MA'DIN POLYTECHNIC COLLEGE, MALAPPURAM

ENGINEERING MATHEMATICS I

SOLVED QUESTION PAPER OCTOBER 2019

COMMON FOR ALL BRANCHES

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PART - A

I

1. $\cos^2 A - \sin^2 A = 2\cos^2 A - 1$

LHS $\Rightarrow \cos^2 A - \sin^2 A = (1 - \sin^2 A) - \sin^2 A$

$$= 1 - 2\sin^2 A$$

$$= \underline{2\cos^2 A - 1}$$

or

~~2.~~

$$\cos^2 A - \sin^2 A = \cos^2 A - (1 - \cos^2 A)$$

$$= \cos^2 A - 1 + \cos^2 A$$

$$= \underline{2\cos^2 A - 1}$$

2. $\sin 3A = 3\sin A - 4\sin^3 A$

3. ~~a We~~ $abc = 4R\Delta$.

We know that $\Delta = \frac{1}{2} bc \sin A$

$$\boxed{\frac{a}{\sin A} = 2R. \qquad \frac{a}{2R} = \sin A}$$

$\Delta = \frac{1}{2} bc \times \sin A$.

$$= \frac{1}{2} bc \times \frac{a}{2R}$$

$$= \frac{abc}{4R}.$$

$\underline{4R\Delta = abc}$

4. $y = x \times \sin x$

$$\frac{dy}{dx} = x \times \frac{d}{dx}(\sin x) + \sin x \times \frac{d}{dx}(x)$$

$$= x\cos x + \sin x$$

5. $s = 2t^3 - 3t^2 + 1$

Velocity, $v = \frac{ds}{dt} = \frac{d}{dt}(2t^3 - 3t^2 + 1)$

$$= 6t^2 - 6t$$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(6t^2 - 6t)$

$$= 12t - 6$$

II

PART - B

1) For expressing $4\cos x + 3\sin x$ in the form $R\sin(x+\alpha)$

We have to find out 'R' & 'α'

We have, $4\cos x + 3\sin x = R\sin(x+\alpha)$

$$= R[\sin x \times \cos\alpha + \cos x \times \sin\alpha]$$

$$= R\sin x \times \cos\alpha + R\cos x \times \sin\alpha$$

$4\cos x + 3\sin x = R\sin x \times \cos\alpha + R\cos x \times \sin\alpha$

Equating the similar terms on both sides.

$4\cos x = R\cos x \times \sin\alpha$.

$3\sin x = R\sin x \times \cos\alpha$.

ie; $4 = R\sin\alpha$ —(1)

$3 = R\cos\alpha$ —(2)

Squaring eq^n (1) & (2)

$(1)^2 \Rightarrow 4^2 = R^2\sin^2\alpha$

$16 = R^2\sin^2\alpha$ — (3)

$(2)^2 \Rightarrow 3^2 = R^2\cos^2\alpha$

$9 = R^2\cos^2\alpha$ — (4)

Adding eq^n (3) & (4)

$(3)+(4) \Rightarrow 16+9 = R^2\sin^2\alpha + R^2\cos^2\alpha$

$\Rightarrow 25 = R^2\underbrace{[\sin^2\alpha + \cos^2\alpha]}_{(1)}$

$\Rightarrow 25 = R^2 \times 1$

$\Rightarrow R = \sqrt{25} = \underline{\underline{\pm 5}}$

Divide eq^n (1) by eq^n (2)

$\frac{(1)}{(2)} \Rightarrow \frac{4}{3} = \frac{R\sin\alpha}{R\cos\alpha}$

$\frac{4}{3} = \frac{\sin\alpha}{\cos\alpha}$

$$\frac{4}{3} = \tan\alpha.$$

$$\alpha = \tan^{-1}\left(\frac{4}{3}\right) = \underline{\underline{53.13}}$$

$$\underline{\underline{4\cos x + 3\sin x = \pm 5\sin(x + 53.13)}}$$

2. $\sin 10 \times \sin 50 \times \sin 70 = \frac{1}{8}$.

