + 5\left(w^2\right) - 14\right)$$
$$= 2\left(w^2 + 7\right)\left(w^2 - 2\right).$$

11. Factor $4z^6 + 20z^3 - 56$.

$$4z^6 + 20z^3 - 56$$
$$= 4\left(z^6 + 5z^4 - 14\right)$$
$$= 4\left(\left(z^3\right)^2 + 5\left(z^3\right) - 14\right)$$
$$= 4\left(z^3 + 7\right)\left(z^3 - 2\right).$$

12. Factor $8a^{10000} + 40a^{5000} - 112$.

$$8a^{10000} + 40a^{5000} - 112$$
$$= 8\left(a^{10000} + 5a^{5000} - 14\right)$$
$$= 8\left(\left(a^{5000}\right)^2 + 5\left(a^{5000}\right) - 14\right)$$
$$= 8\left(a^{5000} + 7\right)\left(a^{5000} - 2\right).$$

13. Factor $16b^{16000} + 80b^{11000} - 224$.

$$16b^{16000} + 80b^{11000} - 224$$
$$= 16b^{6000}\left(b^{10000} + 5b^{5000} - 14\right)$$
$$= 16b^{6000}\left(\left(b^{5000}\right)^2 + 5\left(b^{5000}\right) - 14\right)$$
$$= 16b^{6000}\left(b^{5000} + 7\right)\left(b^{5000} - 2\right).$$

14. Factor $y^2 + 8yz + 12z^2$.

$$y^2 + 8yz + 12z^2 = (y + 6z)(y + 2z).$$

15. Factor $y^2 + 8yz^3 + 12z^6$.

$$y^2 + 8yz^3 + 12z^6$$
$$= y^2 + 8y\left(z^3\right) + 12\left(z^3\right)^2$$
$$= \left(y + 6z^3\right)\left(y + 2z^3\right).$$

16. Factor $y^4 + 8y^3z^4 + 12y^2z^7$.

$$y^4 + 8y^3z^4 + 12y^2z^7$$
$$= y^2z\left(y^2 + 8yz^3 + 12z^6\right)$$
$$= y^2z\left(y^2 + 8y\left(z^3\right) + 12\left(z^3\right)^2\right)$$
$$= y^2z\left(y + 6z^3\right)\left(y + 2z^3\right).$$

17. Factor $32j^{16000}k^{2000} + 160j^{11000}k^{3000} - 448k^{4000}$.

$$32j^{16000}k^{2000} + 160j^{11000}k^{3000} - 448k^{4000}$$
$$= 32j^{6000}k^{2000}\left(j^{10000} + 5j^{5000}k^{1000} - 14k^{2000}\right)$$
$$= 32j^{6000}k^{2000}\left(\left(j^{5000}\right)^2 + 5\left(j^{5000}\right)\left(k^{1000}\right) - 14\left(k^{1000}\right)^2\right)$$
$$= 32j^{6000}k^{2000}\left(j^{5000} + 7k^{1000}\right)\left(j^{5000} - 2k^{1000}\right).$$

identities (solutions)

1. Find P, Q in $Px^2 - 7x + Q \equiv (x - 5)(x - 2)$.

$$Px^2 - 7x + Q \equiv (x - 5)(x - 2)$$
$$\equiv x^2 - 7x + 10$$

$$P = 1$$
$$Q = 10.$$

2. Find R, S, T in $2(4x + 1)(Rx - S) \equiv 48x^2 + Tx - 8$.

$$2(4x + 1)(Rx - S) \equiv 48x^2 + Tx - 8$$
$$(8x + 2)(Rx - S) \equiv 48x^2 + Tx - 8$$
$$8Rx^2 - 8Sx + 2Rx - 2S \equiv 48x^2 + Tx - 8$$
$$8Rx^2 + (2R - 8S)x - 2S \equiv 48x^2 + Tx - 8$$

$$R = 6$$
$$S = 4$$
$$T = -20.$$

3. Find A, B in $Ax + B$. When $x = 3$, $Ax + B = 16$. When $x = 5$, $Ax + B = 30$.

$$\begin{cases} 3A + B = 16 & \cdots (1) \\ 5A + B = 30 & \cdots (2) \end{cases}$$
$$(1) - (2)$$
$$(3A + B) - (5A + B) = 16 - 30$$
$$-2A = -14$$
$$A = 7$$

$$B = 16 - 3 \times 7$$
$$= -5.$$

4. Find A, B, C in $Ax^2 + Bx + C$. When $x = 5$ or $x = -4$, $Ax^2 + Bx + C = 0$. When $x = 0$, $Ax^2 + Bx + C = 100$.

$$Ax^2 + Bx + C \equiv F(x - D)(x - E)$$

$$Ax^2 + Bx + C = F(x - 5)(x + 4) \qquad \text{both sides } 0$$
$$= Fx^2 - Fx - 20F$$

$$A(0)^2 + B(0) + C = 100$$
$$C = 100$$
$$F = -5$$

$$A = -5$$
$$B = 5$$
$$C = 100.$$

5. Find X, Y, Z in $Xa^2 + Ya + Z$. When $a = -3$ or $a = 5$, $Xa^2 + Ya + Z = 0$. When $x = 0$, $Xa^2 + Ya + Z = -30$.

$$Xa^2 + Ya + Z \equiv F(a - D)(a - E)$$

$$Xa^2 + Ya + Z = F(a + 3)(a - 5) \qquad \text{both sides } 0$$
$$= Fa^2 - 2Fa - 15F$$

$$X(0)^2 + Y(0) + Z = -30$$
$$Z = -30$$
$$F = 2$$

$$X = 2$$
$$Y = -4$$
$$Z = -30.$$