

Butenko, Commander, and Pardalos have a reduction which we recap in this paper. They consider the following problem:

BROADCAST SCHEDULING PROBLEM

INSTANCE: A undirected graph $G = (V, E)$ and an integer K .

QUESTION: Does there exist a broadcast schedule with frame length $\leq K$?

Theorem 1. BROADCAST SCHEDULING PROBLEM is \mathcal{NP} -complete.

Proof. Recall that the k -coloring problem is, given $G = (V, E)$ and an integer k , does there exist a proper coloring of vertices of G that uses $\leq k$ colors?

Given a graph $G = (V, E)$, we will construct the corresponding graph $G' = (V', E')$, where $V' = V \cup E$ and $E' = \{[i, (i, j)] : (i, j) \in E, i, j \in V\} \cup \{(e_1, e_2) : e_1, e_2 \in E\}$. By the construction of graph G' , $(v_1, v_2 \in E)$ if and only if $v_1, v_2 \in V$ are 2-hop neighbors in G' . Also, $V' \setminus V$ forms a clique in G' , and any vertex in this clique is a 2-hop neighbor of any vertex in V . Thus no other vertex can transmit in the same time slot with a vertex from the clique, so any broadcast schedule in G' will require m time slots just for vertices from the clique to transmit.

The remaining vertices in V' (*i.e.*, vertices from V) can transmit according to any proper coloring in G , where different time slots in the resulting broadcast schedule will correspond to different colors in the coloring. \square