Pongpak Techagumthorn Phys 224: Thermal Physics

HW 2

1.22 (a)

$$\overline{P} = \frac{\overline{F_{x,piston}}}{A} = -\frac{\overline{F_{x,molecule}}}{A} = -\frac{m\left(\frac{\overline{\Delta v_x}}{\Delta t}\right)}{A}$$

$$\overline{P} = \frac{P}{N}$$

$$P = -\frac{Nm\left(\frac{\overline{\Delta v_x}}{\Delta t}\right)}{A} = -\frac{Nm\overline{v_x}}{A\Delta t}$$

$$PA\Delta t = -Nm\overline{\Delta v_x}$$

$$\frac{PA\Delta t}{m\overline{\Delta v_x}} = -N$$

$$\overline{\Delta v_x} = \overline{v_f} - \overline{v_i} = -\overline{v_x} - \overline{v_x} = 2\overline{v_x}$$

$$\frac{PA\Delta t}{m\left(-2\overline{v_x}\right)} = -N$$

$$\frac{PA\Delta t}{2m\overline{v_x}} = N$$

(b)

$$\frac{PA\Delta t}{2m\overline{v_x}} = N$$

$$\frac{P}{N} = \frac{2m\overline{v_x}}{A\Delta t}$$

$$\Delta t = \frac{2L}{\overline{v_x}}$$

$$\frac{P}{N} = \frac{2m\overline{v_x}}{A\frac{2L}{\overline{v_x}}}$$

$$= \frac{m\overline{v_x^2}}{V}$$

$$\frac{PV}{N} = m\overline{v_x^2}$$

$$kT = \frac{PV}{N} = m\overline{v_x^2}$$

$$kT = m\overline{v_x^2}$$

$$\frac{kT}{m} = \overline{v_x^2}$$

$$\sqrt{\frac{kT}{m}} = \sqrt{\overline{v_x^2}} = \left(\overline{v_x^2}\right)^{\frac{1}{2}}$$

(c)

$$\frac{dN}{dt} = -\frac{AN}{2V}\sqrt{\frac{kT}{m}}$$
$$\frac{dN}{N} = -\frac{A}{2V}\sqrt{\frac{kT}{m}}dt$$
$$\int \frac{1}{N}dN = -\frac{A}{2V}\sqrt{\frac{kT}{m}}\int dt$$
$$\ln(N) = -\frac{A}{2V}\sqrt{\frac{kT}{m}}t + C$$
$$e^{ln(N)} = e^C e^{\left(-\frac{A}{2V}\sqrt{\frac{kT}{m}}t\right)}$$
$$N = e^C e^{\left(-\frac{A}{2V}\sqrt{\frac{kT}{m}}t\right)}$$

$$\frac{1}{\tau} = \frac{A}{2V}\sqrt{\frac{kT}{m}}$$
$$N = N_0 e^{-\frac{t}{\tau}}$$

(d)

$$\tau = \frac{2V}{A}\sqrt{\frac{m}{kT}}$$

Room Temp = 298° K, V = 1L = 0.001m^3 = $1 \times 10^{-3}\,\text{m}^3$, A = 1mm^2 = $1 \times 10^{-6}\,\text{m}^2$
Air = 78% N_2 + 21% O_2 + 1% Ar
$0.78 \times 2 \times 14.00674 + 0.21 \times 2 \times 15.9994 + 0.01 \times 39.948 = 28.97\text{g}\,\text{mol}^{-1}$
$m_{air} = 28.97 \times 10^{-3}/N_a = 4.81 \times 10^{-26}\text{kg}$

$$\tau = \frac{2\left(1 \times 10^{-3}\right)}{1 \times 10^{-6}}\sqrt{\frac{4.81 \times 10^{-26}}{\left(1.38 \times 10^{-23}\right)(298)}} = 6.32$$

