Stability of Predator-Prey Dynamics

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Lotka-Volterra Consumer-Resource Model

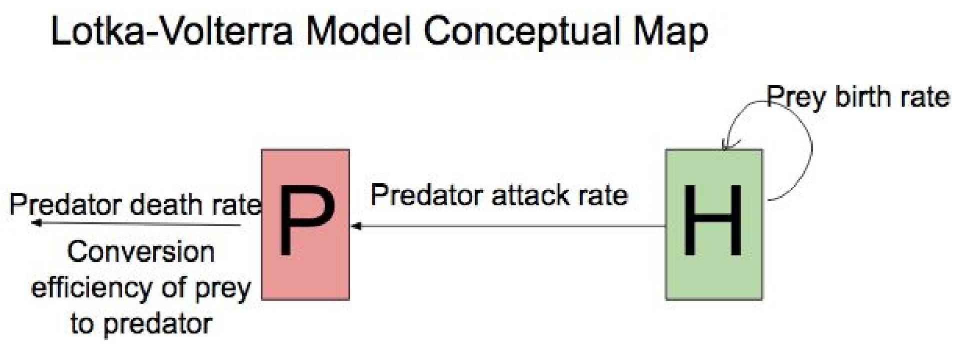


Figure 1: The conceptual map for the Lotka-Volterra Model. The prey birth rate affects prey population. The attack rates impact the conversion of energy from predator to prey, and the predator death rate and conversion efficiency have an impact on the loss of energy in the system.

The simulated model used the following differential equations: $dH/dt = bH-aPH$, $dP/dt=eaPH-sP$. The state variables were P and H. The original parameters for the model were the following: $b = 0.5$, $a = 0.02$, $e = 0.1$, $s = 0.2$, $H0 = 25$, $P0 = 5$. It was found that increasing the prey birth rate increases the size of both populations, and leads to sharper cyclical increases in population size.

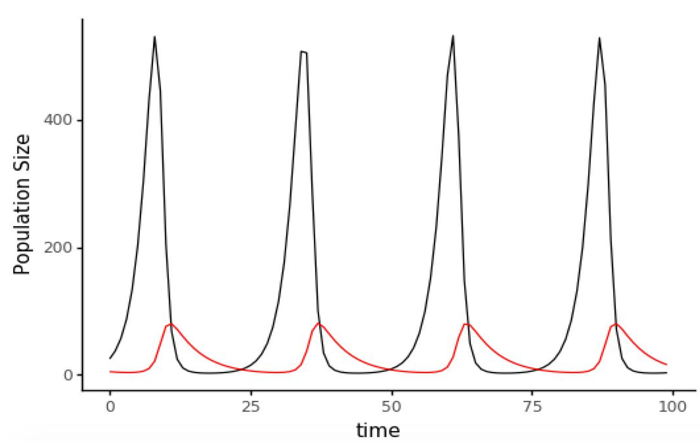


Figure 2: The simulated model for the original conditions of the Lotka-Volterra model. Prey population is represented by the black line and predator population is represented by the red line.

Prey birth rate (*b*):

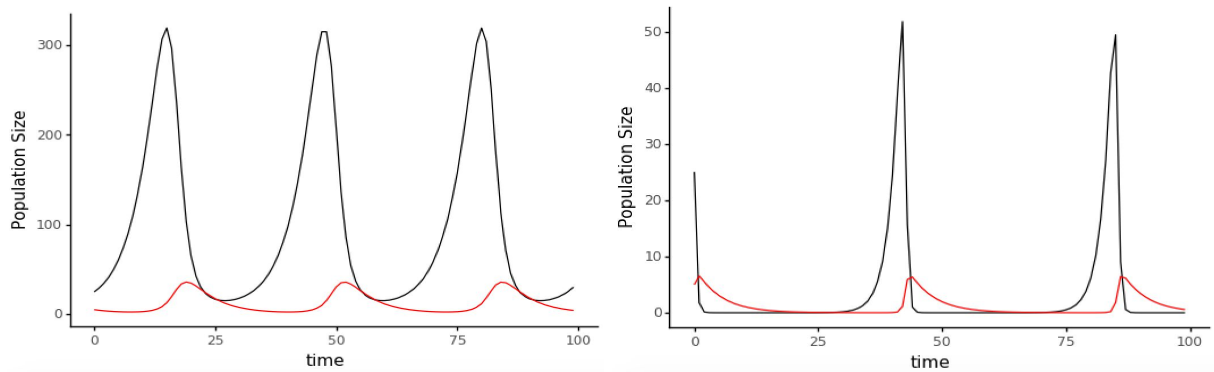


Figure 3: The two values of prey birth rate shown are 0.5 for the graph on the left and 2.0 for the graph on the right which are half of the initial condition and double the initial condition respectively. As we increase the prey birth rate, we can observe that the sharpness of the peaks and dips increases and the width of the oscillations decreases for both the prey and predator. Also, the peaks for both reach much higher population values and less predator-prey cycles are seen throughout the same time interval.

