Elimination of Inertial Sensor Biases by Mechanical Modulation

Aaron Hammond

May 4th, 2021

Abstract

Inertial navigation systems utilise accelerometers and gyroscopes for dead-reckoning when external position inputs are unavailable. However, a long-standing problem for these devices is bias drift - an error which accumulates upon integration within navigation filters, and which typically can not be measured without external odometery.

By physically modulating sensors in the system, additional data may be extracted to resolve the unknown biases, allowing us to eliminate the majority of measurement errors to achieve far greater precision when dead-reckoning.

1 Introduction

Inertial navigation systems typically use a 6 degree of freedom MEMS sensor to measure changes in attitude and velocity, which can be integrated over time to estimate a device's position. While these devices may have relatively low noise, their measurements have inherent biases which appear as an offset in the measurement values from the devices actual motion.

The system of inertial sensors is rank deficient, with each set of measurement consisting of 6 measurements, but 12 unknowns - 6 due to accelerations/rotations, and 6 more as an unknown bias in each axis of measurement.

While these biases can be estimated precisely when another form of external odometry is present - GNSS measurements measuring actual accelerations for example, or zeroing gyroscopes while the system is at rest - these place restrictions on the environment of the device under test.

A naive approach to solving the bias problem may be to add additional inertial sensors, attempting to add more measurements to account for the deficiency, however these additional sensors also contain unknown biases that prevent the variance of estimates from converging.

These sensors can however extract additional data from the system, provided it is designed appropriately.

The biases in inertial MEMS sensors are typically slow-moving, perhaps with a strong temperature dependence, and this characteristic of the sensors make it possible to draw out the data we need to resolve internal biases.

By borrowing modulation techniques from electronics to use in our mechanical system, we can measure inherent biases, rather than merely calibrating or estimating them, for improved inertial performance.

2 Basic Concept - Two Sensors with Relative Motion

Consider the one-dimensional case, using two sensors, with all measurements and motion in the x direction.

At some initial time, both accelerometers are oriented in the same direction. They measure the same acceleration vector, but report different results due to the biases of each sensor.

$$z_a(t_0) = +\ddot{x}(t_0) + \delta^a \tag{1}$$

$$z_b(t_0) = +\ddot{x}(t_0) + \delta^b \tag{2}$$

From our two measurements we have three unknowns, and can not resolve the acceleration, nor the biases. Subsequent measurements also share this rank deficiency, as each measurement carries with it an additional unknown due to the acceleration - this is necessarily unknown as it is the parameter of interest.

Before the next measurement one sensor is rotated 180 degrees to the other, and results in one accelerometer measuring the opposite acceleration vector. However, if the delay between measurements is sufficiently small, the slow-moving biases included in these measurements are the same as the first measurement, allowing us to extract additional data.

$$z_a(t_1) = +\ddot{x}(t_1) + \delta_a \tag{3}$$
$$z_b(t_1) = -\ddot{x}(t_1) + \delta_b \tag{4}$$

This second measurement introduces a further two measurements, while only adding one additional unknown - the acceleration in the second epoch.

By simple substitution of terms, this system of equations can be solved for both the biases and the unbiased accelerations measured by the accelerometers.

$$\ddot{x}(t_0) = \frac{1}{2}\Big(+ z_a(t_0) + z_b(t_0) - z_a(t_1) - z_b(t_1)\Big) \tag{5}$$
$$\ddot{x}(t_1) = \frac{1}{2}\Big(- z_a(t_0) + z_b(t_0) + z_a(t_1) - z_b(t_1)\Big) \tag{6}$$
$$\delta_a = \frac{1}{2}\Big(+ z_a(t_0) - z_b(t_0) + z_a(t_1) + z_b(t_1)\Big) \tag{7}$$
$$\delta_b = \frac{1}{2}\Big(- z_a(t_0) + z_b(t_0) + z_a(t_1) + z_b(t_1)\Big) \tag{8}$$

3 Generalisation

The procedure above presents the most intuitive case, but the requirement for a full 180 degree rotation, and assumption of zero change in the biases can be relaxed in more practical implementations. It can also be extended to three dimensions in a straightforward manner.

