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The $\mathcal{D}_{3h}$ Symmetry of ^{12}C

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Abstract. The $\mathcal{D}_{3h}$ Symmetry of ^{12}C that was predicted by Bijker and Iachello already in 2000 was confirmed by the UConn-Birmingham collaboration in 2014 including a parity doublet in a mixed symmetry rotational band with the spin-parity sequence of: $J^{\pi} = 0^+, 2^+, 3^-, 4^{\pm}$ and 5^-. Such rotational bands including parity doublets are ubiquitous in molecular physics, but it was observed for the first time in nuclear physics in ^{12}C. The Algebraic Cluster Model (ACM) of Bijker and Iachello provides a new exciting chapter in the study of clustering in light nuclei with current and future applications to many light nuclei.

A HISTORICAL REVIEW

In two papers in 2000 [1] and 2002 [2] Bijker and Iachello developed the Algebraic Cluster Model (ACM) with an application to ^{12}C. Briefly, in this model they solved for the eigen states of a U(7) Hamiltonian with an explicit assumption of a $\mathcal{D}_{3h}$ geometrical symmetry in addition to the dynamical symmetry of the U(7) Hamiltonian, expected for the three-particle oblate symmetric spinning top. This model was applied by the (at that time) graduate student Aurora Tumino to calculate form factors of electron scattering from ^{12}C, as she presented in 2002 at the Erice celebration of Iachello's 60^{th} birthday [3]. Ten years later in May 2012 I participated in the celebration of Iachello's 70^{th} birthday at Cocoyoc, Mexico. I chose to present data obtained by our UConn-TUNL collaboration working at the High Intensity gamma Source (HIγS) at Duke University. In this measurement we used a time projection chamber (TPC) detector operating with CO_2 gas. Our main interest originally was the study of the photodissociation of ^{16}O in the $^{16}O(\gamma,\alpha)^{12}C$ reaction, which is the time reverse of the $^{12}C(\alpha,\gamma)$ reaction of importance for stellar evolution theory and nuclear astrophysics. As such the data we obtained on the photodissociation of ^{12}C from the CO_2 gas was considered by us as background. Unbeknownst to me, my data on the discovery of the 2^+_2 of ^{12}C at 10.03 MeV [4] caught the attention of several people at the Cocoyoc 2012 meeting and I was introduced to the ACM of Bijker and Iachello. In August 2013 while visiting the group of Martin Freer in Birmingham I reviewed some data on ^{12}C obtained by his graduate student Daniel Marín-Lámbarri, where he discovered a new 5^- state in ^{12}C at 22.4(2) MeV [5]. I also learned at Birmingham that a state at 13.35 MeV in ^{12}C, previously thought to be a 2^- state, was in fact a 4^- state. Our discovery of the 2^+_2 in ^{12}C [4], the discovery of 5^- state in Birmingham [5] and clarification of the spin of the 4^- state led me to conclude that we discovered evidence of the predicted $\mathcal{D}_{3h}$ symmetry in ^{12}C [5] and that we found the first example of a nuclear rotational band that includes a parity doublets ($4^{\pm}$), as commonly found in molecular physics. I can state without a doubt that my participation in the Iachello 70^{th} birthday symposium in Cocoyoc and my continued intense interactions with Professor Iachello since then, has given me a new direction of research and indeed a new insight on clustering. This continues to be an exciting journey with my friend and dear colleague Professor Iachello.

INTRODCUTION

The structure of ^{12}C has recently attracted much theoretical attention, with the developments of ab initio no-core shell model calculations [6], the no-core symplectic model [7], and effective field theory calculations on the lattice [8]. These calculations attempt to provide a microscopic description of cluster states that are well described in the traditional clustering model [9] and anti-symmetrized molecular dynamics [10], as well as in the more modern

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fermionic molecular dynamics model [11] and more exotic cluster models [12]. However, thus far, ab initio shell model calculations have failed to predict [6] the Hoyle state at 7.654 MeV in ^{12}C that is known to be one of the best examples of alpha clustering in nuclei. The geometrical arrangements of alpha-particles in the Hoyle state on the other hand is a problem of current interest [13]. The linear chain geometry has been ruled out, but on the other hand the obtuse triangular shape [8] Vs. an equilateral triangular shape still need to be unraveled. Determining the structure of the Hoyle state is important since the lattice calculations also attempt to predict the mass of the up and down quarks and the strength of the electromagnetic interactions based on the anthropogenic role of the Hoyle state [8].

The effective field theory lattice calculations [8] and the fermionic molecular dynamics model [11] predict an equilateral arrangement of the three alpha particles in the ground state of ^{12}C as well as for the Hoyle state. These microscopic models provide the foundation for applying the ACM of Bijker and Iachello with the conjectured $\mathcal{D}_{3h}$ symmetry to ^{12}C.