Predator attack rate (*a*):

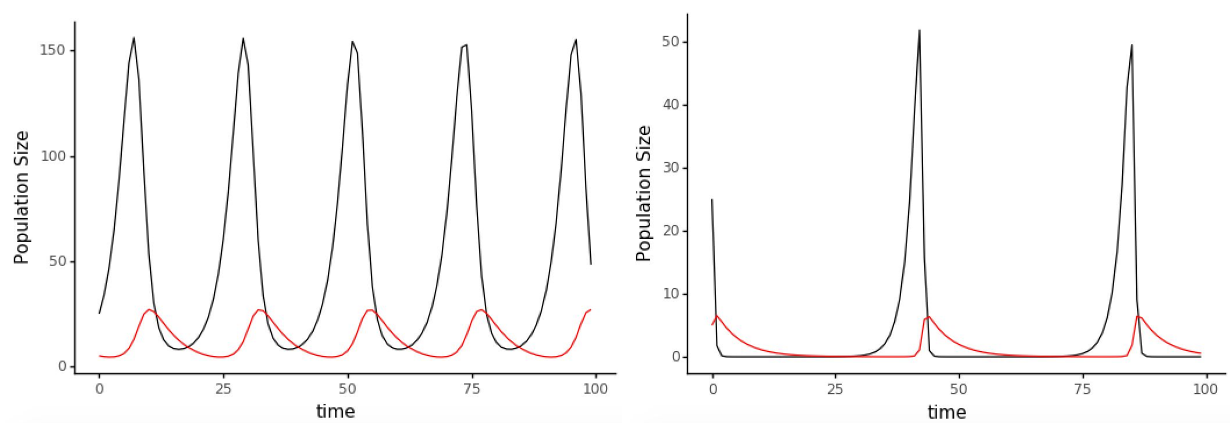


Figure 4: The conditions shown above reflect the original conditions, but the attack rate is 0.04 on the left and 0.5 on the right. It was found that an increase in predator attack rate decreases both population sizes and decreases the cycle time to a certain threshold but then causes longer cycles at extreme attack rates.

Conversion Efficiency (*e*):

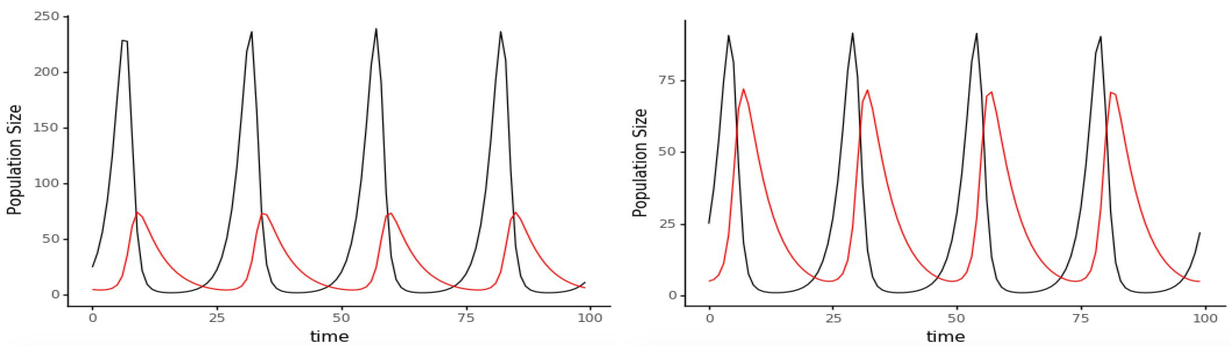


Figure 5: The conditions shown above reflect the original conditions, but the conversion efficiency of prey to predators is 0.2 on the left and 0.5 on the right. It was observed that increasing the conversion efficiency of prey to predators does not alter the cycle length, but predator population size increases and prey population size decreases.

