- The state of the system is given by

$$|\psi(t)\rangle = \sum_{n,k} \beta_{n-k,k}(t)|n-k\rangle_p|k\rangle_s|k\rangle_i$$

- In this case, the expectation value is given by

$$\langle\psi(t)|(a_p^\dagger)^{\pi_2}a_p^{\pi_1}(a_s^\dagger)^{\sigma_2}a_s^{\sigma_1}(a_i^\dagger)^{\lambda_2}a_i^{\lambda_1}|\psi(t)\rangle = \sum_{n,n'=0}^{\infty}\sum_{k=0}^{n}\sum_{k'=0}^{n'} \beta^*_{n'-k',k'}(t)\ \beta_{n-k,k}(t)\ \langle n'-k'|(a_p^\dagger)^{\pi_2}a_p^{\pi_1}|n-k\rangle$$
$$\langle k'|(a_s^\dagger)^{\sigma_2}a_s^{\sigma_1}|k\rangle\ \langle k'|(a_s^\dagger)^{\lambda_2}a_s^{\lambda_1}|k\rangle$$

- We know that

$$\langle m_2|(a^\dagger)^{\lambda_2}a^{\lambda_1}|m_1\rangle = \sqrt{\frac{m_1!}{(m_1-\lambda_1)!}}\sqrt{\frac{(m_2)!}{(m_2-\lambda_1)!}}\delta_{m_2-\lambda_2,m_1-\lambda_1} = \sqrt{\frac{m_1!}{(m_1-\lambda_1)!}}\sqrt{\frac{(m_1+\lambda_2-\lambda_1)!}{(m_1-\lambda_1)!}}\delta_{m_2,m_1+\lambda_2-\lambda_1}$$

- Using the above relation, we obtain

$$\langle\psi(t)|(a_p^\dagger)^{\pi_2}a_p^{\pi_1}(a_s^\dagger)^{\sigma_2}a_s^{\sigma_1}(a_i^\dagger)^{\lambda_2}a_i^{\lambda_1}|\psi(t)\rangle = \sum_{n=0}^{\infty}\sum_{k=0}^{n} \beta^*_{n-k+\Delta\pi,k+\Delta\sigma}(t)\ \beta_{n-k,k}(t)\ \frac{\sqrt{(n-k+\Delta\pi)!(n-k)!}}{(n-k-\pi_1)!}$$
$$\frac{\sqrt{(k+\Delta\sigma)!k!}}{(k-\sigma_1)!}\,\frac{\sqrt{(k+\Delta\lambda)!k!}}{(k-\lambda_1)!}\delta_{\Delta\lambda,\Delta\sigma}$$

where $\Delta\pi = \pi_2-\pi_1$, $\Delta\lambda = \lambda_2-\lambda_1$ and $\Delta\sigma = \sigma_2-\sigma_1$