AE4070 Quiz 1 Writeup

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1 Problem Statement

Design a rotor to provide a thrust of 15000 lbf at a pressure altitude of 4000 ft and an ambient temperature of 95 deg F. The rotor diameter is set at 43 ft, the thrust-weighted solidity σ is set at 0.1, and N_b is set at 3. The tip speed V_{tip} is set at 650 ft/s. A NACA 0012 airfoil is to be used. Determine the linear taper ratio T_r and linear twist rate θ_{tw} that maximizes figure of merit.

2 Solution Description

To solve the problem, a Matlab program was constructed to analyze the performance of the rotor at various taper ratios and twist rates. The Matlab program implements the Blade Element Momentum Theory (BEMT), and uses Prandtl's tip loss model to compensate for tip losses in the results. The basic algorithm implemented by the program is described below.

In the BEMT, each rotor blade is discretized into finite strips, along the radial direction of the blade. At each strip, local properties are computed, including radial position, local solidity, local twist angle, and local mach number. These values are computed using simple geometric relationships. These properties are used to then compute the local inflow velocity ratio λ_i using Equations 1, 2, and 3. The expression for λ_i used in this program accounts for tip losses using Prandtl's tip loss function F. Once the local inflow velocity is known, the effective angle of attack α_i can be computed, and used in conjunction with the local mach number to determine the sectional lift and drag coefficients for the strip. Then, the differential power and thrust coefficients can be computed using Equations 4 and 5.

There are a few things that should be noted about the implementation of the model described above. First, in order to determine the collective pitch angle of the rotor, the coefficient of thrust must be known. However, the collective pitch angle is needed to perform the C_t and C_p calculations. Thus, an iterative procedure is used. The value of C_t, which is known based on the thrust, density, and tip speed, is used to guess a value of collective pitch (actually, 75% radius pitch in this case), using the simple estimate in Equation 6. This estimated value is used in the computation of C_t. The difference between the actual C_t

and this computed one is driven to zero via an iterative procedure. Second, the Prandtl tip loss function F and the local inflow velocity ratio λ_i are mutually-dependent. To break this dependency, λ_i is first computed with F equal to 1, then F is computed, and the process is iterated to a fix-point.

The Matlab program was used to compare a range of values for T_r and θ_{tw}. A test matrix was constructed, with T_r varying from 0.7 to 1.0, and θ_{tw} varying from 0 deg/radius to -30 deg/radius. These limits were chosen given considerations of blade structural issues at low values of taper ratio and tip stall at high values of twist rate. The latter criterion in particular was guided by Figure 3.13 in Leishman. Twenty test points were used along each axis, resulting in a total of 400 test points. The Matlab code was run for each test point to compute $C_{p,meas}$ and Equation 7 was used to compute the Figure of Merit (FOM).

$$\lambda_i = \frac{\sigma C_{l_\alpha}}{16F}\left(\sqrt{1+\frac{32F}{\sigma C_{l_\alpha}}}-1\right) \tag{1}$$

$$F = \left(\frac{2}{\pi}\right)\arccos\exp -f \tag{2}$$

$$f = \frac{N_b}{2}\left(\frac{1-r}{r\phi}\right) \tag{3}$$

$$dC_p = \frac{\sigma_i}{2}r^3(c_d\cos\alpha_i + c_l\sin\alpha_i)dr \tag{4}$$

$$dC_t = \frac{\sigma_i}{2}r^3(c_l\cos\alpha_i - c_d\sin\alpha_i)dr \tag{5}$$

$$\theta_{0.75,guess} = \frac{6C_t}{\sigma c_{l_\alpha}} + \frac{3}{2}\sqrt{\frac{C_t}{2}} \tag{6}$$

$$FOM = \frac{C_t^{\frac{3}{2}}}{\sqrt{2}C_{p,meas}} \tag{7}$$

3 Solution Validation

Since the Matlab program was non-trivial, a validation of its accuracy was desirable. In order to validate the program, Figure 3.18 from the Leishman text was reproduced. The coefficient of thrust and number of blades were set to the values given in the text, though the drag polar, which was not provided in the text, was obtained from the NACA 0012 data used for the rotor design. This particular figure was chosen as a test case because the computation of the thrust differential tests most of the complex parts of the code, including the iteration that determines λ_i and the iteration that causes computed C_t to converge to its assumed value. As can be seen in Figure 1, the results of the simulation are accurate. The generated data agrees with the original figure very closely, accurately reproducing the initial rise in thrust gradient resulting from the tip loss model, as well as the drastic drop in thrust gradient after the 95% radius point. The drop-off happens slightly earlier in the figure in the text, but the small difference could easily be attributed to the difference in the drag polar.