Predator death rate (*s*):

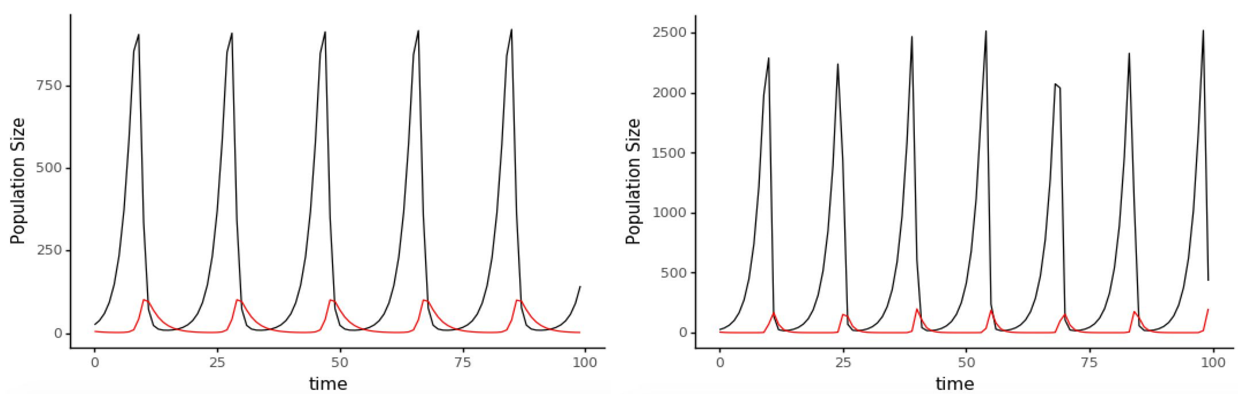


Figure 6: The conditions shown above reflect the original conditions, but the predator death rate is 0.4 on the left and 1.0 on the right. It was observed that increasing the predator death rate causes a decrease in cycle length and predator population size, and an increase in prey population size.

What can you say about the role of predators in the simulation?

In this model, the predator serves to limit the population of the herbivores, and the more efficient, aggressive, and selectively fit they are, the smaller the herbivore population will be.

Rosenzweig-MacArthur Model

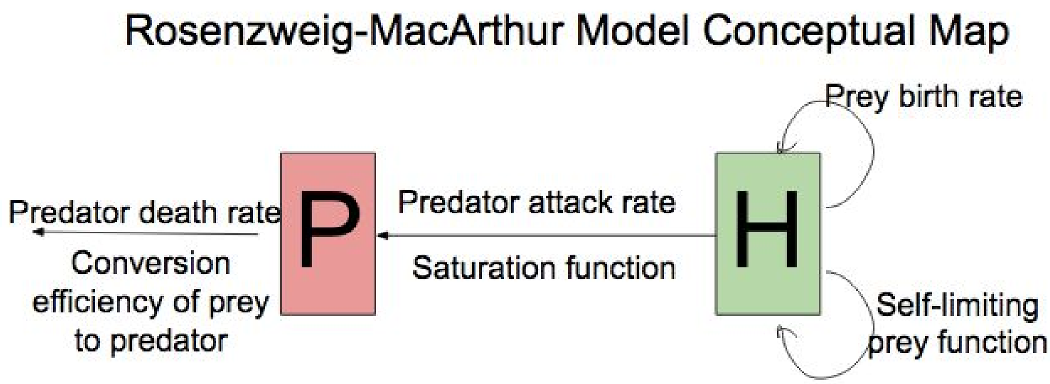


Figure 7: The conceptual map for the Rosenzweig-MacArthur Model. The prey birth rate affects prey population. The attack rate, saturation function, and carrying capacity have an impact on the conversion of energy from predator to prey, and the predator death rate and conversion efficiency have an impact on the loss of energy in the system.

The simulated model used the following differential equations: $dH/dt = bH(1-\alpha H) - \omega HP/(d+H)$, $dP/dt = e\omega HP/(d+H) - sP$. The state variables were P and H. The original parameters for the model were the following: $b = 0.8$, $e = 0.07$, $s = 0.2$, $\omega=5$, $d=400$, $\alpha=.001$, $H0 = 500$, $P0 = 120$.

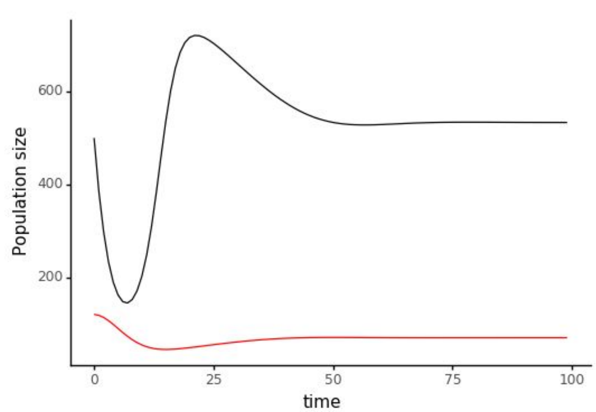


Figure 8: The simulated model for the original conditions of the Rosenzweig-MacArthur model. Prey population is represented by the black line and predator population is represented by the red line.

