

secondary 2 mathematics@2024-12-07

with solutions

expand

1. Expand $-20(7 - w)w^2(6w + 4)$.
2. Expand $-10(5 - 7y)(6y - 3)y^2$.
3. Expand $2(-6y - 2)(2 - 3y)(6 - y)y$.

factor

1. Factor $x^2 - y^2$.
2. Factor $x^2 + y^2$.
3. Factor $x^3 + y^3$.
4. Factor $x^3 - y^3$.
5. Factor $x^2 - 2xy + y^2$.
6. Factor $x^2 + 2xy + y^2$.
7. Factor $u^2 + 5u - 14$.
8. Factor $-476v^2 - 2380v + 6664$.
9. Factor $-952v^{10000} - 4760v^{9999} + 13328v^{9998}$.
10. Factor $2w^4 + 10w^2 - 28$.
11. Factor $4z^6 + 20z^3 - 56$.
12. Factor $8a^{10000} + 40a^{5000} - 112$.
13. Factor $16b^{16000} + 80b^{11000} - 224$.
14. Factor $y^2 + 8yz + 12z^2$.
15. Factor $y^2 + 8yz^3 + 12z^6$.
16. Factor $y^4 + 8y^3z^4 + 12y^2z^7$.
17. Factor $32j^{16000}k^{2000} + 160j^{11000}k^{3000} - 448k^{4000}$.

identities

1. Find P, Q in $Px^2 - 7x + Q \equiv (x - 5)(x - 2)$.
 2. Find R, S, T in $2(4x + 1)(Rx - S) \equiv 48x^2 + Tx - 8$.
 3. Find A, B in $Ax + B$. When $x = 3$, $Ax + B = 16$. When $x = 5$, $Ax + B = 30$.
 4. Find A, B, C in $Ax^2 + Bx + C$. When $x = 5$ or $x = -4$, $Ax^2 + Bx + C = 0$.
When $x = 0$, $Ax^2 + Bx + C = 100$.
 5. Find X, Y, Z in $Xa^2 + Ya + Z$. When $a = -3$ or $a = 5$, $Xa^2 + Ya + Z = 0$.
When $x = 0$, $Xa^2 + Ya + Z = -30$.
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expand (solutions)

1. Expand $-20(7-w)w^2(6w+4)$.

$$\begin{aligned}& -20(7-w)w^2(6w+4) \\&= -20w^2(42w+28-6w^2-4w) \\&= -20w^2(-6w^2+38w+28) \\&= 120w^4-760w^3-560w^2.\end{aligned}$$

2. Expand $-10(5-7y)(6y-3)y^2$.

$$\begin{aligned}& -10(5-7y)(6y-3)y^2 \\&= -10y^2(30y-15-42y^2+21y) \\&= -10y^2(-42y^2+51y-15) \\&= 420y^4-510y^3+150y^2.\end{aligned}$$

3. Expand $2(-6y-2)(2-3y)(6-y)y$.

$$\begin{aligned}& 2(-6y-2)(2-3y)(6-y)y \\&= 2y(-12y+18y^2-4+6y)(6-y) \\&= 2y(18y^2-6y-4)(6-y) \\&= 2y(108y^2-36y-24-18y^3+6y^2+4y) \\&= 2y(-18y^3+114y^2-32y-24) \\&= -36y^4+228y^3-64y^2-48y.\end{aligned}$$

factor (solutions)

1. Factor x^2-y^2 .

$$x^2-y^2=(x-y)(x+y).$$

2. Factor x^2+y^2 .

$$x^2+y^2.$$

3. Factor x^3+y^3 .

$$x^3+y^3=(x+y)(x^2-xy+y^2).$$

4. Factor x^3-y^3 .

$$x^3-y^3=(x-y)(x^2+xy+y^2).$$

5. Factor $x^2-2xy+y^2$.

$$x^2-2xy+y^2=(x-y)^2.$$

6. Factor $x^2+2xy+y^2$.

$$x^2+2xy+y^2=(x+y)^2.$$

7. Factor $u^2 + 5u - 14$.

$$u^2 + 5u - 14 = (u + 7)(u - 2).$$

8. Factor $-476v^2 - 2380v + 6664$.

$$\begin{aligned} & -476v^2 - 2380v + 6664 \\ &= -476(v^2 + 5v - 14) \\ &= -476(v + 7)(v - 2). \end{aligned}$$

9. Factor $-952v^{10000} - 4760v^{9999} + 13328v^{9998}$.

$$\begin{aligned} & -952v^{10000} - 4760v^{9999} + 13328v^{9998} \\ &= -952v^{9998}(v^2 + 5v - 14) \\ &= -952v^{9998}(v + 7)(v - 2). \end{aligned}$$

10. Factor $2w^4 + 10w^2 - 28$.

$$\begin{aligned} & 2w^4 + 10w^2 - 28 \\ &= 2(w^4 + 5w^2 - 14) \\ &= 2((w^2)^2 + 5(w^2) - 14) \\ &= 2(w^2 + 7)(w^2 - 2). \end{aligned}$$

11. Factor $4z^6 + 20z^3 - 56$.

$$\begin{aligned} & 4z^6 + 20z^3 - 56 \\ &= 4(z^6 + 5z^3 - 14) \\ &= 4((z^3)^2 + 5(z^3) - 14) \\ &= 4(z^3 + 7)(z^3 - 2). \end{aligned}$$

