SfePy Documentation

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Contents

1 Notation

Ω	volume (sub)domain
Γ	surface (sub)domain
t	time

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y	any function
$\underline{y}$	any vector function
$\underline{n}$	unit outward normal
q, s	scalar test function
p, r	scalar unknown or parameter function
$\bar{p}$	scalar parameter function
$\underline{v}$	vector test function
$\underline{w}$, $\underline{u}$	vector unknown or parameter function
$\underline{b}$	vector parameter function
$\underline{\underline{e}}(\underline{u})$	Cauchy strain tensor ($\frac{1}{2}((\nabla u) + (\nabla u)^T)$)
$\underline{f}$	vector volume forces
f	scalar volume force (source)
ρ	density
ν	kinematic viscosity
c	any constant
δ_{ij}, $\underline{\underline{I}}$	Kronecker delta, identity matrix

The suffix "$_0$" denotes a quatity related to a previous time step.
Term names are prefixed according to the following conventions:

dw	discrete weak	terms having a virtual (test) argument and zero or more unknown arguments, used for FE assembling
d	discrete	terms having all arguments known, the result is the scalar value of the integral
di	discrete integrated	like 'd' but the result is not a scalar (e.g. a vector)
dq	discrete quadrature	terms having all arguments known, the result are the values in quadrature points of elements
de	discrete element	terms having all arguments known, the result is a vector of integral averages over elements (element average of 'dq')

2 List of all terms

section	name	definition
(12.1)	d_biot_div	$\int_\Omega r\ \alpha_{ij} e_{ij}(\underline{w})$
(6.1)	d_diffusion	$\int_\Omega K_{ij} \nabla_i \bar{p} \nabla_j r$
(7.1)	d_div	$\int_\Omega \bar{p}\ \nabla \cdot \underline{w}$
(11.1)	d_lin_elastic	$\int_\Omega D_{ijkl}\ e_{ij}(\underline{b}) e_{kl}(\underline{w})$
(5.1)	d_surface_dot	$\int_\Gamma pr$, $\int_\Gamma \underline{u} \cdot \underline{w}$
(5.2)	d_surface_integrate	$\int_\Gamma y$, for vectors: $\int_\Gamma \underline{y} \cdot \underline{n}$
(5.3)	d_volume	$\int_\Omega 1$
(5.4)	d_volume_dot	$\int_\Omega pr$, $\int_\Omega \underline{u} \cdot \underline{w}$
(5.5)	d_volume_integrate	$\int_\Omega y$
(5.6)	d_volume_wdot	$\int_\Omega ypr$, $\int_\Omega y\underline{u} \cdot \underline{w}$
(11.2)	de_cauchy_strain	vector of $\forall K \in \mathcal{T}_h$: $\int_{T_K} \underline{\underline{e}}(\underline{w}) / \int_{T_K} 1$
(11.3)	de_cauchy_stress	vector of $\forall K \in \mathcal{T}_h$: $\int_{T_K} D_{ijkl} e_k l(\underline{w}) / \int_{T_K} 1$
(6.2)	de_diffusion_velocity	vector of $\forall K \in \mathcal{T}_h$: $\int_{T_K} K_{ij} \nabla_j r / \int_{T_K} 1$
(5.7)	de_volume_average_mat	$\forall K \in \mathcal{T}_h : \int_{T_K} m / \int_{T_K} 1$
(5.8)	di_volume_integrate_mat	$\int_\Omega m$
(7.2)	dq_grad	$(\nabla p)\vert_{qp}$
(7.3)	dq_lin_convect	$((\underline{b} \cdot \nabla)\underline{u})\vert_{qp}$
(12.2)	dw_biot_div	$\int_\Omega q\ \alpha_{ij} e_{ij}(\underline{u})$
(12.3)	dw_biot_div_dt	$\int_\Omega q\ \alpha_{ij} \frac{e_{ij}(\underline{u}) - e_{ij}(\underline{u}_0)}{\Delta t}$
(12.4)	dw_biot_div_th	$\int_\Omega \left[\int_0^t \alpha_{ij}(t-\tau) \frac{\mathrm{d} e_{kl}(\underline{u}(\tau))}{\mathrm{d}\tau}\ \mathrm{d}\tau\right] q$
(12.5)	dw_biot_grad	$\int_\Omega p\ \alpha_{ij} e_{ij}(\underline{v})$
(12.6)	dw_biot_grad_dt	$\int_\Omega \frac{p - p_0}{\Delta t}\ \alpha_{ij} e_{ij}(\underline{v})$
(12.7)	dw_biot_grad_th	$\int_\Omega \left[\int_0^t \alpha_{ij}(t-\tau)\, p(\tau))\ \mathrm{d}\tau\right]\ e_{ij}(\underline{v})$
(7.4)	dw_convect	$\int_\Omega ((\underline{u} \cdot \nabla)\underline{u}) \cdot \underline{v}$
(6.3)	dw_diffusion	$\int_\Omega K_{ij} \nabla_i q \nabla_j p$
(7.5)	dw_div	$\int_\Omega q\ \nabla \cdot \underline{u}$
(7.6)	dw_div_grad	$\int_\Omega \nu\ \nabla \underline{v} : \nabla \underline{u}$
(7.7)	dw_grad	$\int_\Omega p\ \nabla \cdot \underline{v}$
(7.8)	dw_grad_dt	$\int_\Omega \frac{p - p_0}{\Delta t} \nabla \cdot \underline{v}$
(6.4)	dw_laplace	$c \int_\Omega \nabla s \cdot \nabla r$ or $\sum_{K \in \mathcal{T}_h} \int_{T_K} c_K\ \nabla s \cdot \nabla r$

