# secondary 2 mathematics@2024-12-01 with solutions

## exponents

1. Simplify $a \times a^2$.
2. Simplify $a^3 \times a^4$.
3. Simplify $a^a \times a^b$.
4. Simplify $a^5 \div a^2$.
5. Simplify $a^2 \div a^5$.
6. Simplify $\left(a^3\right)^9$.
7. Simplify $\left(a^9\right)^x$.
8. Simplify $(x^y)^x$.
9. Simplify $\left(\frac{c}{d}\right)^4$.
10. Simplify $\left(\frac{c^2}{d}\right)^4$.
11. Simplify $(xy)^a$.
12. Simplify $xy^a$.
13. Simplify $((xy)^a)^b$.
14. Simplify $\left(\frac{ac}{bd}\right)^e$.
15. Simplify $\left(cd^2\left(\frac{a^2}{b^3}\right)^e\right)^f$.

## expand

1. Expand $(7-3y)(y-3)$.
2. Expand $(-7w-4)(2w-5)$.
3. Expand $(3v+7)(6v-4)$.
4. Expand $(4-2m)(m+6)$.
5. Expand $(1-3x)(3x+3)$.
6. Expand $4y(7y+5)(7y+7)$.
7. Expand $2(-7n-6)(-n-5)n$.
8. Expand $21s^2(3s+7)$.
9. Expand $-6v\left(35v^2-39v-14\right)$.
10. Expand $7w(3w+2)(4w+5)(7w+6)$.

## factor

1. Factor $a^2-b^2$.
2. Factor $a^2-2a-15$.

3. Factor $a^2 + a - 20$.
4. Factor $n^2 - 10n + 21$.
5. Factor $2y^2 + 16y + 14$.
6. Factor $n^2 + 6n + 5$.
7. Factor $x^2 - 5xz - 6z^2$.
8. Factor $3m^2 + 12mn - 36n^2$.
9. Factor $u^4 + 8u^2 + 15$.
10. Factor $m^4 + 2m^2n^2 + n^4$.
11. Factor $(3x)^3 - (4y)^3$.
12. Factor $(3z)^3 + (4w)^3$.
13. Factor $27a^3 - 64b^3$.
14. Factor $27c^3 + 64d^3$.
15. Factor $a^2 + 2ab + b^2$.
16. Factor $4c^2 + 4cd + d^2$.
17. Factor $8e^2 + 8ef + 2f^2$.
18. Factor $v^3 - v^2 - 6v$.
19. Factor $2a^4 + 12a^3 + 16a^2$.
20. Factor $2n^4m - 22n^2m^2 + 56m^3$.

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# exponents (solutions)

1. $a \times a^2 = a^{1+2} = a^3$
2. $a^3 \times a^4 = a^{3+4} = a^7$
3. $a^a \times a^b = a^{a+b}$
4. $a^5 \div a^2 = a^{5-2} = a^3$
5. $a^2 \div a^5 = \frac{1}{a^{5-2}} = \frac{1}{a^3}$
6. $\left(a^3\right)^9 = a^{3\times 9} = a^{27}$
7. $\left(a^9\right)^x = a^{9\times x} = a^{9x}$
8. $\left(x^y\right)^x = x^{y\times x} = x^{yx}$
9. $\left(\frac{c}{d}\right)^4 = \frac{c^4}{d^4}$
10. $\left(\frac{c^2}{d}\right)^4 = \frac{\left(c^2\right)^4}{d^4} = \frac{c^{2\times 4}}{d^4} = \frac{c^8}{d^4}$
11. $(xy)^a = x^a y^a$
12. $xy^a$ is already simplified.
13. $((xy)^a)^b = (xy)^{a\times b} = (xy)^{ab} = x^{ab}y^{ab}$
14. $\left(\frac{ac}{bd}\right)^e = \frac{(ac)^e}{(bd)^e} = \frac{a^e c^e}{b^e d^e}$

15. $$\begin{aligned}&\left(cd^2\left(\frac{a^2}{b^3}\right)^e\right)^f\\&=\left(cd^2\left(\frac{\left(a^2\right)^e}{\left(b^3\right)^e}\right)\right)^f\\&=\left(cd^2\left(\frac{a^{2e}}{b^{3e}}\right)\right)^f\\&=\left(\frac{cd^2a^{2e}}{b^{3e}}\right)^f\\&=\frac{\left(cd^2a^{2e}\right)^f}{\left(b^{3e}\right)^f}\\&=\frac{c^fd^{2f}e^{2ef}}{b^{3ef}}\end{aligned}$$