THE $\mathcal{D}_{3H}$ SYMMETRY

Geometrical equilateral triangular configurations [1,2] have been identified in the triatomic H_3^+ molecule [14] where the predicted spectrum of a triangular oblate spinning top with a $\mathcal{D}_{3h}$ symmetry was observed [1,2,14]. Such triangular configurations are described by the two Jacobi vectors ($\boldsymbol{\lambda}$ and $\boldsymbol{\rho}$) as shown in Fig. 1. It was suggested [1,2] that the three alpha particle system of ^{12}C should lead to similar "triatomic like" structure in nuclei. The application to ^{12}C of the $\mathcal{D}_{3h}$ symmetry, a mathematical tool that was developed to describe molecular structure, emphasizes the role of symmetry across very different energy scales. Specifically, it leads to a model of ^{12}C that classify rotations-vibrations in ^{12}C with predicted mixed-parity rotational bands and parity doublets as shown in Fig. 2. Such a poly-atomic like description of light nuclei should lead to a better understanding of the clustering phenomena in light nuclei and indeed it was successfully applied to ^{16}O [15].

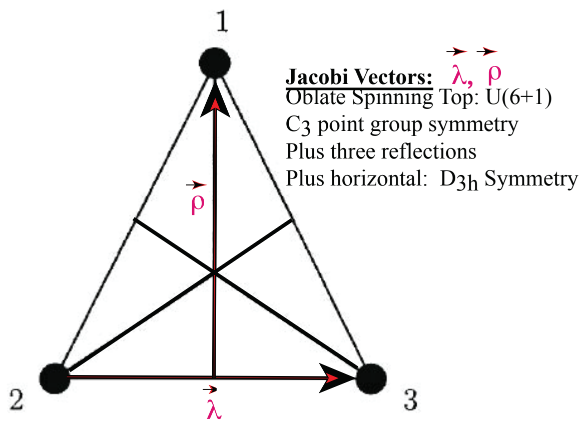

Figure 1: A schematic depiction of triangular symmetric spinning top of three particles placed on an **equilateral triangle**. The triatomic-like configuration is fully described by the two Jacobi vectors ($\boldsymbol{\lambda}$ and $\boldsymbol{\rho}$) with the indicated symmetries.

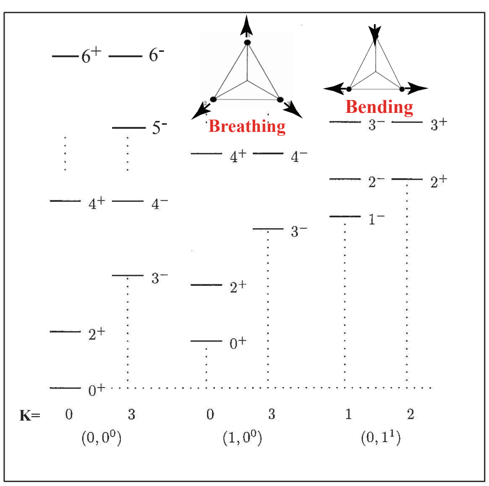

Figure 2: The classification of the rotation vibration spectrum of a three alpha-particles in an equilateral triangular configuration [2].

THE DISCOVERY OF NEW STATES IN ^{12}C

The UConn-TUNL collaboration measurements of the photo-dissociation of ^{12}C revealed an unambiguous evidence for the second 2^+ in ^{12}C at 10.03 MeV [4]. In this measurement we employed an optical readout TPC detector (O-TPC) operating with the gas mixture of CO_2 (80%) + N_2 (20%) at 100 torr [16]. The O-TPC allowed measurements of complete angular distributions with unprecedented accuracy including closed to 30 data points as shown in Fig. 3. These data allowed for unambiguous identification of the 2^+_2 in ^{12}C at 10.03 MeV [4].

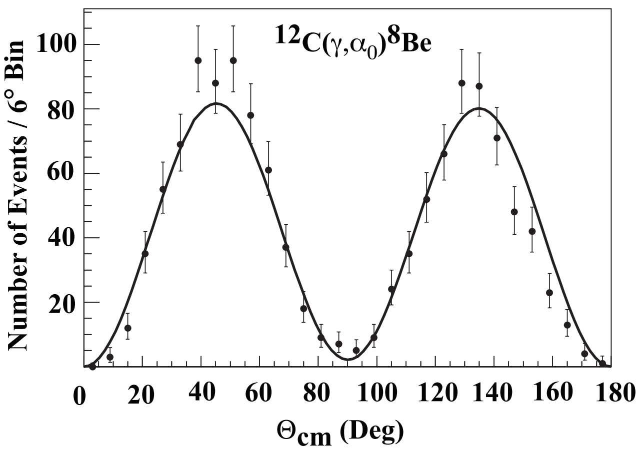

Figure 3: The measured angular distribution of the $^{12}C(\gamma,\alpha_0)^8Be$ reaction that provided unambiguous identification of the second 2^+ in ^{12}C at 10.03 MeV [4].

A measurement of the $^{12}C(^4He,3\alpha)^4He$ reaction performed on the cyclotron at U Birmingham in the UK revealed a new 5- state at 22.4(2) MeV [5] as shown in Fig. 4. In the same time the Birmingham group [17] together with a measurement at the Aarhus [18] clarified the spin-parity of the state at 13.3 MeV in ^{12}C to be 4^-.