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(7.9)	dw_lin_convect	$\int_{\Omega} ((\underline{b} \cdot \nabla) \underline{u}) \cdot \underline{v}$
(11.4)	dw_lin_elastic	$\int_{\Omega} D_{ijkl}\ e_{ij}(\underline{v}) e_{kl}(\underline{u})$
(11.5)	dw_lin_elastic_iso	$\int_{\Omega} D_{ijkl}\ e_{ij}(\underline{v}) e_{kl}(\underline{u})$ with $D_{ijkl} = \mu (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \lambda\ \delta_{ij}\delta_{kl}$
(11.6)	dw_lin_viscous	$\int_{\Omega} D_{ijkl}\ e_{ij}(\underline{v}) \frac{e_{kl}(\underline{u}) - e_{kl}(\underline{u}_0)}{\Delta t}$
(11.7)	dw_lin_viscous_th	$\int_{\Omega} \left[\int_0^t \mathcal{H}_{ijkl}(t-\tau)\, \frac{\mathrm{d} e_{kl}(\underline{u}(\tau))}{\mathrm{d}\tau}\ \mathrm{d}\tau \right] e_{ij}(\underline{v})$
(4.1)	dw_mass	$\int_{\Omega} \rho \underline{v} \cdot \frac{\underline{u} - \underline{u}_0}{\Delta t}$
(4.2)	dw_mass_scalar	$\int_{\Omega} qp$
(4.3)	dw_mass_scalar_fine_coarse	$\int_{\Omega} q_h p_H$
(4.4)	dw_mass_scalar_variable	$\int_{\Omega} cqp$
(4.5)	dw_mass_vector	$\int_{\Omega} \rho\ \underline{v} \cdot \underline{u}$
(6.5)	dw_permeability_r	$\int_{\Omega} K_{ij} \nabla_j q$
(8.1)	dw_point_lspring	$\underline{f}^i = -k \underline{u}^i \quad \forall$ FE node i in region
(7.10)	dw_st_grad_div	$\gamma \int_{\Omega} (\nabla \cdot \underline{u}) \cdot (\nabla \cdot \underline{v})$
(7.11)	dw_st_pspg_c	$\sum_{K \in \mathcal{T}_h} \int_{T_K} \tau_K\ ((\underline{b} \cdot \nabla) \underline{u}) \cdot \nabla q$
(7.12)	dw_st_pspg_p	$\sum_{K \in \mathcal{T}_h} \int_{T_K} \tau_K\ \nabla p \cdot \nabla q$
(7.13)	dw_st_supg_c	$\sum_{K \in \mathcal{T}_h} \int_{T_K} \delta_K\ ((\underline{b} \cdot \nabla) \underline{u}) \cdot ((\underline{b} \cdot \nabla) \underline{v})$
(7.14)	dw_st_supg_p	$\sum_{K \in \mathcal{T}_h} \int_{T_K} \delta_K\ \nabla p \cdot ((\underline{b} \cdot \nabla) \underline{v})$
(10.1)	dw_surface_ltr	$\int_{\Gamma} \underline{v} \cdot \underline{\underline{\sigma}} \cdot \underline{n}$
(5.9)	dw_volume_integrate	$\int_{\Omega} q$
(9.1)	dw_volume_lvf	$\int_{\Omega} \underline{f} \cdot \underline{v}$ or $\int_{\Omega} f q$
(5.10)	dw_volume_wdot	$\int_{\Omega} y q p$, $\int_{\Omega} y \underline{v} \cdot \underline{u}$
(5.11)	dw_volume_wdot_dt	$\int_{\Omega} y q \frac{p - p_0}{\Delta t}$, $\int_{\Omega} y \underline{v} \cdot \frac{\underline{u} - \underline{u}_0}{\Delta t}$
(5.12)	dw_volume_wdot_th	$\int_{\Omega} \left[\int_0^t \mathcal{G}(t-\tau) p(\tau)\ \mathrm{d}\tau \right] q$