$$LHS \Rightarrow \sin 10 \times (\sin 50 \times \sin 70)$$

$$= \sin 10 \times \frac{-1}{2}\left[\cos(50+70) - \cos(50-70)\right]$$

$$= -\tfrac{1}{2} \times \sin 10\left[\cos(120) - \cos(-20)\right]$$

$$= -\tfrac{1}{2} \times \sin 10\left[\cos(180-60) - \cos 20\right]$$

$\cos 120 = \cos($
$= \cos(180-60)$

$$= -\tfrac{1}{2} \times \sin 10\left[-\cos 60 - \cos 20\right]$$

$\cos(180-60) = -\cos 60$

$$= -\tfrac{1}{2} \times \sin 10\left[-\tfrac{1}{2} - \cos 20\right]$$

$$= -\frac{1}{2} \times \sin 10 \times \frac{-1}{2} - \frac{-1}{2} \times \sin 10 \times \cos 20$$

$$= \frac{1}{4}\sin 10 + \frac{1}{2}\sin 10 \times \cos 20$$

$$= \frac{1}{4}\sin 10 + \frac{1}{2}\left[\frac{1}{2}\left(\sin(10+20) + \sin(10-20)\right)\right]$$

$$= \frac{1}{4}\sin 10 + \frac{1}{2}\left[\frac{1}{2}\left(\sin 30 + \sin(-10)\right)\right]$$

$$= \frac{1}{4}\sin 10 + \frac{1}{2} \times \frac{1}{2}\left[\sin 30 - \sin 10\right]$$

$$= \frac{1}{4}\sin 10 + \frac{1}{4}\left[\sin 30 - \sin 10\right]$$

$$= \frac{1}{4}\cancel{\sin 10} + \frac{1}{4}\sin 30 - \frac{1}{4}\cancel{\sin 10}$$

$$= \frac{1}{4}\sin 30$$

$$= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

3). $(a-b)\cos\frac{C}{2} = c \times \sin\frac{A-B}{2}$

LHS $\Rightarrow (a-b)\cos\frac{C}{2}$

$$= (2R\sin A - 2R\sin B) \times \cos\frac{C}{2}$$

$$= 2R(\sin A - \sin B) \times \cos\frac{C}{2}$$

$$= 2R\left(2 \times \cos\frac{A+B}{2} \times \sin\frac{A-B}{2}\right) \times \cos\frac{C}{2}$$

$$= \sin\frac{A-B}{2} \times \left[4R\cos\frac{A+B}{2} \times \cos\frac{C}{2}\right].$$

(rearrange the terms)

$$= \sin\frac{A-B}{2} \times \left[4R \times \cos\left(\frac{A+B}{2} + \frac{C}{2} - \frac{C}{2}\right) \times \cos\frac{C}{2}\right].$$

$$= \sin\frac{A-B}{2} \times \left[4R \times \cos\left(\frac{A+B+C}{2} - \frac{C}{2}\right) \times \cos\frac{C}{2}\right]$$

$$= \sin\frac{A-B}{2} \times \left[4R \times \cos\left(\frac{180}{2} - \frac{C}{2}\right) \times \cos\frac{C}{2}\right].$$

$$= \sin\left(\frac{A-B}{2}\right)\left[4R \times \cos\left(90-\frac{C}{2}\right) \times \cos\frac{C}{2}\right]$$

$$= \sin\left(\frac{A-B}{2}\right)\left[4R \times \sin\frac{C}{2} \times \cos\frac{C}{2}\right]$$

$$= \sin\frac{A-B}{2} \times 2R\left[2 \times \sin\frac{C}{2} \times \cos\frac{C}{2}\right]$$

$$= \sin\frac{A-B}{2} \times 2R \times \sin C$$

$$= \sin\frac{A-B}{2} \times c \quad = \text{RHS}..$$

4.) $y = \sin x$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(\sin x) = \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left[\frac{x+h+x}{2}\right]\sin\left[\frac{x+h-x}{2}\right]}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\frac{2x+h}{2} \times \sin\frac{h}{2}}{h}$$

Dividing both Numerator & denominator by 2.

