A Lazy Man's Approach to GNSS Ambiguity Resolution and Repair

Aaron Hammond

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Abstract

Precision GNSS positioning relies on the accurate resolution of integer ambiguities and biases within signal phase measurements.

With a network of receivers, a continuous chain of pivot recievers and satellites can be formed to form relationships between stations to eliminate rank deficiency in the solution.

When multiple frequencies are present, certain linear combinations of signals may be calculated that reparameterise the problem such that a subset of the parameters may be easily estimated with high certainty, while the integer-valued parameters that provide the higher solution precision are known to be well determined by the so called LAMBDA algorithm.

After initial estimation these integer ambiguities are subject to contamination by cycle-slips in the receiver hardware's tracking loops.

We show that by using the most general form of a robust kalman filter, these pivots, ambiguities, and biases can be resolved, repaired, and broadcast, without regard for the specifics of the GNSS application - instead relying on the power of the covariance matrix for book-keeping between parameters.

1 Introduction

GNSS measurements are well described by the equations below:

$$p^s_{r,f} = \rho^s_r + \xi^s_{r,f} + c(d_{r,f} - d^s_f) + c(dt_r - dt^s + \delta t^{rel}) + I^s_{r,f} + T^s_r + e^s_{r,f} \tag{1}$$

$$\varphi^s_{r,f} = \rho^s_r + \zeta^s_{r,f} + c(\delta_{r,f} - \delta^s_f) + c(dt_r - dt^s + \delta t^{rel}) - I^s_{r,f} + T^s_r + \lambda_f(\omega^s_r + N^s_{r,f}) + \epsilon^s_{r,f} \tag{2}$$

where:

p, φ = pseudorange and carrier phase measurements
ρ = physical range between receiver and satellite
ξ, ζ = code and phase offsets (PCV and PCO)
c = speed of light
d, δ = code and phases biases
dt = clock offsets
δt^{rel} = relativity correction
I, T = slant ionospheric and tropospheric delays
λ = signal wavelength
ω = phase wind-up
N = phase ambiguity
e, ϵ = pseudorange and carrier phase measurement errors
$_r, _f, ^s$ = receiver, frequency and satellite dependent parameters

For accurate user-mode positioning, precise knowledge of satellite clocks and phase biases is required to establish accurate OMC values for use in the user's filter. Such parameters, which are unobservable by measurements from a single isolated receiver, allow the user to perform integer ambiguity resolution with minimal delay, and estimate position with high precision - without waiting for the slow internal convergence of parameters.

Much effort has been placed in determining algorithms to assist in estimating these parameters - including creating pivot chains, special purpose linear combinations, and cycle-slip isolation and repair - but a hands-off approach may yield equivalent or better results by using a more general filter architechture.

2 Biases vs Clocks - The Fractional and Differential Nature of Biases

From equations [1],[2], the δ_f^s and dt^s terms are seen to be unobservable - for any set of measurements there remains a rank deficiency which must be addressed.

For solutions based on least-squares solutions, this indeterminacy may pose a problem, but utilising a kalman filter with apriori variances eliminates the rank deficiency in the first instance. However, the apriori variance is required to be loose to prevent distortion of the solution, and this can only establish the biases to a relatively large confidence interval.

It is commonly taken that the clock terms dt contain the common-mode component of these parameters, and the bias terms δ are small in nature. (check) However, these biases are highly interrelated, and their variances can not be forced to arbitrarily small values without affecting the estimates of other biases in the network.

The uncertainty in these biases presents an obstacle to efficient user-mode positioning - they need to be established to sub-wavelength precision before ambiguity resolution can be performed.

2.1 About pivots

By judicious application of constraints on select biases, the variances of all other biases and clock estimates can be reduced. A so-called pivot chain can be constructed that carefully constrains some biases to zero, tightening the constraints on other parameters, in a manner that progressively traverses the entire network - without distorting the solution.

In cases where the network is partitioned, it may not be possible to generate a continuous pivot chain, and an ad-hoc selection of a new pivot origin must be established. This presents additional problems - a network that is discontinuous has no method of determining the relationship between satellites biases in different halves of the network. Adding constraints to artificially reduce variances will result in a non-physical description of the system, and any user-mode receiver that straddles the partition will suffer with poor performance.

Some work has suggested methods of creating pivot chain based on covariance matrix[], but as we will see, this step too, is unnecessary.