Prey birth rate (*b*):

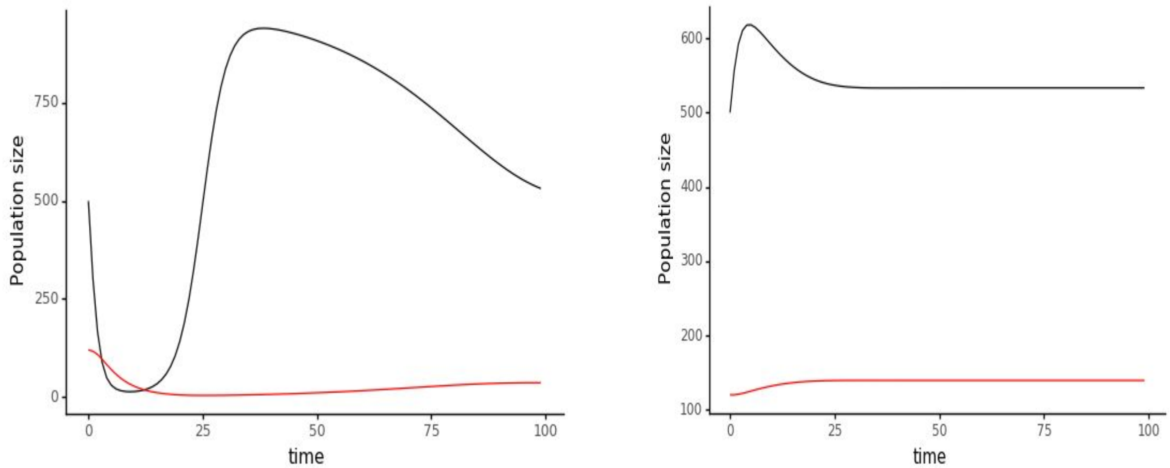


Figure 9: The two values of prey birth rate shown are 0.4 for the graph on the left and 1.6 for the graph on the right which are half of the initial condition and double the initial condition respectively. No cycles of decline and recovery are observed but the maxima of the prey and the predator populations increases as the prey birth rate increases.

Conversion efficiency (*e*):

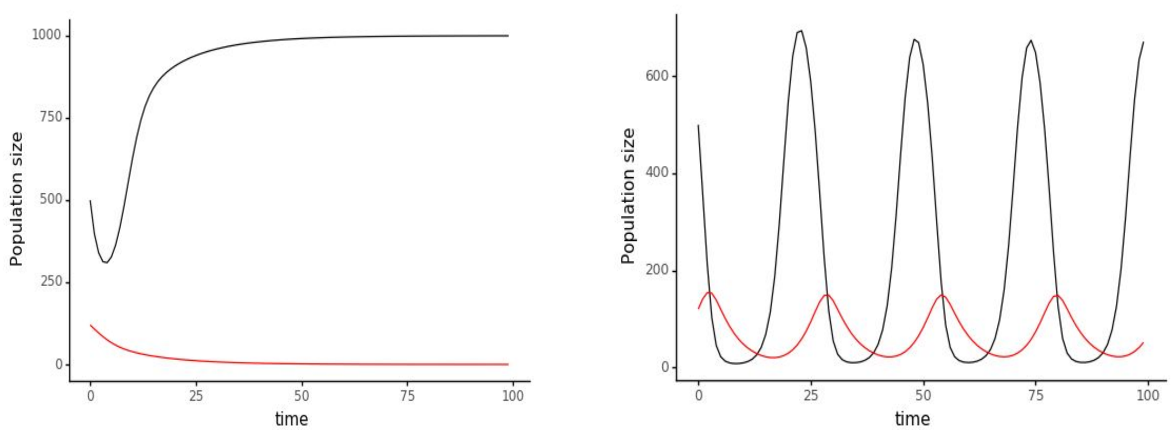


Figure 10: The two values of conversion efficiency shown are 0.035 for the graph on the left and 0.14 for the graph on the right which are half of the initial condition and double the initial condition respectively. Increase in conversion efficiency leads to an increase in predator population and a slight decrease in resource population, but the cycles proceed at the same interval. Also, a small enough conversion efficiency will lead to extinction of the predator.