$$= \lim_{h\to 0} \frac{\frac{2\cos\frac{2x+h}{2}\sin\frac{h}{2}}{2}}{h/2}$$

$$= \lim_{h\to 0} \cos\frac{2x+h}{2} \times \frac{\sin h/2}{h/2}$$

$$= \lim_{h\to 0} \cos\frac{2x+h}{2} \times \underbrace{\lim_{h\to 0} \frac{\sin h/2}{h/2}}_{(1)}$$

$$= \lim_{h\to 0} \cos\frac{2x+h}{2} \times 1$$

$$= \cos_{h\to 0} \frac{2x+h}{2} = \cos\frac{2x+0}{2}$$

$$= \cos\frac{2x}{2} = \underline{\cos x}$$

$$\underline{\frac{d}{dx}(\sin x) = \cos x}$$

5.) $(x^2+y^2)^2 = xy$

Differentiate w.r.to x on both sides

$$\frac{d}{dx}(x^2+y^2)^2 = \frac{d}{dx}(xy)$$

$$\Rightarrow 2(x^2+y^2) \times \frac{d}{dx}(x^2+y^2) = x \times \frac{d}{dx}(y) + y \times \frac{d}{dx}(x)$$

$$\Rightarrow 2(x^2+y^2)\left[2x + 2y \times \frac{dy}{dx}\right] = x\frac{dy}{dx} + y \times 1$$

$$\Rightarrow 4x(x^2+y^2) + 2(x^2+y^2)\times 2y \times \frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\Rightarrow 4x(x^2+y^2) + 4y(x^2+y^2)\times \frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\Rightarrow 4y(x^2+y^2)\times \frac{dy}{dx} - x\frac{dy}{dx} = y - 4x(x^2+y^2)$$

$$\Rightarrow \frac{dy}{dx}\left[4y(x^2+y^2) - x\right] = y - 4x(x^2+y^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - 4x(x^2+y^2)}{4y(x^2+y^2) - x}$$

6.) $y = 3x^2 + x - 2$ at $(1, 2)$

Equation of tangent is $y - y_1 = \frac{dy}{dx}(x - x_1)$

Equation of Normal is $y - y_1 = \frac{-1}{\left(\frac{dy}{dx}\right)}(x - x_1)$

$$y = 3x^2 + x - 2$$

$$\frac{dy}{dx} = 6x + 1$$

$$\left(\frac{dy}{dx}\right)_{(1,2)} = 6\times 1 + 1 = 6 + 1 = 7$$

Equation of tangent is $y-2 = 7(x-1)$

$$(y-2) = 7x-7$$

$$y-2-7x+7=0$$

$$y-7x+5=0$$

Equation of Normal is $y-2 = \frac{-1}{7}(x-1)$

$$\Rightarrow 7(y-2) = -1(x-1)$$

$$\Rightarrow 7y-14 = -x+1$$

$$\Rightarrow 7y-14+x-1=0$$

$$\Rightarrow 7y+x-15=0$$

7.) $\sin A + \sin(120^\circ + A) + \sin(240^\circ + A) = 0$

LHS $\Rightarrow \sin A + \sin(120+A) + \sin(240+A)$

$$= \sin A + \sin 120 \times \cos A + \cos 120 \times \sin A + \sin 240 \times \cos A + \cos 240 \times \sin A$$

$$= \sin A + \sin(90+30) \times \cos A + \cos(90+30) \times \sin A + \sin(180+60) \times \cos A + \cos(180+60) \times \sin A$$

$$= \sin A + \cos 30 \times \cos A + {}^{-}\sin 30 \times \sin A + {}^{-}\sin 60 \times \cos A + {}^{-}\cos 60 \times \sin A$$

$= \sin A + \cancel{\frac{\sqrt{3}}{2} \times \cos A} - \frac{1}{2}\sin A - \cancel{\frac{\sqrt{3}}{2} \times \cos A} - \frac{1}{2}\sin A$

$= \sin A - \frac{1}{2}\sin A - \frac{1}{2}\sin A$

$= \sin A - \cancel{2} \times \frac{1}{\cancel{2}} \sin A$

$= \sin A - \sin A$

$= \underline{\underline{0}}$

PART - C

Unit - I

III

a) $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

LHS $\Rightarrow \dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + (1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