2.2 Alternative to Pivots

Rather than generating pivot chains - with the complexity and possible complications that result - we may simply select differential biases as out parameters of interest.

While a loosely constrained solution's biases have uncertainties that are too large for them to be of use, there are large correlations between biases that are stored in the filter's covariance matrix. Reframing the problem to use differences between biases makes explicit what was only implied by the pivot approach - that bias estimates are only valid relative to one another.

The same mechanism that allowed the pivot chain to progressively reduce bias uncertainty is used to calculate these differential bias estimates with high confidence - the propagation of uncertainty.

[compare pivot chain? against simple differential propagation, actual numbers, and variances.]

put a note here that the partitioned network works as expected in this case.

The effort placed in composing chains of so-called pivot biases is misplaced.

Resolvable biases vs resolvable differential biases (observable) - variance of bias is large - variance of differential bias is well defined - (to float ambiguity level)

As these biases are defined to be small, loosely constraining them around a value of 0.5/0? is sufficient to provide rank sufficiency. - probably dont even need this, becuase its still rank deficient and this is just as arbitrary as setting something to zero..... (check with debug lambda, see how far it searches) -¿ all biases can be constrained to some value around zero. -¿ methods of constraint - initialisation only, fogm elasticity

hammering biases to fractional values - propatation of uncertainty or straight hammer-home (with correlated ones)

3 Wide-lanes, Narrow-lanes, and LAMBDA - Outsourcing Linear Combinations

Much literature is based upon the creation of linear combinations of measurements that seeks to illuminate particular components of signals, including wide and narrow lane combinations which contain noise characteristics that are more amenable for use in solving integer ambiguities within the system. The use of these combinations however, while useful from a visualisation and debugging perspective, attempts to impose an unnecessary culture on the filter, and may in fact dispose of useful data that could otherwise improve the filter's performance and outlier rejection capacity.

3.1 About Wide-Lanes

Wide lanes are defined as linear combinations that have an effective wavelength larger than that of the individual measurements. A signal of large wavelength may be ... Put another way, a large wavelength when discussing ambiguity resolution, relative sizes of ambiguity covariances to the measurement variances is reduced, allowing resolution with higher confidence.

Again, at first appearance, simply applying the Jacobian of the measurement equations in [] results in relatively undefined states with extremely large variances, which conceivably would prevent efficient solving of ambiguities. This is the motivation for composing wide-lanes and narrow-lanes - linear combinations betweeen signal measurementss that have much lower variances, that can be quickly searched.

While these wide-lanes may provide meaningful insight into the nature of the signals for the researcher, they are but a particular choice of linear combination of signals, and choosing the wrong linear combination may destroy data, and miss further insight that might have been gained.

3.2 Alternative to Wide-Lanes

When individual measurements are provided to the filter in an uncombined representation, no single viewpoint is forced on the estimator. By appropriate propagation of uncertainty of the states in an undifferenced-uncombined solution, it can be readily demonstrated that the data of any pseudo signals that might have been formed by selective linear combinations of measurements are all contained within the covariance matrix of the filter.

[section on Wide lanes reproduced from UDUC P matrix]

When resolving integer ambiguities using the LAMBDA method, the first step is the decorrelation adjustment - unskewing the covariance matrix to create an elliptical search space containing the ambiguities that is largely oriented to the new parameters' axes. This is, in effect, a reparameterisation of the states, and these reparameterised states can be inspected during the operation of the algorithm.

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-4 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -4 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 5 \\
0 & 0 & -4 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -4 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 5 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{bmatrix} : \begin{bmatrix}
1040.83 \\ 1040.83 \\ 1040.83 \\ 1040.83 \\ 1040.83 \\ 1040.83 \\ 1040.83 \\ 1040.83 \\ 1040.83 \\ 1.70702 \\ 1.70702 \\ 1.70702 \\ 1.70702 \\ 1.70702 \\ 1.70702 \\ 1.70702 \\ 1.70702 \\ 1.70702
\end{bmatrix} \tag{3}$$

The matrix above is a permutation matrix and corresponding variances after the covariance matrix from a single epoch's filtered uncombined measurements have been provided to LAMBDA.

It is clear that the LAMBDA algorithm has determined without intervention that combinations exist with relatively low variances, (the $\begin{bmatrix}1 & -1\end{bmatrix}$ pairs), corresponding with the wide-lane combination typically generated manually.