$$\tau = 6.32\text{s}$$

(e) Lunar Command Module (LCM) Volume = 6.2m^3
LCM Hatch Size = $0.74\text{m} \times 0.86\text{m} = 0.63\text{m}^2$
LCM Air = 100% O_2
$m_{O_2} = 2 \times 15.9994/N_a = 5.31 \times 10^{-23}\text{g} = 5.31 \times 10^{-26}\text{kg}$

$$\tau = \frac{6.2}{6.3}\sqrt{\frac{5.31 \times 10^{-26}}{\left(1.38 \times 10^{-23}\right)(298)}} = 0.035$$

$$\tau = 0.035\text{s}$$

1.24 Degree of freedom $f = 6$, Temperature = 298° K

$$U_{therm} = \frac{f}{2}NkT$$

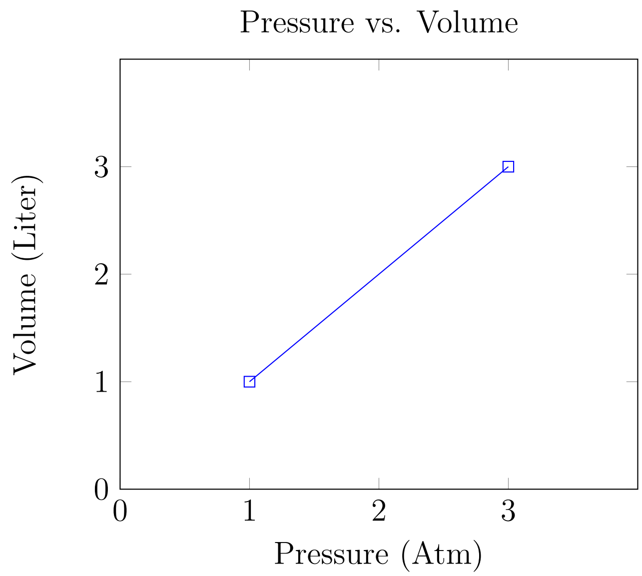
$$N_{Pb} = \frac{1}{207.2} \times 6.022 \times 10^{23} = 2.906 \times 10^{21}$$

$$\begin{aligned} U_{therm} &= \frac{f}{2}NkT \\ &= \left(\frac{6}{2}\right)\left(2.906 \times 10^{21}\right)\left(1.381 \times 10^{-23}\right)(298) \\ &= 35.878\text{J} \end{aligned}$$

1.27 (a) Exothermic chemical reactions such as a thermite reaction.

(b) Phase transitions and latent heat.

1.31 (a) Graph:



(b)

$$W = -\int_{v_i}^{V_f} P(V)dV$$

$$P(V) = \frac{2\text{atm}}{2\text{L}}V = \frac{101.3 \times 10^3\text{Pa}}{0.001\text{m}^3}V = 101.3 \times 10^6 V$$

$$\begin{aligned}
W &= -\int_{1\times10^{-3}}^{3\times10^{-3}} 101.3\times10^{6} V dV \\
&= -101.3\times10^{6}\int_{0.001}^{0.003} V dV \\
&= -101.3\times10^{6}\left[\frac{V^2}{2}\right]_{0.001}^{0.003} \\
&= -101.3\times10^{6}\left[\frac{0.003^2}{2}-\frac{0.001^2}{2}\right] \\
&= -405.2\text{J}
\end{aligned}$$

(c)

$$\Delta U = \frac{f}{2}NkT = \frac{f}{2}PV$$

$$\begin{aligned}
\Delta U &= \frac{f}{2}\Delta(PV) \\
&= \frac{f}{2}\left(P_f V_f - P_i V_i\right)
\end{aligned}$$

Degree of freedom $f = 3$ for Helium at room temperature

$$\begin{aligned}
\Delta U &= \frac{3}{2}\left(\left(303.9\times10^{3}\right)(0.003)-\left(101.3\times10^{3}\right)(0.001)\right) \\
&= \frac{3}{2}(810.4) \\
\Delta U &= 1215.6\text{J}
\end{aligned}$$

(d)

$$Q = \Delta U - W = 1215.6 - -405.2 = 1621\text{J}$$

(e) Add a lot of heat in a short amount of time, such as an explosion.