

**DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/
MANAGEMENT/COMMERCIAL PRACTICE — APRIL, 2019**

ENGINEERING MATHEMATICS - I

[Time : 3 hours]

(Maximum marks : 100)

PART — A

(Maximum marks : 10)

Marks

I Answer *all* questions. Each question carries 2 marks.

1. Prove that $\cos^2 A - \sin^2 A = 1 - 2 \sin^2 A$.
2. If $\cos A = \frac{4}{5}$ and A is acute, find $\cos 3A$.
3. Find the area of the triangle ABC, given $b = 3\text{cm}$, $c = 2\text{cm}$, $A = 30^\circ$.
4. If $y = x \cos x$, Find $\frac{dy}{dx}$.
5. Find the velocity and acceleration at time 't' of a particle moving according to $s = t^3 - 2t^2 + 1$.

(5×2 = 10)

PART — B

(Maximum marks : 30)

II Answer any *five* of the following questions. Each question carries 6 marks.

1. Express $3 \cos x + 4 \sin x$ in the form $R \sin (x + \alpha)$, where α is acute.
2. Prove that $\cos \frac{\pi}{8} + \cos \frac{3\pi}{8} + \cos \frac{5\pi}{8} + \cos \frac{7\pi}{8} = 0$.
3. Prove that $(a + b) \sin \frac{c}{2} = c \cos \frac{A-B}{2}$.
4. Differentiate $\cos x$ by the method of first principles.
5. Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3axy$.
6. Find the equation to the tangent and normal to the curve $x^2 + y^2 = 25$ at (3,-4).
7. Prove that $\sin 120^\circ \cos 330^\circ + \cos 240^\circ \sin 330^\circ = 1$.

(5×6 = 30)

PART — C

(Maximum marks : 60)

(Answer one full question from each unit. Each full question carries 15 marks.)

UNIT — I

- III (a) Prove that $\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = 2 \sec\theta$. 5
- (b) If $\sin A = \frac{3}{5}$ and A is acute, find $\sin 2A$ and $\cos 2A$. 5
- (c) Show that $\tan 75^\circ + \cot 75^\circ = 4$. 5

OR

- IV (a) Prove that $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2 \sec^2 A$. 5
- (b) If $\sin A = \frac{8}{17}$, $\sin B = \frac{3}{5}$; A, B are acute, find $\sin (A-B)$ and $\cos (A+B)$. 5
- (c) From the top of a light house 90m high, the angles of depression of two boats on the sea level are 45° and 60° . Find the distance between the boats. 5

UNIT — II

- V (a) Prove that $\frac{\sin 2A + \sin 5A - \sin A}{\cos 2A + \cos 5A + \cos A} = \tan 2A$. 5
- (b) Prove that $\cos 55^\circ + \cos 65^\circ + \cos 175^\circ = 0$. 5
- (c) Solve $\triangle ABC$, given $a = 5\text{cm}$, $b = 8\text{cm}$ and $C = 30^\circ$. 5

OR

- VI (a) Prove that $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 0$. 5
- (b) Prove that $\cos 3A + \cos 5A + \cos 9A + \cos 17A = 4 \cos 4A \cos 6A \cos 7A$. 5
- (c) Solve $\triangle ABC$, given $a = 2\text{cm}$, $b = 3\text{cm}$ and $c = 4\text{cm}$. 5

UNIT — III

- VII (a) Evaluate (i) $\lim_{x \rightarrow 0} \frac{\sin 2x \cdot \cos x}{x}$ (ii) $\lim_{x \rightarrow \infty} \frac{2x^2 + x + 1}{x^2 - 2x + 1}$ (3 + 3 = 6)
- (b) If $x = a \cos^3 \theta$, $y = b \sin^3 \theta$, find $\frac{dy}{dx}$. 4
- (c) If $y = a \sin mx$, Prove that $\frac{d^2y}{dx^2} + m^2y = 0$. 5

OR

- VIII (a) Evaluate (i) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$ (ii) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$ (4 + 2 = 6)
- (b) If $y = \log (\sec x - \tan x)$, show that $\frac{dy}{dx} = -\sec x$. 4
- (c) If $y = A \sin x + B \cos x$ (A, B are constants), Show that $\frac{d^2y}{dx^2} + y = 0$ 5

UNIT — IV

- IX (a) Find the equations to the tangent and normal to the curve $y = 3x^2 + x + 2$ at $(1, 2)$. 5
- (b) The radius of a circular plate is increasing in length at 0.1 cm/sec when heated. What is the rate at which the area is increasing when the radius is 12 cm ? 5
- (c) An open box is to be made out of a square sheet of side 18 cm by cutting off equal squares at each corner and turning up the sides. What size of the squares should be cut in order that the volume of the box may be maximum ? 5

OR

- X (a) Find the velocity and acceleration of a particle at $t = 4$ seconds whose displacement is given by $S = \frac{1}{2} t^2 + \sqrt{t}$. 5
- (b) A circular patch of oil spreads out on water, the area growing at the rate of 6cm^2 per minute. How fast is the radius increasing when the radius is 2 cms ? 5
- (c) Find the maximum value of $2x^3 - 9x^2 + 12x + 5$. 5
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