ELEC 4110 Homework Fall 2025: Homework 3

1 M-QAM Error Performance

1. (20 pts) A wireless communication link employs 64-QAM modulation to transmit data over an AWGN channel. The system operates with average symbol energy of 2×10^{-4} J and experiences noise with power spectral density 10^{-7} W/Hz. Each symbol spans 0.5 ms, and Gray coding maps bits to constellation points.

   (a) (10 pts) Calculate the symbol error rate using the approximation:

   $$P_M \approx 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{E_s}{N_0}}\right)$$

   (b) (10 pts) Estimate the resulting bit error rate, assuming the Gray coding ensures that symbol errors typically cause only single bit errors.

2 Small-Scale Fading and Diversity Techniques

2. (30 pts) A vehicle moving at 100 km/h receives wireless signals from a base station operating at 1.8 GHz. The propagation environment exhibits RMS delay spread of 8 μs, and the communication system uses BPSK modulation with symbols of duration 40 μs.

   (a) (5 pts) Determine the Doppler frequency, coherence time, and coherence bandwidth of this channel.

   (b) (5 pts) Determine whether the fading is flat or frequency selective. Then determine whether it is slow or fast fading relative to the symbol duration.

   (c) (10 pts) The system designer needs to support real-time video streaming requiring BER of 10^{-4} at distances up to 3 km. Using the free-space path loss model and considering a receiver noise figure of 8 dB, calculate the minimum transmit power needed.

   (d) (10 pts) To extend the operational range, the system implements L-order diversity with maximal ratio combining. Determine the minimum diversity order needed to achieve BER of 10^{-8} when the original SNR at the target distance is 20 dB.

Hint:
(1) The coherence bandwidth and delay spread are related by $B_c \approx \frac{1}{\sigma_\tau}$, where σ_τ is the RMS delay spread.

(2) The required received power can be calculated using the formula:

$$P_r(\text{dBm}) = -174 + 10\log_{10}(W) + NF(\text{dB}) + \text{SNR}(\text{dB})$$

where P_r is the received power in dBm, W is the signal bandwidth in Hz, NF is the receiver noise figure in dB, and SNR is the required signal-to-noise ratio in dB.

(3) The BER performance of BPSK under flat Rayleigh fading channel is approximated by $P_b \approx \frac{1}{4 \cdot \text{SNR}_{\text{lin}}}$, where SNR_{lin} is the linear signal-to-noise ratio.

(4) The free-space path loss is given by $\text{FSPL(dB)} = 20\log_{10}\left(\frac{4\pi d}{\lambda}\right)$, where d is the transmission distance and λ is the signal wavelength.

(5) The error probability with L-th order diversity here is given by

$$P_M \approx \binom{2L-1}{L} \cdot \left(\frac{1}{4\gamma_0}\right)^L$$

where P_M is the symbol error probability, and γ_0 is the received SNR.

3 ISI Mitigation

3. (30 pts) A multipath channel has impulse response characterized by three paths with relative delays of 0, 3 μs, and 8 μs, and relative amplitudes of 1, 0.6, and 0.4 respectively.

   (a) (10 pts) Calculate the RMS delay spread of this channel. For a communication system using 15 μs symbols, determine whether the delay spread exceeds 10% of the symbol duration and state the implications for ISI.

   (b) (10 pts) When direct sequence spread spectrum with processing gain of 50 dB is applied, compute the improvement in effective SINR and the resulting bit error probability for BPSK modulation, assuming initial SNR of 15 dB and no diversity combing is performed.

4 Resource Partitioning

4. (20 pts) A CDMA cellular system serves 25 users simultaneously, with each user transmitting at 20 kbps. The system employs direct sequence spreading with chip rate of 5 MHz and BPSK modulation.

   (a) (10 pts) Compute the processing gain and determine the antijam margin in dB for reliable operation at bit error rate of 10^{-6}. **Hint:** The antijam margin is defined as: $M_j = G_p(\text{dB}) - \left(\frac{E_b}{N_0}\right)_{\text{req}}(\text{dB})$, where $G_p(\text{dB})$ is the processing gain in dB and $\left(\frac{E_b}{N_0}\right)_{\text{req}}(\text{dB})$ is the required E_b/N_0 in dB.

   (b) (10 pts) Calculate the signal-to-interference ratio per bit and verify whether the system meets the performance requirement when considering only multiuser interference.