HW 1

1.8 (a) Assumptions: Cold day = 0° C, Hot Day = 37° C

$$\alpha = \frac{\Delta L/L}{\Delta T}$$

$$\alpha \Delta TL = \Delta L$$

$$(1.1 \times 10^{-5})(37)(1000) = 0.407$$

Total expansion for a 1km steel beam is 0.407m or 40.7cm

(b) The two metals that are laminated together have different coefficients of thermal expansion. This causes the laminated strip to curl and uncurl due to the metals expanding and contracting at slightly different rates.

(c)
$$\alpha \equiv \frac{\Delta L/L}{\Delta T} \quad \beta \equiv \frac{\Delta V/V}{\Delta T}$$

$$V + \Delta V = (L + \Delta L)^{3}$$

$$= L^{3} + 3L^{2}\Delta L + 3L\Delta L^{2} + \Delta L^{3}$$

$$V + \Delta V \approx L^{3} + 3L^{2}\Delta L^{*}$$

$$= V + 3V\frac{\Delta L}{L}$$

$$\Delta V = 3V\frac{\Delta L}{L}$$

$$\frac{\Delta V}{V} = 3\frac{\Delta L}{L}$$

$$\beta = \frac{\Delta V/V}{\Delta T} = \frac{3\frac{\Delta L}{L}}{\Delta T} = 3\left(\frac{\Delta L}{\Delta T}\right) = 3\alpha$$

1.10 Assumptions: Room = 10 m × 10 m × 3 m, Pressure = 1 atm = 101.3 ×10³ Pa, Temperature = 293°K

$$PV = NkT$$

$$\frac{PV}{kT} = N$$

$$\frac{(101.3 \times 10^3)(300)}{(1.38 \times 10^{-23})(293)} = 7.516 \times 10^{27}$$

 $7.516~\times 10^{27}$ molecules of air in average room.

^{*} For small ΔL , $\Delta L^2 \approx 0$ and $\Delta L^3 \approx 0$

- 1.11 Room B will have a greater mass of air. Between the two rooms the pressure and volume are constant and when we look at the Ideal Gas Law: PV = NkT when P and V are constant a decrease in T will increase N, the number of air molecule, so there will be a higher mass in the room with a lower temperature.
- 1.13 Water (H_2O) : $(1.00794 \times 2) + 15.9994 = 18.0153g$ Nitrogen (N_2) : $14.00674 \times 2 = 28.01348g$

Lead (Pb): 207.2g

Quartz (SiO_2) : $28.0855 + (15.9994 \times 2) = 60.0843g$

1.16 (a)

$$F = PA \quad \rho V = m$$

$$F = -mg$$

$$PA = -\rho Vg$$

$$PA = -\rho Azg$$

$$P = -\rho zg$$

$$\Delta P = -\rho g\Delta z$$

$$dP = -\rho gdz$$

$$\frac{dP}{dz} = -\rho g$$

(b)
$$\rho = N \frac{m}{V} \Rightarrow \rho = \frac{Pm}{kT}$$

$$\frac{dP}{dz} = -\rho g \Rightarrow \frac{dP}{dz} = -\frac{Pmg}{kT}$$

The negative coefficient in the equation supports the fact that pressure decreases as the altitude increases.

(c)
$$\frac{dP}{dz} = -\frac{Pmg}{kT} \Rightarrow \frac{dP}{P} = -\frac{mg}{kT}dz$$

$$\int \frac{dP}{P} = \int -\frac{mg}{kT}dz$$

$$\ln(P) = -\left(\frac{mg}{kT}z + C_1\right)$$

$$e^{\ln(P)} = e^{-\frac{mg}{kT}z}C_2$$

$$P = P_0e^{-\frac{mg}{kT}z}$$

The negative exponent in the equation, similar to the previous part, aligns with decreasing pressure, and thus density, as altitude increases.

 $1.19 \ H_2 \ \text{mass} \ m_H = 2.01588 \ \text{g/mol}, \ O_2 \ \text{mass} \ m_O = 31.9988 \ \text{g/mol}$

$$v_{rms} \equiv \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}$$

Gasses are in thermal equilibrium so T is constant.

$$C_1 = \sqrt{3kT} \quad v_{rms} = C_1 \sqrt{\frac{1}{m}}$$

$$H_2 : \sqrt{\frac{1}{m_H}} = \sqrt{\frac{1}{2.01588}} \quad O_2 : \sqrt{\frac{1}{m_O}} = \sqrt{\frac{1}{31.9988}}$$

 v_{rms} for H_2 is faster since the average velocity is inversely proportional to the mass of the particles.

The factor factor for the average velocity of H_2 to O_2 is:

$$\frac{\sqrt{1/m_H}}{\sqrt{1/m_O}} = \frac{\sqrt{1/2.01588}}{\sqrt{1/31.9988}}$$
$$= \sqrt{\frac{31.9988}{2.01588}} \approx 3.989$$

1.25 Water (H_2O) has a quadratic degree of freedom of 12

- 3 translational in (x, y, z)
- 3 rotational in roll, pitch, and yaw
- 6 vibrational, 2 each (1 potential and 1 kinetic) for the following vibrational modes:
 - Symmetric stretching
 - Asymmetric stretching
 - Bending