$= \dfrac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$

$= \dfrac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta}$

$$= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$$

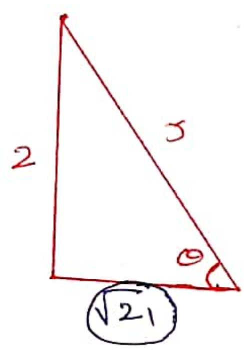
$$= \frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2\cancel{(1+\cos\theta)}}{\cancel{(1+\cos\theta)}\sin\theta} = \frac{2}{\sin\theta}$$

$$= \underline{\underline{2\operatorname{cosec}\theta}} = \underline{\underline{RHS}}$$

b) $\sin\theta = 0.4 = \frac{4}{10} = \frac{2}{5}$



$\text{Base} = \sqrt{\text{Hypotenuse}^2 - \text{Altitude}^2}$

$= \sqrt{5^2 - 2^2} = \sqrt{25-4}$

$= \underline{\underline{\sqrt{21}}}$

$\therefore \cos\theta = \frac{\sqrt{21}}{5}$ ie, $\sec\theta = \frac{5}{\sqrt{21}}$

$\tan\theta = \frac{2}{\sqrt{21}}$

$$\therefore \sec\theta + \tan\theta = \frac{5}{\sqrt{21}} + \frac{2}{\sqrt{21}}$$

$$= \frac{7}{\sqrt{21}}$$

c) $A+B = 45^\circ$

$\tan(A+B) = \tan 45$

$\Rightarrow \dfrac{\tan A + \tan B}{1 - \tan A \times \tan B} = 1$

$\Rightarrow \tan A + \tan B = 1 - \tan A \times \tan B$

$\Rightarrow \tan A + \tan B + \tan A \times \tan B = 1$

$\Rightarrow 1 + \tan A + \tan B + \tan A \times \tan B = 1+1$

$\Rightarrow (1+\tan A) + \tan B(1+\tan A) = 2$

$\Rightarrow (1+\tan A) + (1+\tan B) = 2$

Hence proved.

IV

a) $\frac{1+\cos\theta}{\sin\theta} = \frac{\sin\theta}{1-\cos\theta}$

LHS $\Rightarrow \frac{1+\cos\theta}{\sin\theta} = \frac{1+\cos\theta}{\sin\theta} \times \frac{1-\cos\theta}{1-\cos\theta}$

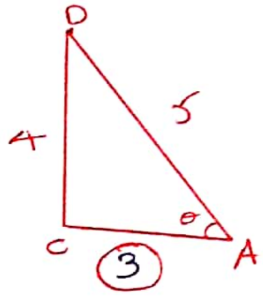
$= \frac{(1+\cos\theta)(1-\cos\theta)}{\sin\theta(1-\cos\theta)}$