3 Introduction

Equations in SfePy are built using terms, which correspond directly to the integral forms of weak formulation of a problem to be solved. As an example, let us consider the Laplace equation:

$$c \Delta t = 0 \text{ in } \Omega, \quad t = \bar{t} \text{ on } \Gamma\ . \tag{1}$$

The weak formulation of (1) is: Find $t \in V$, such that

$$\int_{\Omega} c\ \nabla t : \nabla s = 0, \quad \forall s \in V_0\ . \tag{2}$$

In the syntax used in SfePy input files, this can be written as

```
dw_laplace.i1.Omega( coef, s, t ) = 0 ,
```
(3)

which directly corresponds to the discrete version of (2): Find $\boldsymbol{t} \in V_h$, such that

$$\boldsymbol{s}^T (\int_{\Omega_h} c \, \boldsymbol{G}^T \boldsymbol{G}) \boldsymbol{t} = 0, \quad \forall \boldsymbol{s} \in V_{h0} \; ,$$

where $\nabla u \approx \boldsymbol{G} \boldsymbol{u}$. The integral over the discrete domain Ω_h is approximated by a numerical quadrature, that is named `i1` in our case.

3.1 Term call syntax

In general, the syntax of a term call in SfePy is:

```
<term_name>.<i>.<r>( <arg1>, <arg2>, ... ),
```

where `<i>` denotes an integral name (i.e. a name of numerical quadrature to use) and `<r>` marks a region (domain of the integral). In the following, `<virtual>` corresponds to a test function, `<state>` to a unknown function and `<parameter>` to a known function arguments. We will now describe all the terms available in SfePy to date.

4 Terms in termsMass

4.1 dw_mass

Class: MassTerm
Description: Inertial forces term (constant density).
Definition:

$$\int_\Omega \rho \underline{v} \cdot \frac{\underline{u} - \underline{u}_0}{\Delta t}$$

Arguments:

material.rho	ρ
ts.dt	Δt
parameter	$\underline{u}_0$

Syntax: `dw_mass.<i>.<r>( <ts>, <material>, <virtual>, <state>, <parameter> )`

4.2 dw_mass_scalar

Class: MassScalarTerm
Description: Scalar field mass matrix/rezidual.
Definition:

$$\int_\Omega qp$$

Syntax: `dw_mass_scalar.<i>.<r>( <virtual>, <state> )`

4.3 dw_mass_scalar_fine_coarse

Class: MassScalarFineCoarseTerm
Description: Scalar field mass matrix/rezidual for coarse to fine grid interpolation. Field p_H belong to the coarse grid, test field q_h to the fine grid.
Definition:

$$\int_\Omega q_h p_H$$

Syntax: dw_mass_scalar_fine_coarse.<i>.<r>(<virtual>, <state>, <iemaps>, <pbase>)

4.4 dw_mass_scalar_variable

Class: MassScalarVariableTerm
Description: Scalar field mass matrix/rezidual with coefficient c defined in nodes.
Definition:

$$\int_\Omega cqp$$

Syntax: dw_mass_scalar_variable.<i>.<r>(<material>, <virtual>, <state>)

4.5 dw_mass_vector

Class: MassVectorTerm
Description: Vector field mass matrix/rezidual.
Definition:

$$\int_\Omega \rho \, \underline{v} \cdot \underline{u}$$

Syntax: dw_mass_vector.<i>.<r>(<material>, <virtual>, <state>)