Predator Death Rate (s)

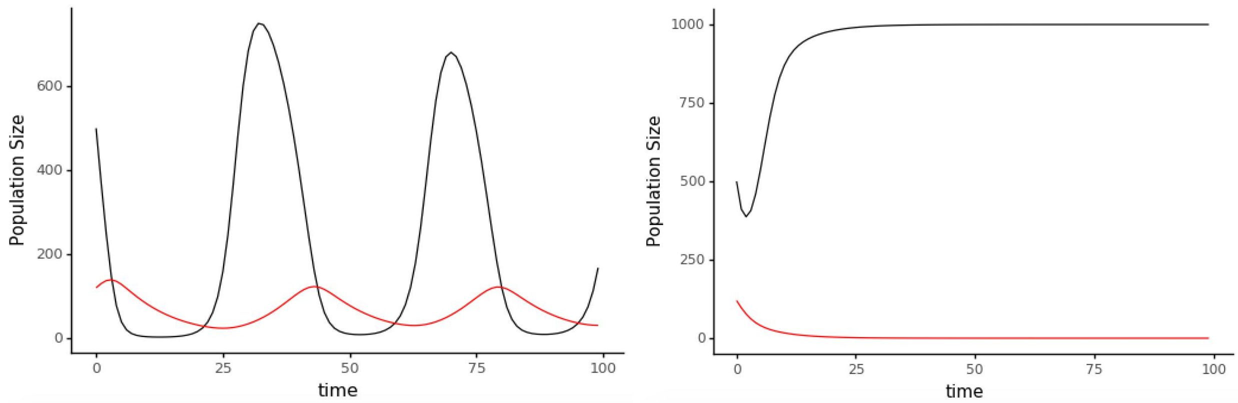


Figure 11: The two values of predator death rates shown are 0.1 for the graph on the left and 0.4 for the graph on the right which are half of the initial condition and double the initial condition respectively. It is observed that as the predator death rate increases, the prey population reaches much higher values while the predator population reaches much lower values. Also, the cycles of decline and recovery do not appear once the the death rate gets high enough as the prey and predator populations quickly stabilize around a certain value.

Self-limiting Prey (*w*)

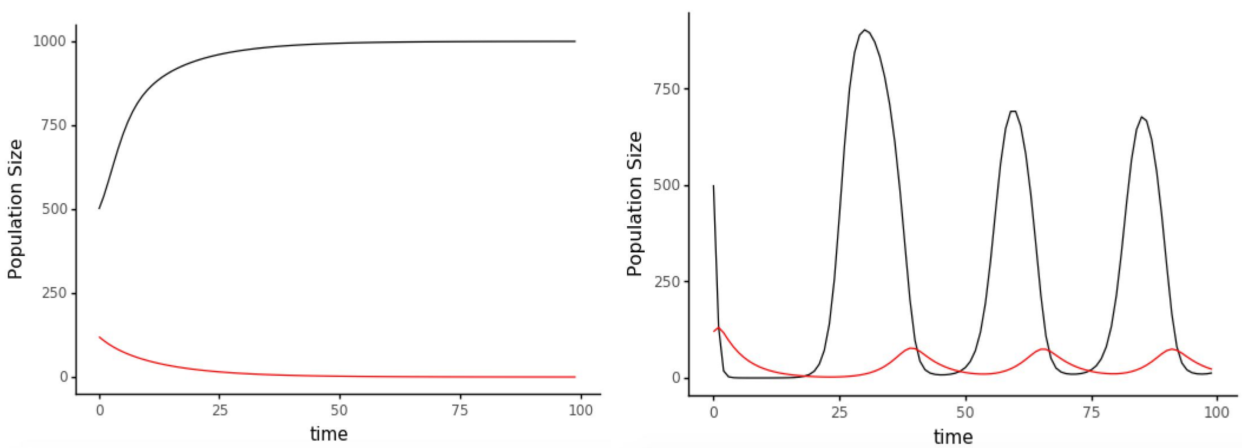


Figure 12: The two values of self-limiting prey shown are 2.5 for the graph on the left and 10 for the graph on the right which are half of the initial condition and double the initial condition respectively. One observation that can be made is that increasing the self-limiting nature of the prey causes cycles of decline and recovery to emerge for both the prey and predator populations. The peaks remain the same size for both prey and predator but they occur less often as the value

of this parameter is further increased. Another observation is that peaks are fairly consistent with regards to the highest population value the prey and predator reach.

