Lab 8

Q1 A.

Start with an empty stack of integers. You will attempt to do a sequence of pushes and pops so that the sequence of pops will be a specified permutation of 1, 2, 3 ,4 , 5, 6. You will be able to do exactly 6 push operations and 6 pop operations. The first push pushes 1 onto the stack; the next pushes 2; and so forth. The sixth push pushes 6 onto the stack.

For this exercise, we will let S denote a push operation and X a pop operation. Example: The sequence SSSSSSXXXXXX outputs 654321.

a. Describe a sequence of pushes and pops that would produce output 325641 (or explain why it is not possible)
b. Describe a sequence of pushes and pops that would produce output 154623 (or explain why it is not possible)

Ans.
Solution to (a) SSSXXSSXSXXX

Solution to (b) No sequence of pops and pushes as described in the instructions could output 154623 because 2 will be pushed before 3, so 3 will be popped before 2 – therefore, the order of the last two digit of 154623 cannot be realized.

Q1 B. Suppose we store n keys in a hash table of size m = n^2 using a hash function h randomly chosen from a Universal class H of hash functions. Assume that X is a random variable that counts the number of collisions. Show that the Expected number of Collisions is < 1/2.

Ans. There are $\binom{n}{2}$ pairs of keys that may collide; each pair collides with probability 1/m if h is chosen at random from a Universal hash function, H. Let X be a random variable that counts the number of collisions. So, the expected number of collisions is

E[X] = (n C 2) . 1/n^2 = ½[(n^2 –n) /n^2 = ½ [1 – 1/n] < 1/2.

Q2. For each integer n = 1,2,3,…, 7, determine whether there exists a red-black tree having exactly n nodes, with *all of them black.* Fill out the chart below to tabulate the results: (Ans)

Num nodes *n*	**Does there exist a red-black tree with *n* nodes, all of which are black?**
1	Yes

2	No
3	Yes
4	No
5	No
6	No
7	Yes

Q3. For each integer n = 1,2,3,…, 7, determine whether there exists a red-black tree having exactly n nodes and exactly one red node. Fill out the chart below to tabulate the results: (Ans)

Num nodes *n*	**Does there exist a red-black tree with *n* nodes that has exactly one red node?**
1	No
2	Yes
3	No
4	Yes
5	Yes
6	No
7	No