# expand (solutions)

1. $(7-3y)(y-3)=7y-21-3y^2+9y=-3y^2+16y-21$
2. $(-7w-4)(2w-5)=-14w^2+35w-8w+20=-14w^2+27w+20$
3. $(3v+7)(6v-4)=18v^2-12v+42v-28=18v^2+30v-28$
4. $(4-2m)(m+6)=4m+24-2m^2-12m=-2m^2-8m+24$
5. $(1-3x)(3x+3)=3x+3-9x^2-9x=-9x^2-6x+3$
6. $$\begin{aligned}&4y(7y+5)(7y+7)\\&=4y\left(49y^2+49y+35y+35\right)\\&=4y\left(49y^2+84y+35\right)\\&=196y^3+336y^2+140y\end{aligned}$$
7. $$\begin{aligned}&2(-7n-6)(-n-5)n\\&=2n(-7n-6)(-n-5)\\&=2n\left(7n^2+35n+6n+30\right)\\&=2n\left(7n^2+41n+30\right)\\&=14n^3+82n^2+60n\end{aligned}$$
8. $21s^2(3s+7)=63s^3+147s^2$
9. $-6v\left(35v^2-39v-14\right)=-210v^3+234v^2+84v$
10. $$\begin{aligned}&7w(3w+2)(4w+5)(7w+6)\\&=7w(3w+2)\left(28w^2+24w+35w+30\right)\\&=7w(3w+2)\left(28w^2+59w+30\right)\\&=7w\left(84w^3+177w^2+90w+56w^2+118w+60\right)\\&=7w\left(84w^3+233w^2+208w+60\right)\\&=588w^4+1631w^3+1456w^2+420w\end{aligned}$$

# factor (solutions)

1. $a^2 - b^2 = (a + b)(a - b)$
2. $a^2 - 2a - 15 = (a + 3)(a - 5)$
3. $a^2 + a - 20 = (a + 5)(a - 4)$
4. $n^2 - 10n + 21 = (n - 3)(n - 7)$
5. $2y^2 + 16y + 14 = 2\left(y^2 + 8y + 7\right) = 2(y + 7)(y + 1)$
6. $n^2 + 6n + 5 = (n + 5)(n + 1)$
7. $x^2 - 5xz - 6z^2 = (x + z)(x - 6z)$
8. $3m^2 + 12mn - 36n^2 = 3\left(m^2 + 4mn - 12n^2\right) = 3(m + 6n)(m - 2n)$
9. $u^4 + 8u^2 + 15 = \left(u^2\right)^2 + 8\left(u^2\right) + 15 = \left(u^2 + 5\right)\left(u^2 + 3\right)$
10. $m^4 + 2m^2n^2 + n^4 = \left(m^2\right)^2 + 2\left(m^2\right)\left(n^2\right) + \left(n^2\right)^2 = \left(m^2 + n^2\right)^2$
11. $$\begin{aligned} &(3x)^3 - (4y)^3 \\ &= (3x - 4y)\left((3x)^2 + (3x)(4y) + (4y)^2\right) \\ &= (3x - 4y)\left(9x^2 + 12xy + 16y^2\right) \end{aligned}$$
12. $$\begin{aligned} &(3z)^3 + (4w)^3 \\ &= (3z + 4w)\left((3z)^2 - (3z)(4w) + (4w)^2\right) \\ &= (3z + 4w)\left(9z^2 - 12zw + 16w^2\right) \end{aligned}$$
13. $$\begin{aligned} &27a^3 - 64b^3 \\ &= 3^3a^3 - 4^3b^3 \\ &= (3a)^3 - (4b)^3 \\ &= (3a - 4b)\left((3a)^2 + (3a)(4b) + (4b)^2\right) \\ &= (3a - 4b)\left(9a^2 + 12ab + 16b^2\right) \end{aligned}$$
14. $$\begin{aligned} &27c^3 + 64d^3 \\ &= 3^3c^3 + 4^3d^3 \\ &= (3c)^3 + (4d)^3 \\ &= (3c + 4d)\left((3c)^2 - (3c)(4d) + (4d)^2\right) \\ &= (3c + 4d)\left(9c^2 - 12cd + 16d^2\right) \end{aligned}$$
15. $a^2 + 2ab + b^2 = (a + b)^2$
16. $4c^2 + 4cd + d^2 = (2c)^2 + 2(2c)d + d^2 = (2c + d)^2$
17. $8e^2 + 8ef + 2f^2 = 2\left(4e^2 + 4ef + f^2\right) = 2\left((2e)^2 + 2(2e)f + f^2\right) = 2(2e + f)^2$
18. $v^3 - v^2 - 6v = v\left(v^2 - v - 6\right) = v(v + 2)(v - 3)$
19. $2a^4 + 12a^3 + 16a^2 = 2a^2\left(a^2 + 6a + 8\right) = 2a^2(a + 4)(a + 2)$

20.

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\begin{aligned}
& 2n^4m - 22n^2m^2 + 56m^3 \\
&= 2m\left(n^4 - 11n^2m + 28m\right) \\
&= 2m\left(\left(n^2\right)^2 - 11\left(n^2\right)m + 28m^2\right) \\
&= 2m\left(n^2 - 4m\right)\left(n^2 - 7m^2\right)
\end{aligned}
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