5 Terms in termsBasic

5.1 d_surface_dot

Class: DotProductSurfaceTerm
Description: Surface $L^2(\Gamma)$ dot product for both scalar and vector fields.
Definition:

$$\int_\Gamma pr, \int_\Gamma \underline{u} \cdot \underline{w}$$

Syntax: d_surface_dot.<i>.<r>(<parameter_1>, <parameter_2>)

5.2 d_surface_integrate

Class: IntegrateSurfaceTerm
Definition:

$$\int_\Gamma y, \text{ for vectors: } \int_\Gamma \underline{y} \cdot \underline{n}$$

Syntax: d_surface_integrate.<i>.<r>(<parameter>)

5.3 d_volume

Class: VolumeTerm
Description: Volume of a domain. Uses approximation of the parameter variable.
Definition:

$$\int_\Omega 1$$

Syntax: d_volume.<i>.<r>(<parameter>)

5.4 d_volume_dot

Class: DotProductVolumeTerm
Description: Volume $L^2(\Omega)$ dot product for both scalar and vector fields.
Definition:

$$\int_\Omega pr,\ \int_\Omega \underline{u} \cdot \underline{w}$$

Syntax: `d_volume_dot.<i>.<r>( <parameter_1>, <parameter_2> )`

5.5 d_volume_integrate

Class: IntegrateVolumeTerm
Definition:

$$\int_\Omega y$$

Syntax: `d_volume_integrate.<i>.<r>( <parameter> )`

5.6 d_volume_wdot

Class: WDotProductVolumeTerm
Description: Volume $L^2(\Omega)$ weighted dot product for both scalar and vector fields.
Definition:

$$\int_\Omega ypr,\ \int_\Omega y\underline{u} \cdot \underline{w}$$

Arguments:

material	weight function y

Syntax: `d_volume_wdot.<i>.<r>( <material>, <parameter_1>, <parameter_2> )`

5.7 de_volume_average_mat

Class: AverageVolumeMatTerm
Description: Material parameter m averaged in elements. Uses approximation of y variable.
Definition:

$$\forall K \in \mathcal{T}_h : \int_{T_K} m / \int_{T_K} 1$$

Arguments:

material	m (can have up to two dimensions)
parameter	y
shape	shape of material parameter parameter
mode	'const' or 'vertex' or 'element_avg'

Syntax: `de_volume_average_mat.<i>.<r>( <material>, <parameter>, <shape>, <mode> )`

5.8 di_volume_integrate_mat

Class: IntegrateVolumeMatTerm
Description: Integrate material parameter m over a domain. Uses approximation of y variable.
Definition:

$$\int_\Omega m$$

Arguments:

material	m (can have up to two dimensions)
parameter	y
shape	shape of material parameter parameter
mode	'const' or 'vertex' or 'element_avg'

Syntax: `di_volume_integrate_mat.<i>.<r>( <material>, <parameter>, <shape>, <mode> )`

5.9 dw_volume_integrate

Class: IntegrateVolumeOperatorTerm
Definition:

$$\int_\Omega q$$

Syntax: `dw_volume_integrate.<i>.<r>( <virtual> )`

5.10 dw_volume_wdot

Class: WDotProductVolumeOperatorTerm
Description: Volume $L^2(\Omega)$ weighted dot product operator for scalar and vector (not implemented!) fields.
Definition:

$$\int_\Omega yqp,\ \int_\Omega y\underline{v} \cdot \underline{u}$$

Arguments:

material	weight function y

Syntax: `dw_volume_wdot.<i>.<r>( <material>, <virtual>, <state> )`

5.11 dw_volume_wdot_dt

Class: WDotProductVolumeOperatorDtTerm

Description: Volume $L^2(\Omega)$ weighted dot product operator for scalar and vector (not implemented!) fields.
Definition:

$$\int_\Omega y q \frac{p - p_0}{\Delta t} \;,\; \int_\Omega y \underline{v} \cdot \frac{\underline{u} - \underline{u}_0}{\Delta t}$$

Arguments:

material	weight function y

Syntax: `dw_volume_wdot_dt.<i>.<r>( <ts>, <material>, <virtual>, <state>, <parameter> )`

5.12 dw_volume_wdot_th

Class: WDotProductVolumeOperatorTHTerm
Definition:

$$\int_\Omega \left[\int_0^t \mathcal{G}(t-\tau) p(\tau) \,\mathrm{d}\tau\right] q$$