Saturation Effect (*d*)

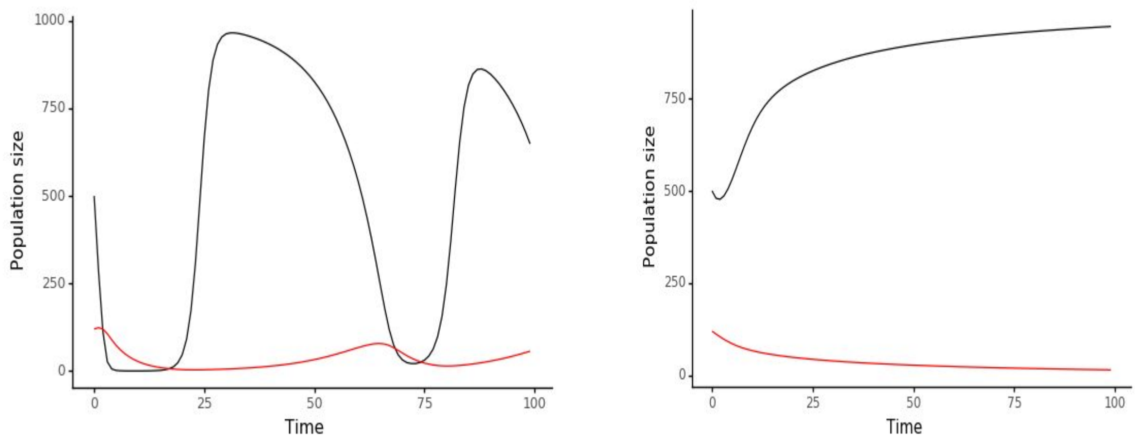


Figure 13: The two values for saturation effect are 200 for the graph on the left and 800 for the graph on the right which are half of the initial condition and double the initial condition respectively. When the saturation effect increases , the prey population becomes more damp as it reaches a final value. Both the prey and predator population also begin to oscillate but the predator population is only slightly oscillating. When the saturation effect is decreased, the prey population is able to reach higher population values.

Inverse carrying capacity (α)

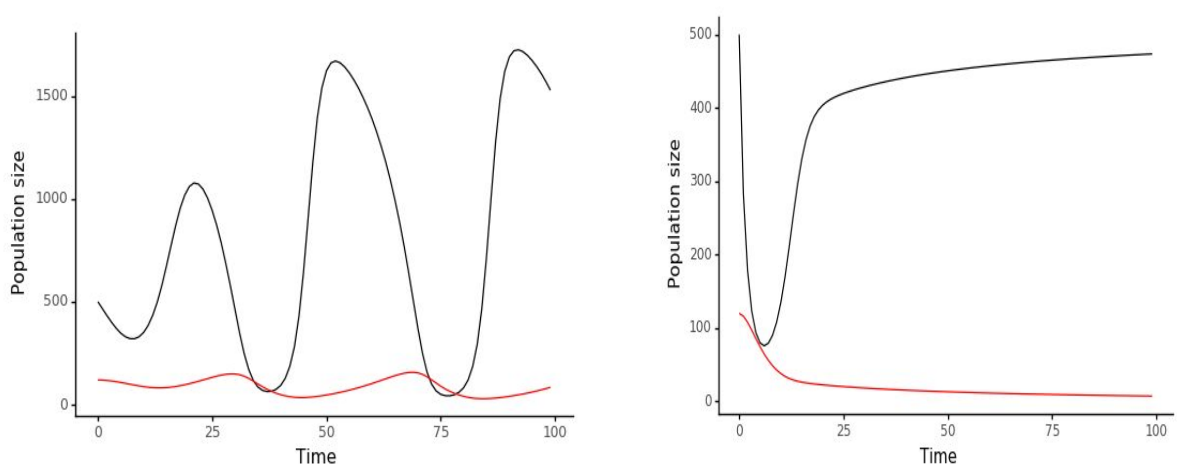


Figure 14: The two values for inverse carrying capacity are 0.0005 for the graph on the left and 0.002 for the graph on the right which are half of the initial condition and double the initial condition respectively. When the inverse carrying capacity decreases, the prey and predator populations begin to oscillate with the predator population being slightly less oscillatory. Also, the prey population attains higher population values. As the inverse carrying capacity increases, the prey population obtains a steeper fall in the beginning as well as a lower final value and the predator population has no noticeable effect.

How do the dynamics differ from Lotka-Volterra?

The dynamics of the Rosenzweig-MacArthur model differs from the Lotka-Volterra model due to additional parameters included. The R-M model contains a carrying capacity which must be upheld by the prey population. This is best seen by looking at the differential equation for the prey population from each of the models. The L-V model has the equation $dH/dt = bH - aPH$, which shows that if there is no predator population, the prey population would exponentially increase without limit. This is vastly different in the R-M model as the equation is $dH/dt = bH(1-\alpha H) - \omega HP/(d+H)$ which limits the prey population by the inverse carrying capacity α. In addition the predator-prey interaction dynamics is different between the models. In the L-V model equations shows that the predators decrease the prey in larger numbers as the prey population increases. The R-M model serves as a contrast as the R-M model removes this issue by having the interaction become $\omega HP/(d+H)$ instead which includes a self limiting prey factor and saturation parameter.

