$$(f * g)(t) := \int_{-\infty}^{\infty} f(\tau) g(t-\tau)\, d\tau = \int_{-\infty}^{\infty} f(t-\tau) g(\tau)\, d\tau$$

$$(f \star g)(t) = \int_{-\infty}^{\infty} f(\tau+t) \overline{f(\tau)}\, d\tau = \int_{-\infty}^{\infty} f(\tau) \overline{f(\tau - t)}\, d\tau$$

$SNR_{dB} = 10 \log_{10} SNR$
(20 if amplitude)

$$(2\pi)^{-k/2} \det(\Sigma)^{-1/2} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

$$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

WSS: constant mean
autocorr/cov depends on time()
$E[X_t^2] < \infty$

white Gaussian process

$Cov(X,Y) = E(XY) - E(X)E(Y)$

binary ...

$$P_e(V_{th}) = p_0 Q\left(\frac{AT+V_{th}}{\sigma_{nT}}\right) + p_1 Q\left(\frac{AT-V_{th}}{\sigma_{nT}}\right)$$

$$BER = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$Q(x) \approx \frac{p(x)}{x} = \frac{1}{\sqrt{2\pi}\, x} e^{-x^2/2}$$

$$Q(x) = \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right) \quad \operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2}\, dt$$

$$V_{th} = \frac{\sigma_{nT}^2}{S_{01} - S_{00}} \ln \frac{p_0}{p_1} + \frac{S_{00} + S_{01}}{2}$$

$S_{01} = AT$

$S_{02} = AT$

$$P_e(V_{th}) = Q\left(\sqrt{\frac{(S_{01} - S_{00})^2}{4\sigma_{nT}^2}}\right) = Q\left(\sqrt{\frac{E_y}{2N_0}}\right)$$

$$\rho = \frac{(S_{01} - S_{00})^2}{\sigma_{nT}^2} \le \int_{\mathbb{R}} \frac{|G(f)|^2}{S_n(f)}\, df$$

$$H_{opt}(f) = c\, \frac{G^*(f) e^{-j2\pi f T}}{S_n(f)}$$

$$h_{opt}(t) = g^*(T-t)$$

$$E_y = E_0 + E_1 - 2\sqrt{E_0 E_1}\, \rho_{01}$$

$$\phi_1(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f t)$$

$$\phi_2(t) = -\sqrt{\frac{2E}{T}} \sin(2\pi f t)$$

σ^2 $\sigma^2 =$

$$C_{ij} = \int_{-\infty}^{\infty} S_n(f)\, \Phi_i^*(f)\, \Phi_j(f)\, df$$

minimum distance

$\Rightarrow$ max (project onto signal directly, unnormalized)

$$\int_0^T y_n(t) s_i^*(t)\, dt - \frac{E_i}{2}$$

pairwise $P_{ij} = Q\left(\frac{\|s_i - s_j\|}{2\sigma}\right) = Q\left(\sqrt{\frac{\|s_i - s_j\|^2}{2N_0}}\right)$

$$Q\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right) \le P_{eM} \le (M-1)\, Q\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right)$$

$$\text{PAPR} = \frac{\text{peak } A^2}{\text{RMS}^2}$$

M-FSK

$$P_{eM} = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(1 - Q\left(x + \sqrt{\frac{2E_s}{N_0}}\right)\right)^{M-1} \exp\left(-\frac{x^2}{2}\right) dx$$

$$P_e \approx \frac{1}{2} P_{eM} = \frac{P_{eM}}{2} \frac{M}{M-1}$$

~~app for $x>3$~~

$$P_{eM} \le (M-1)\, Q\left(\sqrt{\frac{E_s}{N_0}}\right) \quad \boxed{E_s}$$

$$P_e \le \frac{M}{2} Q\left(\sqrt{\frac{E_s}{N_0}}\right) \quad \boxed{E_s}$$

$$\approx \frac{M}{4} \exp\left(\frac{kE_b}{2N_0}\right) \quad \boxed{E_b}$$

$x > 3$
$Q(x) \le \frac{1}{2} e^{-x^2/2}$
approx

M-PSK

$$P_{eM} = \frac{1}{\pi} \int_0^{\pi(M-1)/M} \exp\left(\frac{E_s}{N_0} \frac{\sin^2(\pi/M)}{\sin^2\phi}\right) d\phi$$

$$\boxed{1}\, Q\left(\sqrt{\frac{2E_s \sin^2(\pi/M)}{N_0}}\right) \le P_{eM} \le \boxed{2}\, Q(\text{xxx}) \quad \approx \text{tight as SNR} \uparrow$$