Syntax: `dw_volume_wdot_th.<i>.<r>( <ts>, <material>, <virtual>, <state>, <parameter> )`

6 Terms in termsLaplace

6.1 d_diffusion

Class: DiffusionIntegratedTerm
Description: Integrated general diffusion term with permeability K_{ij} constant or given in mesh vertices.
Definition:

$$\int_\Omega K_{ij} \nabla_i \bar{p} \nabla_j r$$

Syntax: `d_diffusion.<i>.<r>( <material>, <parameter_1>, <parameter_2> )`

6.2 de_diffusion_velocity

Class: DiffusionVelocityTerm
Description: Diffusion velocity averaged in elements.
Definition: vector of

$$\forall K \in \mathcal{T}_h : \int_{T_K} K_{ij} \nabla_j r / \int_{T_K} 1$$

Syntax: `de_diffusion_velocity.<i>.<r>( <material>, <parameter> )`

6.3 dw_diffusion

Class: DiffusionTerm
Description: General diffusion term with permeability K_{ij} constant or given in mesh vertices.
Definition:

$$\int_\Omega K_{ij} \nabla_i q \nabla_j p$$

Syntax: `dw_diffusion.<i>.<r>( <material>, <virtual>, <state> )`

6.4 dw_laplace

Class: LaplaceTerm
Description: Laplace term with c constant or constant per element.
Definition:

$$c \int_\Omega \nabla s \cdot \nabla r \text{ or } \sum_{K \in \mathcal{T}_h} \int_{T_K} c_K \ \nabla s \cdot \nabla r$$

Syntax: `dw_laplace.<i>.<r>( <material>, <virtual>, <state> )`

6.5 dw_permeability_r

Class: PermeabilityRTerm
Description: Special-purpose diffusion-like term with permeability K_{ij} constant or given in mesh vertices (to use on a right-hand side).
Definition:

$$\int_\Omega K_{ij} \nabla_j q$$

Syntax: `dw_permeability_r.<i>.<r>( <material>, <virtual>, <index> )`

7 Terms in termsNavierStokes

7.1 d_div

Class: DivIntegratedTerm
Description: Integrated divergence term (weak form).
Definition:

$$\int_\Omega \bar{p} \ \nabla \cdot \underline{w}$$

Syntax: `d_div.<i>.<r>( <parameter_1>, <parameter_2> )`

7.2 dq_grad

Class: GradQTerm
Description: Gradient term (weak form) in quadrature points.
Definition:

$$(\nabla p)|_{qp}$$

Syntax: `dq_grad.<i>.<r>( <state> )`

7.3 dq_lin_convect

Class: LinearConvectQTerm
Description: Linearized convective term evaluated in quadrature points.
Definition:

$$((\underline{b} \cdot \nabla) \underline{u})|_{qp}$$

Syntax: `dq_lin_convect.<i>.<r>( <parameter>, <state> )`

7.4 dw_convect

Class: ConvectTerm
Description: Nonlinear convective term.
Definition:

$$\int_\Omega ((\underline{u} \cdot \nabla) \underline{u}) \cdot \underline{v}$$

Syntax: `dw_convect.<i>.<r>( <virtual>, <state> )`

7.5 dw_div

Class: DivTerm
Description: Divergence term (weak form).
Definition:

$$\int_{\Omega} q \; \nabla \cdot \underline{u}$$

Syntax: dw_div.<i>.<r>(<virtual>, <state>)

7.6 dw_div_grad

Class: DivGradTerm
Description: Diffusion term.
Definition:

$$\int_{\Omega} \nu \; \nabla \underline{v} : \nabla \underline{u}$$

Syntax: dw_div_grad.<i>.<r>(<material>, <virtual>, <state>)

7.7 dw_grad

Class: GradTerm
Description: Gradient term (weak form).
Definition:

$$\int_{\Omega} p \; \nabla \cdot \underline{v}$$

Syntax: dw_grad.<i>.<r>(<virtual>, <state>)

7.8 dw_grad_dt

Class: GradDtTerm
Description: Gradient term (weak form) with time-discretized $\dot{p}$.
Definition:

$$\int_{\Omega} \frac{p - p_0}{\Delta t} \nabla \cdot \underline{v}$$

Arguments:

ts.dt	Δt
parameter	p_0

Syntax: dw_grad_dt.<i>.<r>(<ts>, <virtual>, <state>, <parameter>)

