We consider an n-dimensional random variable $w \sim f_w$, where $f_w : \mathbb{R}^n \to \mathbb{R}$ is a probability density function. Additionally, we consider the following function.

$$x = g(w) := \Delta_w w + b, \tag{1}$$

where Δ_w is a regular matrix and $b \in \mathbb{R}^n$ is a constant. Then, we obtain

$$w = \Delta_w^{-1}(x - b) =: g^{-1}(x). \tag{2}$$

It is assumed that

- $X = g(W) = (g_1(X_1, ..., X_n), ..., g_n(X_1, ..., X_n))$.
- The integral range of a cumulative distribution function $F_x(x)$ is $A \in (-\infty, x_1] \times \cdots (-\infty, x_n]$.
- The Jacobian of g is denoted by $|J|$.

Then,

$$\begin{aligned} F_x(x) &= P(X \in A) \\ &= P(W \in g^{-1}(A)) \\ &= \int_{g^{-1}(A)} f_w(w)\mathrm{d}w \\ &= \int_A f_w(g^{-1}(x))|J|\mathrm{d}x, \end{aligned} \tag{3}$$

and we obtain the following probability density function.

$$f_x(x) = \frac{\partial}{\partial x_1 \partial x_2 \cdots \partial x_n} F_x(x) = f_w(g^{-1}(x))|J|. \tag{4}$$

In this problem, the Jacobian matrix is

$$J = \frac{\partial w}{\partial x} = \Delta_w^{-1}.$$

Therefore,

$$f_x(x) = f_w(\Delta_w^{-1}(x - b))|\Delta_w^{-1}|. \tag{5}$$