Note that the wide-lane ambiguities, while easy to solve manually, will also be automatically solved by LAMBDA. Due to the low variance these are prioritised internally, and are unlikely to add significant processing burden.

thats easy to solve - the wide lane, gets solved easily in lambda anyway, but has the potential to find it if its a little ways away, as opposed to integer rounding. - we dont gain anything by doing two chunks.

The underlying mathematics is ambivalent to the effort placed in conjuring combinations that appear mofe informative to the researcher.

4 Reset vs repair - A Distinction Without a Difference

Much effort has been placed in the attribution and repair of phase observations and their ambiguities after cycle slip events. An uncorrected cycle-slip will contaminate state elements that are contained in phase measurements, including the biases and clocks fundamental to accurate user-mode PPP.

When the system is well-modelled, a statistical test on phase measurements such as (4 sigma) is sufficient to isolate spurious signals from statistically reasonable measurements. To prevent contamination of the filter, any detection of an outlying value can be addressed by de-weighting the measurement - by increasing its variance, or discarding it entirely.

A second (or sequence) of these spurious measurements however, indicates a persistent mis-modelling of the parameters in that signal. Process noise may absorb some errors in stochastic parameters, however a large discontinuity in a parameter with low variance and low process noise - such as a cycle slip and the resultant incorrect ambiguity term - must be corrected by repairing the parameter.

4.1 About Repair

If the nature of spurious measurements can be determined - eg. a cycle-slip of 5 wavelengths on L1 frequency - future measurements can be repaired to maintain consistency with the state estimates. Alternatively, the filter state may be amended to capture the change in ambiguity term.

To achieve this end, accurate determination of the location and size of the slip is required. This is often achieved by implementing time-series analysis of signals, employing linear combinations of measurements that seek to isolate the phase ambiguity term, while rejecting the other components of measurements. (Melbourne-Wubenna?)

To repair a signal accurately however, the measurement noise and filter covariances must be sufficiently low to precisely attribute the source of the mismodelling.

4.2 Alternative to Repair

As we have seen, while linear combinations may make a useful diagnostic tool for the researcher, these linear combinations are all embedded within an undifferenced-uncombined filter's covariance matrix - a statistcal test that evaluates measurements against expected values using the filter's covariance matrix will automatically encompass the Melbourne Wubenna combinations that were desired.

Furthermore, rather than performing statistical tests solely on the researcher's select choice of linear combinations, this more general approach tests all possible linear combinations as modelled by the system's jacobian and covariance matrix, allowing far greater capacity for outlier rejection.

When a cycle-slip occurs in a well-modelled system, we might be tempted to back-calculate the error and make corrections to the state. However, if we have provided all measurements to the filter in an uncombined form, it too has the data required to make these corrections.

By simply temporarily unconstraining and reducing the confidence of the ambiguity parameter - by reinitialising, or temporarily assigning to it a large process noise - the health of the filter is rectified. The data in the covariance matrix acts to reconstrain the ambuiguity term with subsequent measurements.

As most parameters retained their confidence levels during the reinitialisation, their low uncertainties are combined and propagated according to the measurement equations to establish the best estimate of the unconstrained ambiguity parameter.

This method of repair takes into account much more data than would be obtained using a simple linear combination, and incorporates further knowledge of the system, such as process noise in bias parameters, the measurement noise, and correlations between parameters that may not be obvious to the researcher a-priori. This general approach not only reconstrains the ambiguity to the best possible estimate, but retains a meaningful estimate of its uncertainty (this is destroyed with a naive apprach to repair).

[Get an analytical covariance matrix of many observations and show that adding a new ambiguity gets smashed to zero when added. (despite the biases being ill-constrained in terms of absolute variance)]

[why we cant do better]

5 TODO

initialisation - 0.5

constraining to zero long term state transition - tie biases to 0.5 cycles with elasticity
what happens if you dont constrain after initialisation?
parameters of interest - satellite biases are of use to end user, receiver biases not so much
biases as differential pairs - vs biases as differential to pseudo satellite with bias of zero.
review of pivots - analysis as a special case of general case.
reason to keep biases around zero is to ensure linearity?

6 Proposal and Results

The concepts described were implemented as the basis of the PPP processor in Geoscience Australia's Ginarn ACS toolkit.

1. No linear combinations.
2. No pivot chain.
3. (Biases constrained to fractions?)
4. Report biases as differential pairs
5. Statistical rejection of measurements

blah, blah, images and tables.

References

Figure 1. Figure caption. To get a figure to span two columns, use the environment figure* rather than figure.