7.9 dw_lin_convect

Class: LinearConvectTerm
Description: Linearized convective term.
Definition:

$$\int_{\Omega} ((\underline{b} \cdot \nabla) \underline{u}) \cdot \underline{v}$$

Syntax: dw_lin_convect.<i>.<r>(<virtual>, <parameter>, <state>)

7.10 dw_st_grad_div

Class: GradDivStabilizationTerm
Description: Grad-div stabilization term (γ is a global stabilization parameter).
Definition:

$$\gamma \int_{\Omega} (\nabla \cdot \underline{u}) \cdot (\nabla \cdot \underline{v})$$

Syntax: `dw_st_grad_div.<i>.<r>( <material>, <virtual>, <state> )`

7.11 dw_st_pspg_c

Class: PSPGCStabilizationTerm
Description: PSPG stabilization term, convective part (τ is a local stabilization parameter).
Definition:

$$\sum_{K \in \mathcal{T}_h} \int_{T_K} \tau_K \ ((\underline{b} \cdot \nabla) \underline{u}) \cdot \nabla q$$

Syntax: `dw_st_pspg_c.<i>.<r>( <material>, <virtual>, <parameter>, <state> )`

7.12 dw_st_pspg_p

Class: PSPGPStabilizationTerm
Description: PSPG stabilization term, pressure part (τ is a local stabilization parameter), alias to Laplace term dw_laplace.
Definition:

$$\sum_{K \in \mathcal{T}_h} \int_{T_K} \tau_K \ \nabla p \cdot \nabla q$$

Syntax: `dw_st_pspg_p.<i>.<r>( <material>, <virtual>, <state> )`

7.13 dw_st_supg_c

Class: SUPGCStabilizationTerm
Description: SUPG stabilization term, convective part (δ is a local stabilization parameter).
Definition:

$$\sum_{K \in \mathcal{T}_h} \int_{T_K} \delta_K \ ((\underline{b} \cdot \nabla) \underline{u}) \cdot ((\underline{b} \cdot \nabla) \underline{v})$$

Syntax: `dw_st_supg_c.<i>.<r>( <material>, <virtual>, <parameter>, <state> )`

7.14 dw_st_supg_p

Class: SUPGPStabilizationTerm
Description: SUPG stabilization term, pressure part (δ is a local stabilization parameter).
Definition:

$$\sum_{K \in \mathcal{T}_h} \int_{T_K} \delta_K \ \nabla p \cdot ((\underline{b} \cdot \nabla) \underline{v})$$

Syntax: `dw_st_supg_p.<i>.<r>( <material>, <virtual>, <parameter>, <state> )`

8 Terms in termsPoint

8.1 dw_point_lspring

Class: LinearPointSpringTerm
Description: Linear springs constraining movement of FE nodes in a reagion; use as a relaxed Dirichlet boundary conditions.
Definition:

$$\underline{f}^i = -k \underline{u}^i \quad \forall \text{ FE node } i \text{ in region}$$

Syntax: `dw_point_lspring.<i>.<r>( <material>, <virtual>, <state> )`

9 Terms in termsVolume

9.1 dw_volume_lvf

Class: LinearVolumeForceTerm
Description: Vector or scalar linear volume forces (weak form) — a right-hand side source term.
Definition:

$$\int_\Omega \underline{f} \cdot \underline{v} \text{ or } \int_\Omega f q$$

Syntax: `dw_volume_lvf.<i>.<r>( <material>, <virtual> )`

10 Terms in termsSurface

10.1 dw_surface_ltr

Class: LinearTractionTerm
Description: Linear traction forces (weak form), where, depending on dimension of 'material' argument, $\underline{\underline{\sigma}} \cdot \underline{n}$ is $\bar{p}\underline{\underline{I}} \cdot \underline{n}$ for a given scalar pressure, $\underline{f}$ for a traction vector, and itself for a stress tensor.
Definition:

$$\int_\Gamma \underline{v} \cdot \underline{\underline{\sigma}} \cdot \underline{n}$$