$= \frac{1-\cos^2\theta}{\sin\theta(1-\cos\theta)}$

$= \frac{\sin^2\theta}{\sin\theta(1-\cos\theta)}$

$= \frac{\sin\theta}{1-\cos\theta} = RHS$

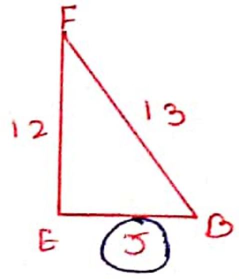
b) $\sin A = \frac{4}{5}$



$AC = \sqrt{5^2-4^2} = \sqrt{25-16}$

$= \sqrt{9} = 3$

$\cancel{\cos B = \frac{5}{13}}$ $\sin B = \frac{12}{13}$



$BE = \sqrt{13^2-12^2} = \sqrt{169-144}$

$= \sqrt{25} = 5$

$$\cos A = \frac{3}{5} \qquad \cos B = \frac{5}{13}$$

$$\sin(A+B) = \sin A \times \cos B + \cos A \times \sin B$$

$$= \frac{4}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{12}{13}$$

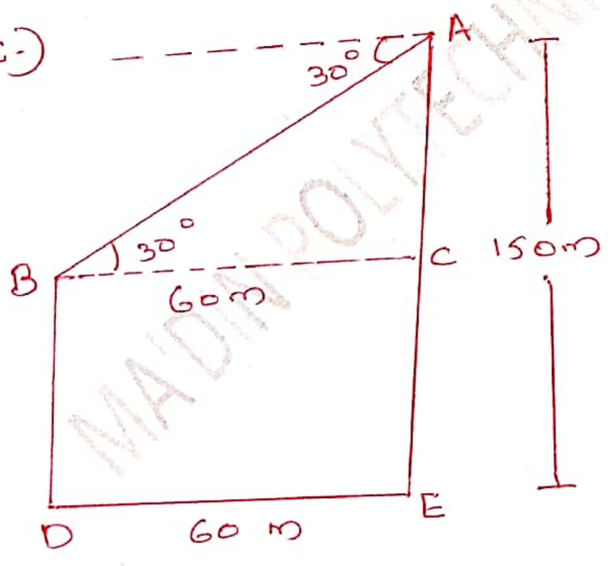
$$= \frac{20}{65} + \frac{36}{65} = \underline{\underline{\frac{56}{65}}}$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13}$$

$$= \frac{15}{65} + \frac{48}{65} = \underline{\underline{\frac{63}{65}}}$$

c.)



BD => First tower

AE => 2nd tower.

Consider ΔABC,

$$\tan 30^\circ = \frac{AC}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AC}{60}$$

$$\Rightarrow 60 \times \frac{1}{\sqrt{3}} = AC$$

$$\therefore AC = \frac{60}{\sqrt{3}} m$$

$\therefore$ Height of the 1st tower ~~AE~~ $CE = AE - AC$

$$CE = 150 - \frac{60}{\sqrt{3}}$$

$$= 115.36 m$$

CE & BD are equal

$\therefore$ Height of the 1st tower $= 115.36 m$

Unit - II

V) a) $\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = 2$.

$$LHS \Rightarrow \frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A}$$

$$= \frac{3\sin A - 4\sin^3 A}{\sin A} - \frac{4\cos^3 A - 3\cos A}{\cos A}$$

$$= \frac{\cancel{\sin A}(3 - 4\sin^2 A)}{\cancel{\sin A}} - \frac{\cancel{\cos A}(4\cos^2 A - 3)}{\cancel{\cos A}}$$

$$= 3 - 4\sin^2 A - (4\cos^2 A - 3)$$

$$= 3 - 4\sin^2 A - 4\cos^2 A + 3$$

$$= 6 - 4\underbrace{(\sin^2 A + \cos^2 A)}_{(1)}$$

$$= 6 - 4 = \underline{\underline{2}}$$

b) $\tan A + \cot A = 2\operatorname{cosec} 2A$

LHS $\Rightarrow \tan A + \cot A$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \times \cos A} = \frac{1}{\sin A \times \cos A}.$$

Multiply Nr. & Dr. by 2.

$$= \frac{2}{2\sin A \times \cos A} = \frac{2}{\sin 2A}$$

$$= 2 \times \frac{1}{\sin 2A}$$

$$= \underline{\underline{2\operatorname{cosec} 2A}} = \underline{\text{RHS}}$$

Hence proved.

c) $\frac{\sin 2A}{1+\cos 2A} = \tan A$

LHS $\Rightarrow$ $\frac{\sin 2A}{1+\cos 2A} = \frac{2\sin A \times \cos A}{1+2\cos^2 A - 1}$

$= \frac{\cancel{2}\sin A \times \cancel{\cos} A}{\cancel{2}\cos^{\cancel{2}} A} = \frac{\sin A}{\cos A}$

$= \underline{\underline{\tan A}} = \text{RHS}.$

Put $A = 15°$.

$\therefore \tan 15 = \frac{\sin(2\times 15)}{1+\cos(2\times 15)} = \frac{\sin 30}{1+\cos 30}$

$= \frac{1/2}{1+\frac{\sqrt{3}}{2}} = \frac{\frac{1}{\cancel{2}}}{\frac{2+\sqrt{3}}{\cancel{2}}} = \frac{1}{2+\sqrt{3}}$

$= \frac{1}{2+\sqrt{3}} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})} = \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$

