

Lab 6

1. Show that any comparison-based algorithm to sort 4 elements requires at least 5 comparisons in the worst case.

Solution. We showed in class that a binary tree T having L leaves must have node of depth at least $\lceil \log L \rceil$. In particular, if T is the decision tree for a sorting algorithm, sorting n distinct values, there must be a branch in T having length at least $\lceil \log n! \rceil$; this means that the sorting algorithm requires, in the worst case, at least $\lceil \log n! \rceil$ comparisons.

When $n = 4$, therefore, at least $\lceil \log n! \rceil$ comparisons are necessary. But

$$\lceil \log 4! \rceil = \lceil \log 24 \rceil = \lceil 3.2 \rceil = 4.$$

Therefore, at least 5 comparisons are needed.

2.

Explain how RadixSort can be used to sort the following $S = \{125, 27, 729, 1, 27, 8, 64, 343, 216\}$

using radix = 9.

Soln.

APPROACH A:

Do it in class using Radix as shown for the IBM solution.

APPROACH B: Use multiple Buckets.

Here the range is very large and one bucket will not work. Even 3 buckets will not work just because of the highest number 729. The max number with 3 buckets with radix 9 is

$728 = 9^2 \cdot q_2 + 9 \cdot q_1 + r$ where $q_2 = q_1 = r = 8$. The range for q_2 , q_1 and r are, of course 0 to 8.

Alternatively, we can use radix 10. To make use of radix 9, let us CHANGE the max number to 728.

The formulae we need to use are as follows:

1. Bucket r - use input values x and remainder using mod 9.
2. Bucket q_1 needs to be filled by reading values (left to right) from bucket r but divided by 9 and then mod 9 $\rightarrow (y / 9) \bmod 9$.
3. Bucket q_2 needs to be filled by reading values (left to right) from bucket q_1 but divided by 9^2 and no mod operation $\rightarrow z / 81$.

Then read data from q1 left to right. See the table below for details.

	125	27	728 (changed from 729)	1	27	8	64	343	216
Bucket r	216 27 27	343 64 1							8 728 125
	0	1	2	3	4	5	6	7	8
Bucket q1	8 1			27 27	125		216	64	728
	0	1	2	3	4	5	6	7	8
Bucket q2 (RETURN RSEULT - Left to top to bottom to right)	64 27 27 8 1								728
	0	1	2	3	4	5	6	7	8
Div by 9 Quotient (scratch pad)	13	3	80	0	3	0	7	38	24

3. Carry out the steps of RadixSort to sort the following {80, 27, 72, 1, 27, 8, 64, 34, 16} – Hint: use 9 for your radix.

Solution - please note that in $q[0]$, the first value is 1 and not 0. The same is true in the sorted output i.e the first number is 1 followed by 8.

3. $S = \{80, 27, 72, 1, 27, 8, 64, 34, 16\}$

$r[]$:

	27							16	8
	72	64						34	80
	27	1							
	0	1	2	3	4	5	6	7	8

$q[]$:

	8			34				80	
	0	16		27				64	72
	0	1	2	3	4	5	6	7	8

read $q[]$ from ~~bottom~~ left to right, bottom to top

0 8 16 27 27 34 64 72 80