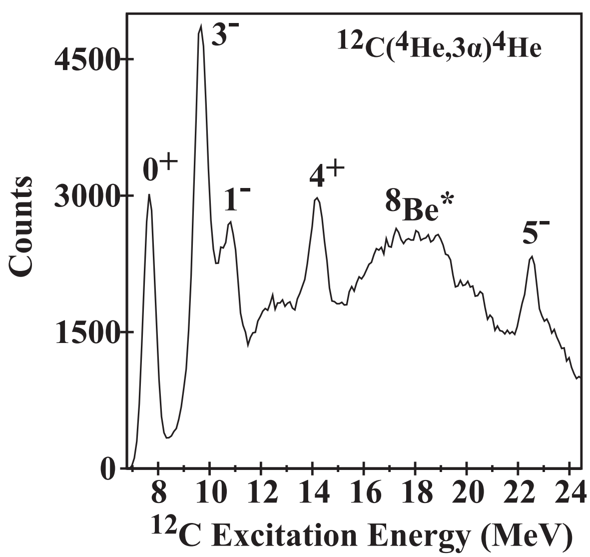

Figure 4: The measured spectrum of correlated alpha-particles from the $^{12}C(^4He,3\alpha)^4He$ reaction that provided identification of the 5^- in ^{12}C at 22.4(2) MeV [5].

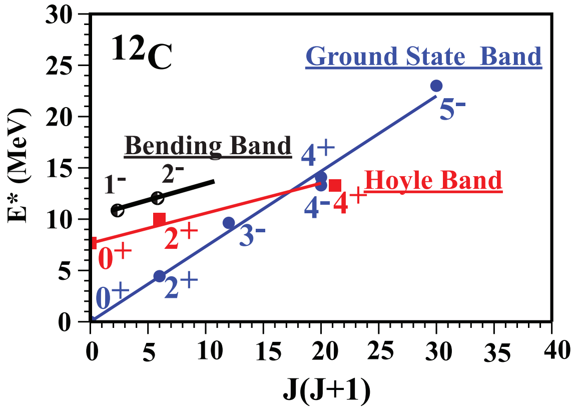

Figure 5: Rotational band structure of the ground state band, the Hoyle band, and the bending vibration in ^{12}C. [5].

The discovery of the new states in ^{12}C discussed above, together with the predicted spectrum of the triangular symmetric spinning top shown in Fig. 2 led to the discovery of the mixed parity band including a parity doublet in the ground state band of ^{12}C with $J^{\pi} = 0^+, 2^+, 3^-, 4^{\pm}$ and 5^- as shown in Fig. 5. First evidence for a rotational band built on top of the Hoyle state is also shown in Fig. 5. The prediction of the ACM cluster model for the structure of ^{12}C is shown in Fig. 6. Note that all observed cluster states below 15 MeV in ^{12}C are predicted by the ACM except for two 1^+ states that are not cluster states.

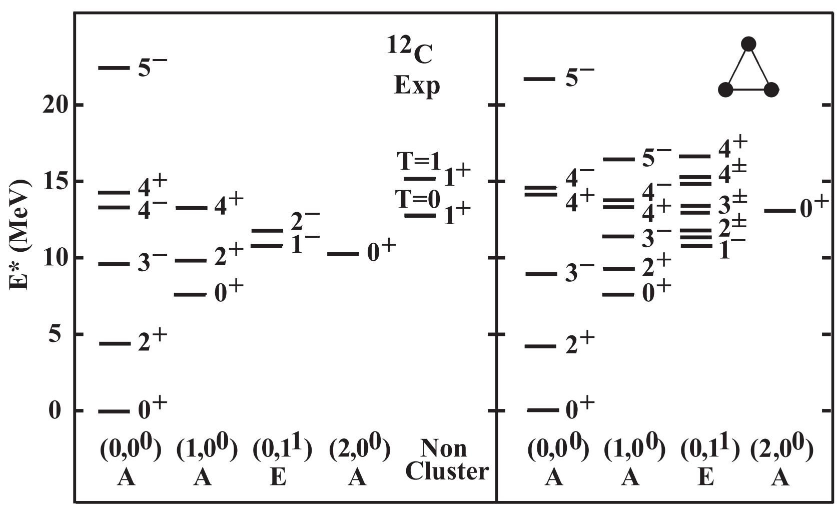

Figure 6: Comparison between the low-lying experimental spectrum of ^{12}C and the energies of the oblate symmetric top. The levels are organized in columns corresponding to the ground state band and the vibrational bands with A and E symmetry of an oblate top [5] with triangular symmetry. The last column on the left-hand side, shows the lowest observed noncluster (1^+) levels.

CONCLUSIONS

The “missing” 3^- and 4^- states predicted in the Hoyle band shown in Fig. 6, are essential for an unambiguous identification of a geometrical equilateral arrangements of the three alpha-particles in the Hoyle state. This search is ongoing now and it should allow for determining the structure of the Hoyle state.

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