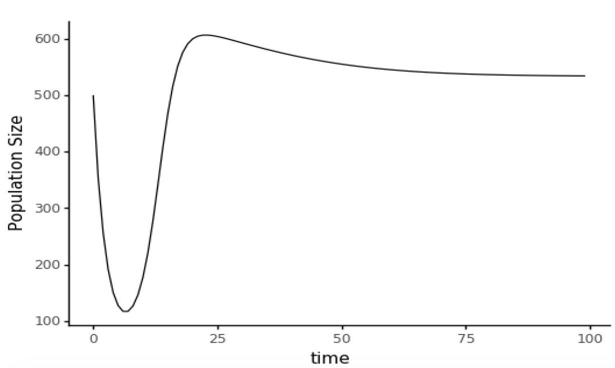
What can you say about the "role" of each parameter, especially what causes the dynamics to differ between the L-V and R-M models?

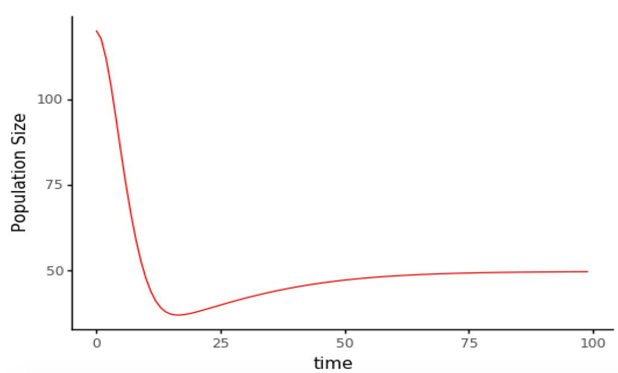
An increase in the prey birth rate increases the maxima for the prey by a significant amount while only increasing the predator population by a vastly smaller margin. This is shown in Figure 9 and serves as a contrast to the L-V model which shows both population having a significant change in numbers. This is expected from the dH/dt equation as for the R-M model the population of the predators is less sensitive to changes in the prey population. An increase in conversion efficiency of prey to predator results in oscillatory changes in both populations. The dynamics differ from the L-V model due to the R-M model not having as high of an amplitude in the oscillations for the predator population as seen in Figure 10 which may be attributed to not having predator population be so closely dependent in the prey population like in the L-V model. An increase in predator death results in the decrease of the predator population along with an increase in the prey population as seen in Figure 11. This is similar to the L-V model expect without the oscillations due to the two populations not being as closely related to each other before as well as the R-M model having the carrying capacity have a significant role in determining the population of the prey that was absent previously. An increase in the self-limiting parameter results in increased oscillation for the both population without a noticeable increase in highest magnitude achieved which is seen in Figure 12. This differs from the L-V model as it does not take this variable into account and assumes that the predator interacts with the prey solely by the values of the population. A decrease in the saturation effect parameter results in increased oscillation in the populations with more noticable changes to the prey than to the predator as seen in Figure 13. This role differs as once again the L-V model does not consider this parameter in its calculations. A decrease in the inverse carrying capacity reduces oscillations in populations as well as making the prey population go to a constant maximum value as seen in Figure 14. This is different from the L-V model as in theory there is no limit to the prey population as a carrying capacity is not specified rather it uses the predator population for limiting.

What is the relationship between parameter values and predator abundance?

The predator population increases with a decrease in the inverse carrying capacity, saturation effect, and predator death rate. The predator population increases with an increase conversion efficiency of prey to predator and self-limiting prey. There is no significant change in predator abundance as there is a change in the prey birth rate as values stay in a similar range.

