

Lab 1 Solutions

For Problems 1 and 2, see the code in folders prob1, prob2.

Problem 3. *Greedy Strategies.* See if you can solve SubsetSum problems using the following greedy strategy. With a greedy strategy, at each step in an algorithm a value that is optimal *at that time* is chosen. Decide whether the following greedy strategy works: Begin by sorting the input set S ; assume that S in sorted order is as follows: $\{s_0, s_1, \dots, s_{n-1}\}$. Initialize an empty set T ; we will add elements to T as we scan S . As you scan S , if $s_0 \leq k$, put s_0 in T ; otherwise leave it out. Then, if the sum of the elements of T together with s_1 is $\leq k$, then put s_1 in T ; otherwise leave it out. Then, if the sum of the elements of T together with s_2 is $\leq k$, put s_2 in T ; otherwise, leave it out. Continue this way until every number in S has been checked.

For this problem, decide if this strategy always works. If not, give an example of a SubsetSum problem for which the algorithm gives an incorrect result. If you think it does work, give an argument to support your idea.

Solution. The algorithm does not work. Consider $S = \{2, 4, 5\}$, $k = 7$. Using the greedy strategy, the algorithm populates the set T with 2, 4 and then cannot continue, so the final value is $T = \{2, 4\}$. Since the sum of elements in T is not 7, the return is (incorrectly) null.

Problem 4. You are given a solution T to a SubsetSum problem with a $S = \{s_0, s_1, \dots, s_{n-1}\}$ and k some non-negative integer. (Recall that T is a solution if it is a subset of S the sum of whose elements is equal to k .) Suppose that s_{n-1} belongs to T . Is it necessarily true that the set $T - \{s_{n-1}\}$ is a solution to the SubsetSum problem with inputs S', k' where $S' = \{s_0, s_1, \dots, s_{n-2}\}$ and $k' = k - s_{n-1}$? Explain.

Solution. This is correct. We must show that the sum of the elements of $T' = T - \{s_{n-1}\}$ is $k - s_{n-1}$. But the sum of the elements of T (which is the set $T' \cup \{s_{n-1}\}$) is k . Since $s_{n-1} \notin T'$, the sum of the elements of T' must be s_{n-1} less than the sum of the elements of T ; that is the sum of the elements of T' is $k - s_{n-1}$. (Note that it is possible that the only element of T is s_{n-1} . In that case the sum of elements of T is s_{n-1} (so it must be that $k = s_{n-1}$). Then the sum of elements of $T' = T - \{s_{n-1}\}$, which is now empty, must be $k - k = 0$; since the sum of an empty set of integers is 0, this result is still correct.)