4 Data Analysis

The figures of merit computed in the test were used to plot a surface, with T_r and θ_{tw} as the X and Y axes, and FOM as the Z axis. This surface is shown in Figure 2. As can be seen, FOM seems to increase linearly with decreasing taper ratio, while it responds quadratically to linear twist rate. The peak in this surface lies at an FOM of 0.883, which occurs at $T_r = 0.7$ and $\theta_{tw} =$ -23.68 deg/radius. These values of taper ratio and twist rate are entirely reasonable, with the taper ratio lying at a structurally feasible point and the twist rate lying well under the stall points shown in Figure 3.13 in Leishman. The computed FOM seems quite high, however, by about a factor of 10%. However, the Figure of Merit of the rotor using the simple estimation and $\kappa = 1.07$ is 0.832, so our computed value seems to be in the correct neighborhood. Moreover, the solution validation presented above suggests the computed value to be accurate as well, though it is obvious that in a real helicopter, losses would exist that would prevent the achievement of such a high figure of merit.

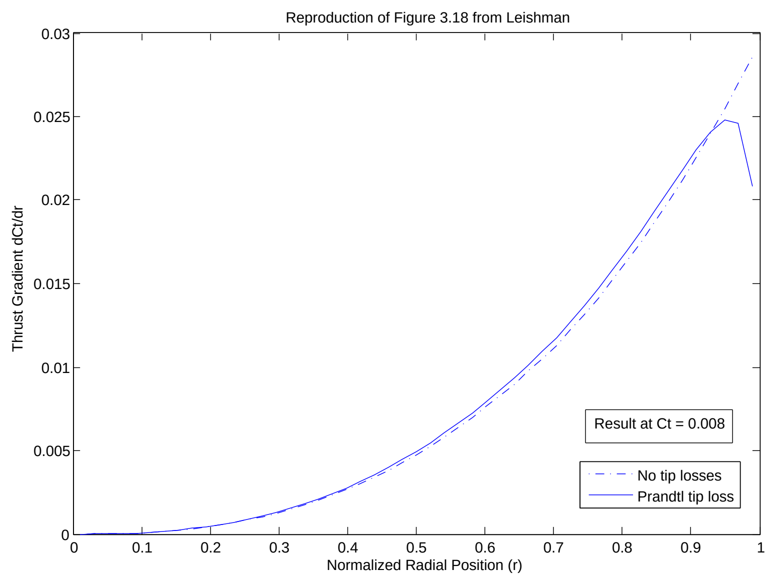

Figure 1: Spanwise variation of blade thrust using tip loss model.

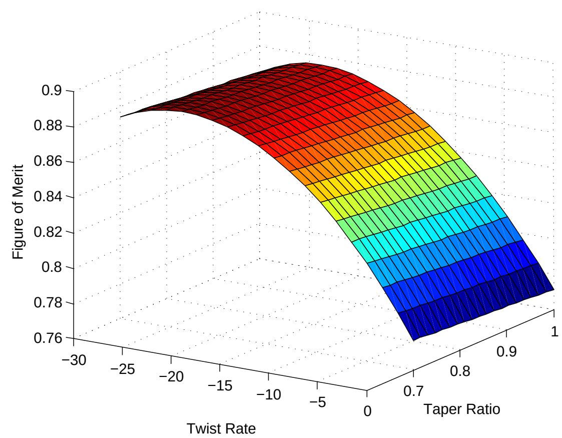

Figure 2: Variation in Figure of Merit with T_r and θ_{tw}.