Syntax: `dw_surface_ltr.<i>.<r>( <material>, <virtual> )`

11 Terms in termsLinElasticity

11.1 d_lin_elastic

Class: LinearElasticIntegratedTerm
Description: Integrated general linear elasticity term.
Definition:

$$\int_\Omega D_{ijkl}\ e_{ij}(\underline{b}) e_{kl}(\underline{w})$$

Syntax: `d_lin_elastic.<i>.<r>( <material>, <parameter_1>, <parameter_2> )`

11.2 de_cauchy_strain

Class: CauchyStrainTerm
Description: Cauchy strain tensor averaged in elements.
Definition: vector of

$$\forall K \in \mathcal{T}_h : \int_{T_K} \underline{\underline{e}}(\underline{w}) / \int_{T_K} 1$$

Syntax: `de_cauchy_strain.<i>.<r>( <parameter> )`

11.3 de_cauchy_stress

Class: CauchyStressTerm
Description: Cauchy stress tensor averaged in elements.
Definition: vector of

$$\forall K \in \mathcal{T}_h : \int_{T_K} D_{ijkl} e_k l(\underline{w}) / \int_{T_K} 1$$

Syntax: `de_cauchy_stress.<i>.<r>( <material>, <parameter> )`

11.4 dw_lin_elastic

Class: LinearElasticTerm
Description: General linear elasticity term, with D_{ijkl} given in the usual matrix form exploiting symmetry: in 3D it is 6×6 with the indices ordered as $[11, 22, 33, 12, 13, 23]$, in 2D it is 3×3 with the indices ordered as $[11, 22, 12]$.
Definition:

$$\int_\Omega D_{ijkl}\ e_{ij}(\underline{v}) e_{kl}(\underline{u})$$

Syntax: `dw_lin_elastic.<i>.<r>( <material>, <virtual>, <state> )`

11.5 dw_lin_elastic_iso

Class: LinearElasticIsotropicTerm
Description: Isotropic linear elasticity term.
Definition:

$$\int_\Omega D_{ijkl}\ e_{ij}(\underline{v}) e_{kl}(\underline{u}) \mbox{ with } D_{ijkl} = \mu (\delta_{ik} \delta_{jl}+\delta_{il} \delta_{jk}) + \lambda \ \delta_{ij} \delta_{kl}$$

Syntax: `dw_lin_elastic_iso.<i>.<r>( <material>, <virtual>, <state> )`

11.6 dw_lin_viscous

Class: LinearViscousTerm
Description: General linear viscosity term, with D_{ijkl} given in the usual matrix form exploiting symmetry: in 3D it is 6×6 with the indices ordered as $[11, 22, 33, 12, 13, 23]$, in 2D it is 3×3 with the indices ordered as $[11, 22, 12]$.
Definition:

$$\int_\Omega D_{ijkl}\ e_{ij}(\underline{v}) \frac{e_{kl}(\underline{u}) - e_{kl}(\underline{u}_0)}{\Delta t}$$

Arguments:

ts.dt	Δt
material	D_{ijkl}
virtual	$\underline{v}$
state	$\underline{u}$ (displacements of current time step)
parameter	$\underline{u}_0$ (known displacements of previous time step)

Syntax: `dw_lin_viscous.<i>.<r>( <ts>, <material>, <virtual>, <state>, <parameter> )`

11.7 dw_lin_viscous_th

Class: LinearViscousTHTerm
Definition:

$$\int_{\Omega} \left [\int_0^t \mathcal{H}_{ijkl}(t-\tau)\,\frac{\mathrm{d} e_{kl}(\underline{u}(\tau))}{\mathrm{d}\tau} \, \mathrm{d}\tau \right]\,e_{ij}(\underline{v})$$

Syntax: `dw_lin_viscous_th.<i>.<r>( <ts>, <material>, <virtual>, <state>, <parameter> )`

12 Terms in termsBiot

12.1 d_biot_div

Class: BiotDivRIntegratedTerm
Description: Integrated Biot divergence-like term (weak form) with α_{ij} given in vector form exploiting symmetry: in 3D it has the indices ordered as $[11, 22, 33, 12, 13, 23]$, in 2D it has the indices ordered as $[11, 22, 12]$.
Definition:

$$\int_\Omega r\ \alpha_{ij} e_{ij}(\underline{w})$$

Syntax: `d_biot_div.<i>.<r>( <material>, <parameter_1>, <parameter_2> )`

12.2 dw_biot_div

Class: BiotDivTerm
Description: Biot divergence-like term (weak form) with α_{ij} given in vector form exploiting symmetry: in 3D it has the indices ordered as $[11, 22, 33, 12, 13, 23]$, in 2D it has the indices ordered as $[11, 22, 12]$.
Definition:

$$\int_\Omega q\ \alpha_{ij} e_{ij}(\underline{u})$$

Syntax: `dw_biot_div.<i>.<r>( <material>, <virtual>, <state> )`

12.3 dw_biot_div_dt

Class: BiotDivDtTerm
Description: Biot divergence-like rate term (weak form) with α_{ij} given in vector form exploiting symmetry: in 3D it has the indices ordered as $[11, 22, 33, 12, 13, 23]$, in 2D it has the indices ordered as $[11, 22, 12]$.
Definition:

$$\int_\Omega q\ \alpha_{ij} \frac{e_{ij}(\underline{u}) - e_{ij}(\underline{u_0})}{\Delta t}$$

Syntax: `dw_biot_div_dt.<i>.<r>( <ts>, <material>, <virtual>, <state>, <parameter> )`

12.4 dw_biot_div_th

Class: BiotDivTHTerm
Definition:

$$\int_\Omega \left[\int_0^t \alpha_{ij}(t-\tau) \frac{\mathrm{d} e_{kl}(\underline{u}(\tau))}{\mathrm{d}\tau}\ \mathrm{d}\tau\right] q$$

Syntax: `dw_biot_div_th.<i>.<r>( <ts>, <material>, <virtual>, <state>, <parameter> )`

12.5 dw_biot_grad

Class: BiotGradTerm
Description: Biot gradient-like term (weak form) with α_{ij} given in vector form exploiting symmetry: in 3D it has the indices ordered as $[11, 22, 33, 12, 13, 23]$, in 2D it has the indices ordered as $[11, 22, 12]$.
Definition:

$$\int_\Omega p\ \alpha_{ij} e_{ij}(\underline{v})$$

Syntax: `dw_biot_grad.<i>.<r>( <material>, <virtual>, <state> )`

12.6 dw_biot_grad_dt

Class: BiotGradDtTerm
Description: Biot gradient-like term (weak form) with time-discretized $\dot{p}$ and α_{ij} given in vector form exploiting symmetry: in 3D it has the indices ordered as $[11, 22, 33, 12, 13, 23]$, in 2D it has the indices ordered as $[11, 22, 12]$.
Definition:

$$\int_{\Omega} \frac{p - p_0}{\Delta t} \; \alpha_{ij} e_{ij}(\underline{v})$$

Arguments:

ts.dt	Δt
parameter	p_0

Syntax: `dw_biot_grad_dt.<i>.<r>( <ts>, <material>, <virtual>, <state>, <parameter> )`

12.7 dw_biot_grad_th

Class: BiotGradTHTerm
Definition:

$$\int_{\Omega} \left[\int_0^t \alpha_{ij}(t - \tau) \, p(\tau)) \; \mathrm{d}\tau \right] \, e_{ij}(\underline{v})$$

Syntax: `dw_biot_grad_th.<i>.<r>( <ts>, <material>, <virtual>, <state>, <parameter> )`

13 Term caches in cachesBasic

13.1 cauchy_strain

Class: CauchyStrainDataCache

```
cache = term.getCache( 'cauchy_strain', <index> )
data = cache( <data name>, <ig>, <ih>, state )
```

13.2 div_vector

Class: DivVectorDataCache

```
cache = term.getCache( 'div_vector', <index> )
data = cache( <data name>, <ig>, <ih>, state )
```

13.3 grad_scalar

Class: GradScalarDataCache

```
cache = term.getCache( 'grad_scalar', <index> )
data = cache( <data name>, <ig>, <ih>, state )
```

13.4 mat_in_qp

Class: MatInQPDataCache

```
cache = term.getCache( 'mat_in_qp', <index> )
data = cache( <data name>, <ig>, <ih>, mat, ap, assumedShapes, modeIn )
```

13.5 state_in_surface_qp

Class: StateInSurfaceQPDataCache

```
cache = term.getCache( 'state_in_surface_qp', <index> )
data = cache( <data name>, <ig>, <ih>, state )
```

13.6 state_in_volume_qp

Class: StateInVolumeQPDataCache

```
cache = term.getCache( 'state_in_volume_qp', <index> )
data = cache( <data name>, <ig>, <ih>, state )
```

13.7 volume

Class: VolumeDataCache

```
cache = term.getCache( 'volume', <index> )
data = cache( <data name>, <ig>, <ih>, region, field )
```