Gray code, divide by $k = \log_2 M$

M-QAM

$\alpha = \sqrt{E}$

$$E_s = \frac{4E}{\sqrt{M}} \sum_{k=1}^{\sqrt{M}/2} (2k-1)^2$$

$$= \frac{4E}{\sqrt{M}} \frac{\sqrt{M}(\sqrt{M}+1)(\sqrt{M}-1)}{3}$$

$$= \frac{2}{3}E(M-1)$$

$$\alpha^2 = E = \frac{3E_s}{2(M-1)}$$

$$P_{\sqrt{M}\text{-PAM}} = \cancel{2(1-k)}\ 2(1-1/\sqrt{M})\, Q\left(\sqrt{\frac{2E}{N_0}}\right)$$

$$= \frac{2(\sqrt{M}-1)}{\sqrt{M}}\, Q\left(\sqrt{\frac{3E_s}{(M-1)N_0}}\right)$$

$$P_{e,M} \approx 2P_{\sqrt{M}\text{-PAM}}$$

gain over M-PSK $= \dfrac{3(M-1)}{2\sin^2(\pi/M)}$ M>4 always better

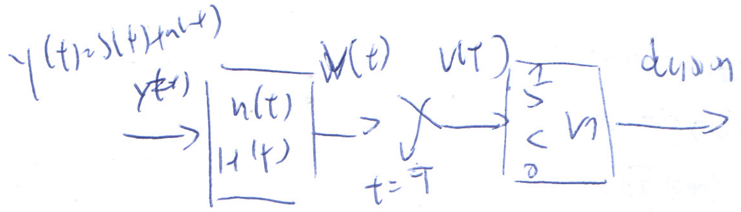
≈ tight as SNR↑

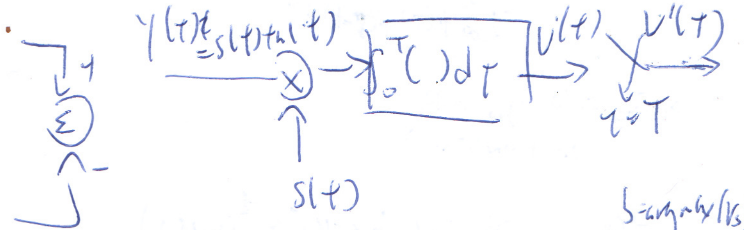
(2) $Q\left(\sqrt{\frac{3E_s}{(M-1)N_0}}\right) \le P_{e,M} \le 4Q(\ldots)$

$$\frac{d_{min}}{2} = \sqrt{E} = \sqrt{\frac{3E_s}{2(M-1)}}$$

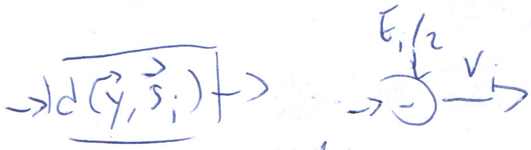
$$P_{e,b} = \frac{P_{e,M}}{k} = \frac{P_{e,M}}{\log_2 M}$$

0000	0100	0101	0001
1000	1100	1101	1001
1010	1110	1111	1011
0010	0110	0111	0011





$\hat{s} = \arg\max(v_i)$



$C = B\log_2\left(1+\frac{S}{N}\right)$ $N = BN_0$, S is power

$\frac{C}{B} = \log_2\left(1+\frac{S}{N}\right)$

$$\frac{E_b}{N_0} = \frac{ST_b}{N/B} = \frac{B}{C}\frac{S}{N} = \frac{B}{C}\left(2^{C/B}-1\right)$$

as $\frac{C}{B} \to 0$ $\frac{E_b}{N_0} \to \ln 2 = 0.693 = -1.59$ dB

Fubini's theorem

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha-\beta) + \cos(\alpha+\beta))$

$\sin\alpha\sin\beta = \frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta))$

$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$

$\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$

$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$

$\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\alpha-\beta)$

FT	f	ω
$f(x-a)$	$e^{-j2\pi af}\hat{f}(f)$	$e^{-ja\omega}\hat{f}(\omega)$
$f(x)e^{iax}$	$\hat{f}\left(f-\frac{a}{2\pi}\right)$	$\hat{f}(\omega-a)$
$\int_{-\infty}^{t} f(t)\,dt$	$\frac{\hat{f}(f)}{i2\pi f} + C\delta(f)$	$\frac{\hat{f}(\omega)}{i\omega} + 2\pi C\delta(\omega) = \pi X(0)\delta(\omega)$
$u(t)$	$\underbrace{\frac{1}{2}\left(\frac{1}{i\pi f}\right.}_{\text{sgn sgn}} + \underbrace{\left.\delta(f)\right)}_{1}$	$\pi\left(\frac{1}{i\pi\omega} + \delta(\omega)\right)$
$e^{-at}u(t)$	$\frac{1}{a+i2\pi f}$	$\frac{1}{a+i\omega}$
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+4\pi^2 f^2}$	$\cancel{\frac{2a}{a^2+\omega^2}}$ $\frac{2a}{a^2+\omega^2}$