$V = ($

$$V(R,\Theta,\Phi,\theta,\varphi,\chi) = \sum_{F=0}^{\infty}\sum_{m_F=-F}^{F}\sum_{L=0}^{\infty}\sum_{J=0}^{\infty} G_{L,J}^{F,m_F}(R)\,T_{L,J}^{F,m_F}(\Omega,\omega)$$

$$\int_\omega\int_\Omega T_{L',J'}^{F',m_F'*} V(R,\Theta,\Phi,\theta,\varphi,\chi)\,d\omega\, d\Omega = G_{L,J}^{F,m_F}(R)\sum_F\sum_{m_F}\sum_L\sum_J \delta_{FF'}\delta_{m_Fm_F'}\delta_{LL'}\delta_{JJ'} \times \int_\omega\int_\Omega T_{L',J'}^{F',m_F'*}(\Omega,\omega)\,T_{L,J}^{F,m_F}(\Omega,\omega)\,d\Omega\, d\omega$$

$$\int_\omega\int_\Omega T_{L,J}^{F,m_F}(\Omega,\omega)\,V(R,\Theta,\Phi,\theta,\varphi,\chi)\,d\omega\, d\Omega = G_{L,J}^{F,m_F}(R) \times \text{Normalization Factor}$$

$$G_{L,J}^{F,m_F}(R) = \frac{1}{\text{Normalization Factor}}\int_\omega\int_\Omega T_{L,J}^{F,m_F}(\Omega,\omega)\,V(R,\Theta,\Phi,\varphi,\chi)\,d\omega\, d\Omega$$

we are missing this!!!

- Let us find this normalization constant:

Remember that

$$T_{L,J}^{F,m_F}(\Omega,\omega) = \sum_{m_L,m_J}(-1)^{L-J+m_F}\sqrt{2F+1}\begin{pmatrix} L & J & F \\ m_L & m_J & -m_F\end{pmatrix} Y_{m_L}^{L}(\Omega)\,D_{Km_J}^{J}(\omega)$$

Then, we would have that

$$\int_\omega\int_\Omega T_{L',J'}^{F',m_F'*}(\Omega,\omega)\,T_{L,J}^{F,m_F}(\Omega,\omega)$$

$$= \sum_{m_L,m_J}\sum_{m_L',m_J'}\delta_{m_Lm_L'}\delta_{m_Jm_J'}(-1)^{L'-J'+m_F'}(-1)^{L-J+m_F}\sqrt{2F'+1}\sqrt{2F+1}$$

$$\times\begin{pmatrix} L' & J' & F' \\ m_L' & m_J' & -m_F'\end{pmatrix}\begin{pmatrix} L & J & F \\ m_L & m_J & -m_F\end{pmatrix}\int_\Omega Y_{m_L'}^{L'}(\Omega)\,Y_{m_L}^{L}(\Omega)\,d\Omega$$

$$\times\int_\omega D_{K'm_J'}^{J'*}(\omega)\,D_{Km_J}^{J}(\omega)\,d\omega$$

$$= \cancel{(-1)^{2(L-J+m_F)}} = 1 \quad (2F+1) \begin{pmatrix} L & J & F \\ m_L & m_J & -m_F \end{pmatrix}^2$$

$$\Phi^{J\,+}_{k\,m_J}(\omega) = \left[\frac{2J+1}{8\pi^2}\right]^{1/2} D^{J}_{k\,m_J}(\omega)$$

$$D^{*} \cdot D = \frac{2J+1}{8\pi^2}$$

$$\times \frac{2J+1}{8\pi^2}$$

Alors, finalement on a

$$\text{Normalization factor} = \frac{(2F+1)(2J+1)}{8\pi^2} \begin{pmatrix} L & J & F \\ m_L & m_J & -m_F \end{pmatrix}^2$$

c'est-à-dire:

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$$\boxed{\Gamma^{F,m_F}_{L,J}(R) = \frac{(2F+1)(2J+1)}{8\pi^2} \begin{pmatrix} L & J & F \\ m_L & m_J & -m_F \end{pmatrix}^2 \int_{\omega}\int_{\Omega} T^{F,m_F}_{L,J}(\Omega,\omega)\, V(R,\Theta,\Phi,\varphi,\chi)\, d\omega\, d\Omega}$$