ELEC 4110 Homework Fall 2025: Homework 1

1 LTI Systems and Fourier Transform

1. (10 pts) Compute the Fourier transform of the following signals.
   (a) (5 pts) $x_1(t) = e^{-2|t-1|}$.
   (b) (5 pts) $x_2(t) = \frac{\sin(\pi t)}{\pi t} \cdot \cos(4\pi t)$.
2. (20 pts) Consider a LTI system described by the differential equation

$$\frac{d^2 y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt} + x(t).$$

   (a) (5 pts) Find the frequency response of this system.
   (b) (5 pts) Find the impulse response of this system.
   (c) (10 pts) For input $x(t) = e^{-2t}u(t)$, find the output $y(t)$.

2 Random Variables and Random Processes

2. (15 pts) Two random variables are distributed by

$$f_{X,Y}(x,y) = \begin{cases} Kxe^{-y}, & \text{for } 0 \le x \le 1, x \le y < \infty \\ 0, & \text{otherwise} \end{cases}$$

   (a) (5 pts) Find the constant K.
   (b) (5 pts) Find the marginal density functions $f_X(x)$ and $f_Y(y)$.
   (c) (5 pts) Compute $P(X > 0.5 \mid Y = 2)$.
3. (15 pts) Define the random process

$$Z(t) = X\cos(\omega_0 t) + Y\sin(\omega_0 t),$$

   where X and Y are independent Gaussian random variables with mean 0 and variance σ^2.

   (a) (5 pts) Find $m_Z(t)$.
   (b) (5 pts) Compute the autocorrelation $R_Z(t_1, t_2)$.
   (c) (5 pts) Determine whether $Z(t)$ is wide-sense stationary.

3 Digital Communication

4. (20 pts) **Digital Modulation and Optimum Receiver.** A binary communication system transmits signals

$$s_0(t) = A, \quad s_1(t) = -A, \quad 0 \le t \le T.$$

Bits "0" and "1" are equally likely. The channel is AWGN with two-sided power spectral density $N_0/2$.

   (a) (10 pts) Derive the minimum bit error probability P_e.

   (b) (5 pts) Determine the impulse response $h(t)$ of the optimal matched filter.

   (c) (5 pts) Specify the optimal sampling time and decision threshold.

5. (20 pts) **Signal Space Representation.** Consider four signals $s_1(t), s_2(t), s_3(t), x(t)$ defined on $0 \le t \le 3$, each piecewise constant on intervals $[0, 1)$, $[1, 2)$, $[2, 3]$:

$$\begin{aligned} s_1(t) &: \quad [1,\ 1,\ 1], \\ s_2(t) &: \quad [1,\ -1,\ -1], \\ s_3(t) &: \quad [1,\ 1,\ -1], \\ x(t) &: \quad [1,\ -1,\ 1]. \end{aligned}$$

   (a) (10 pts) Apply Gram–Schmidt to obtain an orthonormal basis $\{\varphi_i(t)\}$ with $s_1(t), s_2(t), s_3(t)$.

   (b) (10 pts) Express $x(t)$ as a coordinate vector in the obtained basis $\{\varphi_i(t)\}$.