Paradox of Enrichment





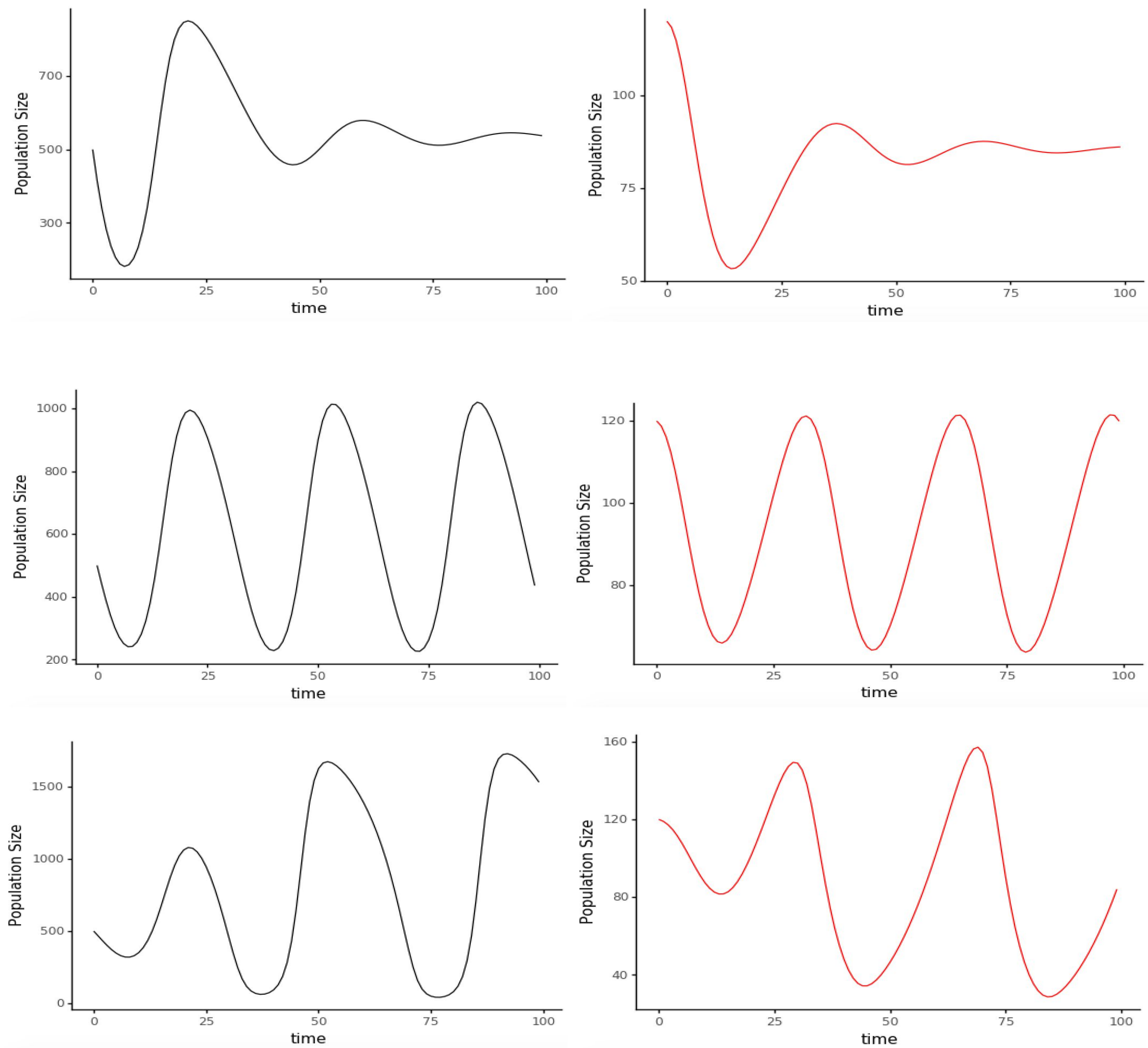


Figure 15: Due to the large difference in scale between the prey and predator populations, the behavior of the predators was best seen by plotting its curve separately. The prey curves are on the left side of the page in black and the predator curves are on the right in red.

What happens as carrying capacity increases?

From top to bottom, the alpha values tested were 0.00125, 0.0008, 0.000625, and 0.0005 which is representative of a change in carrying capacity from 800 to 2,000. As the prey carrying capacity increases, there is an observable change in the magnitude of the relative minima of the predator and the relative maxima of the prey. As expected, the prey population peaks at larger population values as the prey carrying capacity is increased considering this is an indication of greater resource availability. The predator curves behave in an unusual way considering the increase in prey carrying capacity causes there to be larger collapses in the predator population.

Another thing that should be noted is that for lower prey carrying capacity values, the prey and predator population remained steady after the peak or collapse but as the carrying capacity increases a cyclical pattern emerged of decline and recovery.

Why do you think we see the Paradox of Enrichment?

The reason why we see this Paradox of Enrichment is because the predator population becomes unsustainably large due to the fact that there is so much prey to eat. It would appear that having so much food would be a great thing for predators, but the reality is that predators require many other resources to live besides just prey. Once the predators eat enough prey to reach a high enough population, conditions in the predator's natural habitat will become so congested that there is a major depletion in other critical resources such as water, shelter, and a safe place to raise the young. The result is a crash in the population as it becomes destabilized and this could possibly lead to local eradication or complete extinction of the species if the population does not recover quickly enough. It is important to note that the prey follow a similar pattern of collapse as when the predator population becomes so low, they also find themselves in a state of congestion and quickly deplete critical resources.