Lab 1 Solns – Math probles 1 & 2, and GCD & Subsetsum – E.K 9/5/19

Problem 1. Which of the following functions are increasing? eventually nondecreasing? If you remember techniques from calculus, you can make use of those.

(1) $f(x) = -x^2$

(2) $f(x) = x^2 + 2x + 1$

(3) $f(x) = x^3 + x$

Solution. (1) is not eventually nondecreasing. (2) is eventually nondecreasing. (3) is increasing

Problem 2. Consider the following pairs and functions f, g. Decide if it is correct to say that, asymptotically, f grows no faster than g, g grows no faster than f, or both.

(1) $f(x) = 2x^2$, $g(x) = x^2 + 1$

(2) $f(x) = x^2$, $g(x) = x^3$

(3) $f(x) = 4x + 1$, $g(x) = x^2 - 1$

Solution. For (1), each function grows no faster than the other. For (2), f grows no faster than g, but not conversely. For (3), f grows no faster than g, but not conversely.

For Problems 1 and 2, see the code in folders prob1, prob2.

Problem 3. *Greedy Strategies.* See if you can solve SubsetSum problems using the following *greedy* strategy. With a greedy strategy, at each step in an algorithm a value that is optimal *at that time* is chosen. Decide whether the following greedy strategy works: Begin by sorting the input set S; assume that S in sorted order is as follows: $\{s_0, s_1, \ldots, s_{n-1}\}$. Initialize an empty set T; we will add elements to T as we scan S. As you scan S, if $s_0 \leq k$, put s_0 in T; otherwise leave it out. Then, if the sum of the elements of T together with s_1 is $\leq k$, then put s_1 in T; otherwise leave it out. Then, if the sum of the elements of T together with s_2 is $\leq k$, put s_2 in T; otherwise, leave it out Continue this way until every number in S has been checked.

For this problem, decide if this strategy always works. If not, give an example of a SubsetSum problem for which the algorithm gives an incorrect result. If you think it does work, give an argument to support your idea.

Solution. The algorithm does not work. Consider $S = \{2, 4, 5\}, k = 7$. Using the greedy strategy, the algorithms populates the set T with $2, 4$ and then cannot continue, so the final value is $T = \{2, 4\}$. Since the sum of elements in T is not 7, the return is (incorrectly) null.

Problem 4. You are given a solution T to a SubsetSum problem with a $S = \{s_0, s_1, \ldots, s_{n-1}\}$ and k some non-negative integer. (Recall that T is a solution if it is a subset of S the sum of whose elements is equal to k.) Suppose that s_{n-1} belongs to T. Is it necessarily true that the set $T - \{s_{n-1}\}$ is a solution to the SubsetSum problem with inputs S', k' where $S' = \{s_0, s_1, \ldots, s_{n-2}\}$ and $k' = k - s_{n-1}$? Explain.

Solution. This is correct. We must show that the sum of the elements of $T' = T - \{s_{n-1}\}$ is $k - s_{n-1}$. But the sum of the elements of T (which is the set $T' \cup \{s_{n-1}\}$) is k. Since $s_{n-1} \notin T'$, the sum of the elements of T' must be s_{n-1} less than the sum of the elements of T; that is the sum of the elements of T' is $k - s_{n-1}$. (Note that it is possible that the only element of T is s_{n-1}. In that case the sum of elements of T is s_{n-1} (so it must be that $k = s_{n-1}$). Then the sum of elements of $T' = T - \{s_{n-1}\}$, which is now empty, must be $k - k = 0$; since the sum of an empty set of integers is 0, this result is still correct.)