$= \frac{2-\sqrt{3}}{4-3} = \underline{\underline{2-\sqrt{3}}}$

$\boxed{(a+b)(a-b) = a^2 - b^2}$

VI

a) $\dfrac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$

LHS $\Rightarrow \dfrac{\sin 3A + \sin A + \sin 5A}{\cos 3A + \cos A + \cos 5A}$

$$= \frac{\sin 3A + 2\sin\left(\frac{A+5A}{2}\right) \times \cos\left(\frac{A-5A}{2}\right)}{\cos 3A + 2\cos\left(\frac{A+5A}{2}\right) \times \cos\left(\frac{A-5A}{2}\right)}$$

$$= \frac{\sin 3A + 2\sin\left(\frac{6A}{2}\right) \times \cos\left(\frac{-4A}{2}\right)}{\cos 3A + 2\cos\left(\frac{6A}{2}\right) \times \cos\left(\frac{-4A}{2}\right)}$$

$$= \frac{\sin 3A + 2\sin 3A \times \cos(-2A)}{\cos 3A + 2\cos 3A \times \cos(-2A)}$$

$$= \frac{\sin 3A + 2\sin 3A \times \cos 2A}{\cos 3A + 2\cos 3A \times \cos 2A}$$

$\boxed{\cos(-\theta) = \cos\theta}$

$$= \frac{\sin 3A\,\cancel{(1 + 2\cos 2A)}}{\cos 3A\,\cancel{(1 + 2\cos 2A)}} = \frac{\sin 3A}{\cos 3A} = \underline{\underline{\tan 3A}}$$

b) $\sin A+\sin 3A+\sin 5A+\sin 7A = 4\cos A \times \cos 2A \times \sin 4A$.

LHS $\Rightarrow (\sin A+\sin 3A)+(\sin 5A+\sin 7A)$

$= 2\sin\left(\frac{A+3A}{2}\right)\times\cos\left(\frac{A-3A}{2}\right) + 2\sin\left(\frac{5A+7A}{2}\right)\times\cos\left(\frac{5A-7A}{2}\right)$

$= 2\sin \frac{4A}{2} \times \cos\left(\frac{-2A}{2}\right) + 2\sin \frac{12A}{2} \times \cos\left(\frac{-2A}{2}\right)$

$= 2\sin 2A \times \cos(-A) + 2\sin 6A \times \cos(-A)$

$= 2\sin 2A \times \cos A + 2\sin 6A \times \cos A$

$= 2\cos A(\sin 2A + \sin 6A)$

$= 2\cos A\left(2\sin \frac{2A+6A}{2} \times \cos \frac{2A-6A}{2}\right)$

$= 2\cos A \times 2 \times \sin 4A \times \cos(-2A)$

$= 4\cos A \times \sin 4A \times \cos 2A$

$= \underline{4\cos A \times \cos 2A \times \sin 4A} = \underline{RHS}$

c) $a = 4cm$, $b = 5cm$, $c = 7cm$.

$A = \cos^{-1}\left[\frac{b^2+c^2-a^2}{2bc}\right] = \cos^{-1}\left[\frac{5^2+7^2-4^2}{2\times5\times7}\right]$

$= \cos^{-1}\left[\frac{25+49-16}{70}\right] = \cos^{-1}\left[\frac{58}{70}\right] = \underline{34^\circ 03'}$

$$B = \cos^{-1}\left[\frac{a^2+c^2-b^2}{2ac}\right]$$

$$= \cos^{-1}\left[\frac{4^2+7^2-5^2}{2\times4\times7}\right] = \cos^{-1}\left[\frac{16+49-25}{56}\right]$$

$$= 44^\circ 25'$$

$$C = 180 - (A+B)$$

$$= 180^\circ - (34^\circ 03' + 44^\circ 25')$$

$$= 101^\circ 32'$$

UNIT - III

VII

a) $$\lim_{x\to4} \frac{x^4-256}{x^3-64} = \lim_{x\to4} \frac{x^4-4^4}{x^3-4^3}$$

Dividing by $(x-4)$

$$= \lim_{x\to4} \left[\frac{\frac{x^4-4^4}{x-4}}{\frac{x^3-4^3}{x-4}}\right]$$

$$= \frac{\lim_{x\to4} \frac{x^4-4^4}{x-4}}{\lim_{x\to4} \frac{x^3-4^3}{x-4}}$$

$$= \frac{4 \times 4^{4-1}}{3 \times 4^{3-1}} = \frac{4 \times 4^3}{3 \times 4^2}$$

$$= \frac{256}{48} = \frac{16}{3}$$

b) $x = a(\theta - \sin\theta)$

$y = a(1 - \cos\theta$.