12. Factor $8a^{10000} + 40a^{5000} - 112$.

$$\begin{aligned} & 8a^{10000} + 40a^{5000} - 112 \\ &= 8(a^{10000} + 5a^{5000} - 14) \\ &= 8((a^{5000})^2 + 5(a^{5000}) - 14) \\ &= 8(a^{5000} + 7)(a^{5000} - 2). \end{aligned}$$

13. Factor $16b^{16000} + 80b^{11000} - 224$.

$$\begin{aligned} & 16b^{16000} + 80b^{11000} - 224 \\ &= 16b^{6000}(b^{10000} + 5b^{5000} - 14) \\ &= 16b^{6000}((b^{5000})^2 + 5(b^{5000}) - 14) \\ &= 16b^{6000}(b^{5000} + 7)(b^{5000} - 2). \end{aligned}$$

14. Factor $y^2 + 8yz + 12z^2$.

$$y^2 + 8yz + 12z^2 = (y + 6z)(y + 2z).$$

15. Factor $y^2 + 8yz^3 + 12z^6$.

$$\begin{aligned} & y^2 + 8yz^3 + 12z^6 \\ &= y^2 + 8y(z^3) + 12(z^3)^2 \\ &= (y + 6z^3)(y + 2z^3). \end{aligned}$$

16. Factor $y^4 + 8y^3z^4 + 12y^2z^7$.

$$\begin{aligned} & y^4 + 8y^3z^4 + 12y^2z^7 \\ &= y^2z(y^2 + 8yz^3 + 12z^6) \\ &= y^2z(y^2 + 8y(z^3) + 12(z^3)^2) \\ &= y^2z(y + 6z^3)(y + 2z^3). \end{aligned}$$

17. Factor $32j^{16000}k^{2000} + 160j^{11000}k^{3000} - 448k^{4000}$.

$$\begin{aligned} & 32j^{16000}k^{2000} + 160j^{11000}k^{3000} - 448k^{4000} \\ &= 32j^{6000}k^{2000}(j^{10000} + 5j^{5000}k^{1000} - 14k^{2000}) \\ &= 32j^{6000}k^{2000}\left((j^{5000})^2 + 5(j^{5000})(k^{1000}) - 14(k^{1000})^2\right) \\ &= 32j^{6000}k^{2000}(j^{5000} + 7k^{1000})(j^{5000} - 2k^{1000}). \end{aligned}$$

identities (solutions)

1. Find P, Q in $Px^2 - 7x + Q \equiv (x - 5)(x - 2)$.

$$\begin{aligned} Px^2 - 7x + Q &\equiv (x - 5)(x - 2) \\ &\equiv x^2 - 7x + 10 \end{aligned}$$

$$\begin{aligned} P &= 1 \\ Q &= 10. \end{aligned}$$

2. Find R, S, T in $2(4x + 1)(Rx - S) \equiv 48x^2 + Tx - 8$.

$$\begin{aligned} 2(4x + 1)(Rx - S) &\equiv 48x^2 + Tx - 8 \\ (8x + 2)(Rx - S) &\equiv 48x^2 + Tx - 8 \\ 8Rx^2 - 8Sx + 2Rx - 2S &\equiv 48x^2 + Tx - 8 \\ 8Rx^2 + (2R - 8S)x - 2S &\equiv 48x^2 + Tx - 8 \end{aligned}$$

$$\begin{aligned} R &= 6 \\ S &= 4 \\ T &= -20. \end{aligned}$$

3. Find A, B in $Ax + B$. When $x = 3$, $Ax + B = 16$. When $x = 5$, $Ax + B = 30$.

$$\begin{aligned} & \begin{cases} 3A + B = 16 & \dots (1) \\ 5A + B = 30 & \dots (2) \end{cases} \\ & (1) - (2) \\ & (3A + B) - (5A + B) = 16 - 30 \\ & -2A = -14 \\ & A = 7 \end{aligned}$$

$$\begin{aligned} B &= 16 - 3 \times 7 \\ &= -5. \end{aligned}$$

4. Find A, B, C in $Ax^2 + Bx + C$. When $x = 5$ or $x = -4$, $Ax^2 + Bx + C = 0$.
When $x = 0$, $Ax^2 + Bx + C = 100$.

$$Ax^2 + Bx + C \equiv F(x - D)(x - E)$$

$$\begin{aligned} Ax^2 + Bx + C &= F(x - 5)(x + 4) && \text{both sides } 0 \\ &= Fx^2 - Fx - 20F \end{aligned}$$

$$\begin{aligned} A(0)^2 + B(0) + C &= 100 \\ C &= 100 \\ F &= -5 \end{aligned}$$

$$\begin{aligned} A &= -5 \\ B &= 5 \\ C &= 100. \end{aligned}$$

5. Find X, Y, Z in $Xa^2 + Ya + Z$. When $a = -3$ or $a = 5$, $Xa^2 + Ya + Z = 0$.
When $x = 0$, $Xa^2 + Ya + Z = -30$.

$$Xa^2 + Ya + Z \equiv F(a - D)(a - E)$$

$$\begin{aligned} Xa^2 + Ya + Z &= F(a + 3)(a - 5) && \text{both sides } 0 \\ &= Fa^2 - 2Fa - 15F \end{aligned}$$

$$\begin{aligned} X(0)^2 + Y(0) + Z &= -30 \\ Z &= -30 \\ F &= 2 \end{aligned}$$

$$\begin{aligned} X &= 2 \\ Y &= -4 \\ Z &= -30. \end{aligned}$$