We consider the following continuous-time dynamical system in Ver. 1.

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \tag{1}$$

In this study, we design a digital controller with the sampling period $\Delta \geq 0$, that is

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} u_{1,k} \\ u_{2,k} \end{bmatrix}, \ \forall t \in [k\Delta, (k+1)\Delta). \tag{2}$$

It is assumed that the system state at $t = k\Delta$ is $[x_k \ y_k \ \theta_k]^\top$.

Then,

$$\theta_{k+1} = \theta((k+1)\Delta) = \theta_k + \int_{k\Delta}^{(k+1)\Delta} u_{2,k}\mathrm{d}t = \theta_k + u_{2,k}\Delta. \tag{3}$$

Additionally,

$$x_{k+1} = x_k + u_{1,k}\int_{k\Delta}^{(k+1)\Delta} \cos\theta(t)\mathrm{d}t, \tag{4}$$

$$y_{k+1} = y_k + u_{1,k}\int_{k\Delta}^{(k+1)\Delta} \sin\theta(t)\mathrm{d}t, \tag{5}$$

where $\theta(t) = \theta_k + u_{2,k}(t - k\Delta), \ t \in [k\Delta, (k+1)\Delta)$.

1. If $u_{2,k} = 0$,

$$x_{k+1} = x_k + u_{1,k}\cos\theta_k\Delta, \tag{6}$$

$$y_{k+1} = y_k + u_{1,k}\sin\theta_k\Delta, \tag{7}$$

$$\theta_{k+1} = \theta_k. \tag{8}$$

2. If $u_{2,k} \neq 0$,

$$\begin{aligned} \int_{k\Delta}^{(k+1)\Delta} \cos(t)\mathrm{d}t &= \int_{k\Delta}^{(k+1)\Delta} \cos(\theta_k + u_{2,k}(t - k\delta)\mathrm{d}t \\ &= \frac{1}{u_{2,k}}\left[\sin(\theta_k + u_{2,k}(t - k\Delta))\right]_{k\Delta}^{(k+1)\Delta} \\ &= \frac{1}{u_{2,k}}(\sin(\theta_k + u_{2,k}\Delta) - \sin\theta_k), \end{aligned}$$

$$\begin{aligned} \int_{k\Delta}^{(k+1)\Delta} \sin(t)\mathrm{d}t &= \int_{k\Delta}^{(k+1)\Delta} \sin(\theta_k + u_{2,k}(t - k\delta)\mathrm{d}t \\ &= \frac{-1}{u_{2,k}}\left[\cos(\theta_k + u_{2,k}(t - k\Delta))\right]_{k\Delta}^{(k+1)\Delta} \\ &= \frac{-1}{u_{2,k}}(\cos(\theta_k + u_{2,k}\Delta) - \cos\theta_k), \end{aligned}$$

Then,

$$x_{k+1} = x_k + \frac{u_{1,k}}{u_{2,k}}(\sin(\theta_k + u_{2,k}\Delta) - \sin\theta_k), \tag{9}$$

$$y_{k+1} = y_k + \frac{u_{1,k}}{u_{2,k}}(\cos\theta_k - \cos(\theta_k + u_{2,k}\Delta)), \tag{10}$$

$$\theta_{k+1} = \theta_k + u_{2,k}\Delta. \tag{11}$$

If $u_{2,k} \simeq 0$, then,

$$\lim_{u_{2,k}\Delta \to 0} \frac{\sin(\theta_k + u_{2,k}\Delta) - \sin\theta_k}{u_{2,k}\Delta} = \cos\theta_k,$$
$$\lim_{u_{2,k}\Delta \to 0} \frac{\cos(\theta_k + u_{2,k}\Delta) - \cos\theta_k}{u_{2,k}\Delta} = -\sin\theta_k,$$

In this case,

$$x_{k+1} = x_k + u_{1,k}\Delta\cos\theta_k, \tag{12}$$
$$y_{k+1} = y_k + u_{1,k}\Delta\sin\theta_k, \tag{13}$$
$$\theta_{k+1} = \theta_k + u_{2,k}\Delta. \tag{14}$$