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(a(\theta - \sin\theta))$$

$$= a(1 - \cos\theta)$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(a(1-\cos\theta)) = a\sin\theta$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{\cancel{a}\sin\theta}{\cancel{a}(1-\cos\theta)}$$

$$= \frac{\cancel{2}\cancel{\sin\theta/2} \times \cos\theta/2}{\cancel{2}\sin^2\theta/2}$$

$$= \frac{\cos\theta/2}{\sin\theta/2}$$

$$= \cot\frac{\theta}{2}$$

c) $y = A\cos px + B\sin px$.

$$\frac{dy}{dx} = A \times -\sin px \times p + B\cos px \times p$$

$$= -Ap\sin px + Bp\cos px$$

$$\frac{d^2y}{dx^2} = -Ap \times \cos px \times p + Bp \times -\sin px \times p$$

$$= -Ap^2 \cdot \cos px - Bp^2 \sin px$$

$$= -p^2(A\cos px + \sin px)$$

$$= -p^2 y$$

$$\therefore \frac{d^2y}{dx^2} \propto y$$

VIII

a) (i) $$\lim_{x\to 0} \frac{1-\cos 2x}{x^2} = \lim_{x\to 0} \frac{2\sin^2 x}{x^2}$$

$$= 2\left[\lim_{x\to 0} \frac{\sin^2 x}{x^2}\right]$$

$$= 2\left[\lim_{x\to 0} \frac{\sin x}{x} \times \lim_{x\to 0} \frac{\sin x}{x}\right]$$

$$= 2 \times 1 \times 1$$

$$= 2$$

(ii) $\lim_{x \to -1} \frac{x^3+1}{x+1} = \lim_{x \to -1} \frac{x^3-(-1)^3}{x-(-1)}$

$= 3 \times (-1)^{3-1}$

$= 3 \times (-1)^2 = 3 \times 1 = \underline{\underline{3}}$

b) $y = (x^2+x+1)^7 \sin^2 x$

$\frac{dy}{dx} = (x^2+x+1)^7 \times \frac{d}{dx}(\sin^2 x) + \sin^2 x \times \frac{d}{dx}(x^2+x+1)^7$

$= (x^2+x+1)^7 \times 2\sin x \cos x + \sin^2 x \times 7(x^2+x+1)^6 \times \frac{d}{dx}(x^2+x+1)$

$= (x^2+x+1)^7 \times 2\sin x \cos x + 7\sin^2 x \cdot (x^2+x+1)^6 (2x+1)$

$= (x^2+x+1)^6 \left[(x^2+x+1) \times \sin 2x + 7\sin^2 x(2x+1)\right]$

c) $y = Ae^{nx} + Be^{-nx}$

$\frac{dy}{dx} = Ae^{nx} \times \frac{d}{dx}(nx) + Be^{-nx} \times \frac{d}{dx}(-nx)$

$= Ae^{nx} \times n + Be^{-nx} \times -n$

$= \underline{\underline{Ane^{nx} - Bne^{-nx}}}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[Ane^{nx} - Bne^{-nx}\right]$$

$$= Ane^{nx} \times n - Bne^{-nx} \times -n$$

$$= An^2e^{nx} + Bn^2e^{-nx}$$

$$= n^2\left[Ae^{nx} + Be^{-nx}\right]$$

$$= n^2y$$

$$\therefore \frac{d^2y}{dx^2} - n^2y = 0$$

UNIT - IV

IX a)

$$h = 60t - t^2$$

$$\frac{dh}{dt} = \frac{d}{dt}(60t - t^2) = 60 - 2t$$

$$\frac{dh}{dt} = 0 \Rightarrow 60 - 2t = 0$$

$$\Rightarrow 2t = 60$$

$$\Rightarrow t = \frac{60}{2} = 30$$

Greatest height $h = 60 \times 30 - (30)^2$

$$= 1800 - 900$$

$$= 900$$

b) $V = \frac{4}{3}\pi r^3$ $\qquad \frac{dV}{dt} = -10$ cc/sec

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$= \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