By preparing the system of equations as follows, with a design matrix incorportating a rotation transformation between the frame of the moving sensor to that of the stator, arbitrary rotations of the secondary sensor may be commanded, and measurements incorporated into a standard navigational kalman filter.

$$\mathbf{z}(t) = R(t)\ddot{\mathbf{x}}(t) + \delta + \epsilon \tag{9}$$

$$\begin{bmatrix} \mathbf{z_1} \\ \mathbf{z_2} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{I} & 0 \\ \mathbf{R} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}} \\ \delta_\mathbf{1} \\ \delta_\mathbf{2} \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} z_{x_a} \\ z_{y_a} \\ z_{z_a} \\ z_{x_b} \\ z_{y_b} \\ z_{z_b} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ cos(\theta) & -sin(\theta) & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ sin(\theta) & +cos(\theta) & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{x}(t) \\ \ddot{y}(t) \\ \ddot{z}(t) \\ \delta_{x_a} \\ \delta_{y_a} \\ \delta_{z_a} \\ \delta_{x_b} \\ \delta_{y_b} \\ \delta_{z_b} \end{bmatrix} \tag{11}$$

When all state elements are modelled with process noise, the variance of the bias states (and accelerations) will tend to increase over time. If the standard stationary MEMS configuration is used the uncertainty in both will be limited only by the bias characteristics of the sensors, but modulating the rotations included in the R term allow sets of measurements to be created that are not rank deficient, and which constrain the biases over time.

In order to maintain and utilise the rank sufficiency, the two sensors must be actuated (or allowed to move) such that they have a motion of a higher frequency than the bandwidth of their biases. In practice, a rotation or osciallation of as low as 1 RPM is sufficient to keep up with transients in the biases in most MEMS sensors.

For the three dimensional case, rotating the auxiliary sensor around the $\langle 1, 1, 1 \rangle$ vector may provide the best results as measurements from all three axes are combined in the final solution.

4 Extension and Visualisation

In the previous two sections, it has been somewhat explicit that the rank deficiency of the system can be eliminated by rotating two sensors relative to one another. However, by utilising much the same concepts, the same can be accomplished using a single sensor - provided that the dynamics are suitably modelled and it is still able to be modulated.

Consider a stationary observer who uses a biased sensor to measure an unknown gravitational field - the value he records will be offset due to the bias, and reads sligtly higher than the correct value. He then upturns the sensor, so that he is measuring the gravitational field in the negative direction - this time the bias still offsets the measurement to the positive side (less negative), and has the effect of causing the absolute value of the measurement to be slightly lower than the correct value. By taking the average of the absolute value of the two measurements, the bias cancels, and an accurate measurement of the field is obtained.

The situation described above does require the observer to be stationary, as it must be guaranteed to measure the same acceleration in both occasions for this simple technique to work. However, if the dynamics of the system are well modelled, it may be sufficient for the device to be 'mostly stationary.'

By modulating the sensor's orientation at a rate higher than any expected dynamics of the system, subsequent measurements may be viewed as measuring *almost* the same acceleration, albeit from different orientations. Selection of modulation rate and Kalman filter tuning will play a large part in the effectiveness of this approach.

4.1 Active vs Passive Modulation

Commonly with modulation, a signal is applied to the system such that subsequent measurements of the system can be correlated with the applied signal to extract low noise data. The application of this modulation signal however, need not be specified by the device, but merely needs to be known so as to enable the correlation.

In the case of dead-reckoning with inertial sensors, reporting their motion is a fundamental aspect of the sensor, and thus if one allows the sensor to move somewhat arbitrarily with respect to the body of the device under test (eg. attached to a spring), the reported motion can be used to determine the orientation and effective modulation signal. This is sufficient to resolve the position of the device, orientation of the sensor, and the internal biases - although somewhat recursively.

4.2 Conclusion

While the problem of unknown MEMS sensor biases may be resolved with restricted motion - by zeroing device while stationary - this is by no means necessary. By allowing and encouraging motion of the sensors, we effectivly get a better picture of the device by looking at it from different points of view. The techniques described apply equally to accelerometers and gyroscopes, and while in motion, stationary, or free-fall, and provide a method for obtaining extremely precise measurements while dead-reckoning, albeit with the expense of added mechanical complexity.