ie, $-10 = 4\pi r^2 \frac{dr}{dt}$

$$\Rightarrow \frac{dr}{dt} = \frac{-10}{4\pi \times r^2} \qquad r = 15$$

$$= \frac{-10}{4\pi \times 15^2} = \frac{-1}{90\pi} \text{ cm/sec}$$

Surface area $S = 4\pi r^2$

$$\frac{dS}{dt} = 4\pi \frac{d}{dt}(r^2)$$

$$= 4\pi \times 2r \times \frac{dr}{dt}$$

$$= 4\pi \times 2 \times 15 \times \frac{-1}{90\pi}$$

$$= -\frac{4}{3} \text{ sq.cm/sec.}$$

$\therefore$ Surface area is shrinking at the rate of $\frac{4}{3}$ cm^2/sec

c) $S = 2x^3 - 9x^2 + 12x$

$$\frac{ds}{dx} = 6x^2 - 18x + 12$$

At a maxima or a minima $\frac{ds}{dx} = 0$

$$\frac{ds}{dx} = 0 \Rightarrow 6x^2 - 18x + 12 = 0$$

$$\Rightarrow 6(x^2 - 3x + 2) = 0$$

ie, $(x-2)(x-1) = 0$

$$x = 1, 2$$

$$\frac{d^2s}{dx^2} = 12x - 18$$

when $x = 1 \Rightarrow \frac{d^2s}{dx^2} = 12x - 18 = 12 \times 1 - 18$

$$= 12 - 18 = -6 < 0$$

When $x = 2 \Rightarrow \frac{d^2s}{dx^2} = 12 \times 2 - 18 = 24 - 18 = 6 > 0$

$\therefore$ The function is maximum when $x = 1$,

because $\frac{d^2y}{dx^2} < 0$ at $x = 1$.

Maximum deflection is $y = 2 \times 1^3 - 9 \times 1^2 + 12 \times 1$

$$= 5$$

X) a) $S = 3t^3 - t^2 + 9t + 1$

Velocity $v = \frac{ds}{dt} = 9t^2 - 2t + 9$

Acceleration, $a = \frac{dv}{dt} = 18t - 2$

Velocity at $t = 3$ is

$V = 9 \times 3^2 - 2 \times 3 + 9$

$= 9 \times 9 - 6 + 9 = 81 - 6 + 9 = 81 + 3 = \underline{\underline{84}}$

Acceleration at $t = 3$ sec,

$a = 18 \times 3 - 2 = 54 - 2 = \underline{\underline{52}}$

b) Volume of spherical balloon, $V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 25$ cc/sec

$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt} = 4\pi \underline{\underline{r^2}} \frac{dr}{dt}$

$\Rightarrow 25 = 4\pi r^2 \frac{dr}{dt}$

$\Rightarrow 25 = 4\pi \times 15^2 \frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{25}{4\pi 15^2} = \frac{5}{180\pi}$ cm/sec $= \underline{\underline{\frac{1}{36\pi}}}$ cm/sec

c) $y = 2x^3 - 3x^2 - 36x + 10$

$\frac{dy}{dx} = 6x^2 - 6x - 36$

At maximum $\frac{dy}{dx} = 0$

ie, $6x^2 - 6x - 36 = 0$

$6(x^2 - x - 6) = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

ie, $x = -2, 3$

$\frac{d^2y}{dx^2} = 12x - 6$

$\frac{d^2y}{dx^2}$ at $x = 3$ is $12 \times 3 - 6$

$\Rightarrow 36 - 6 = 30 > 0$

$\frac{d^2y}{dx^2}$ at $x = -2$ is $\Rightarrow 12 \times -2 - 6 = -24 - 6 = -30 < 0$

$\therefore$ The function is maximum at $x = -2$, because

$\frac{d^2y}{dx^2} < 0$ at $x = -2$

Maximum value is $y = 2 \times (-2)^3 - 3 \times (-2)^2 - 36 \times -2 + 10$

$= -16 - 12 + 72 + 10$

$= -28 + 82$

$= 54$