ASSIGNMENT 3

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1 QUESTION No 2.58

Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of 60°

2 SOLUTION

Data from the given question :

	Symbols	Circle
Centre	$\mathbf{O}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
Radius	r	5

Let PA and PB be tangents to circle with radius 5 cm which are inclined to each other at an angle 60°. We know a tangent is always perpendicular to the radius .

$$\therefore OA \perp AP \qquad (2.0.1)$$

We know that, line joining the centre and the external point bisect the angle between pair of tangents from that external point.
$\angle APB = 60°$
In $\triangle OAP$,

$$\sin 30° = \frac{OA}{OP} \qquad (2.0.2)$$

$$\frac{1}{2} = \frac{5}{OP} \qquad (2.0.3)$$

$$\implies OP = 10 \qquad (2.0.4)$$

$$\therefore \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \qquad (2.0.5)$$

$$(\mathbf{O} - \mathbf{A})^T(\mathbf{A} - \mathbf{P}) = 0 \quad (\because OA \perp AP) \qquad (2.0.6)$$

$$\mathbf{A}^T(\mathbf{A} - \mathbf{P}) = 0 \quad \left(\because \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) \qquad (2.0.7)$$

$$\mathbf{A}^T\mathbf{A} - \mathbf{A}^T\mathbf{P} = 0 \qquad (2.0.8)$$

$$\|\mathbf{A}\|^2 = \mathbf{A}^T\mathbf{P} \qquad (2.0.9)$$

$$\|\mathbf{A}\|^2 = \mathbf{P}^T\mathbf{A} \quad \left(\because \mathbf{A}^T\mathbf{P} = \mathbf{P}^T\mathbf{A} \right) \qquad (2.0.10)$$

$$\mathbf{P}^T\mathbf{A} = 25 \quad \left(\because \|\mathbf{A}\|^2 = 25 \right) \qquad (2.0.11)$$

$$\begin{pmatrix} 10 & 0 \end{pmatrix}\mathbf{A} = 25 \quad \left(\because \mathbf{P} = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \right) \qquad (2.0.12)$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{A} = \frac{5}{2} \qquad (2.0.13)$$

$$\mathbf{A} = \begin{pmatrix} \frac{5}{2} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (2.0.14)$$

$$\mathbf{A} = \mathbf{a} + \lambda\mathbf{m} \qquad (2.0.15)$$

$$\mathbf{a} = \begin{pmatrix} \frac{5}{2} \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (2.0.16)$$

We know,

$$\|\mathbf{a} + \lambda\mathbf{m}\|^2 = 25 \qquad (2.0.17)$$

$$(\mathbf{a} + \lambda\mathbf{m})^T(\mathbf{a} + \lambda\mathbf{m}) = r^2 \qquad (2.0.18)$$

$$\lambda^2 = \frac{r^2 - \|\mathbf{a}\|^2}{\|\mathbf{m}\|^2} \qquad (2.0.19)$$

$$\lambda = \pm 4.33 \qquad (2.0.20)$$

Substitute λ value in (2.0.14) we get

$$\mathbf{A} = \begin{pmatrix} \frac{5}{2} \\ 4.33 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \frac{5}{2} \\ -4.33 \end{pmatrix} \qquad (2.0.21)$$
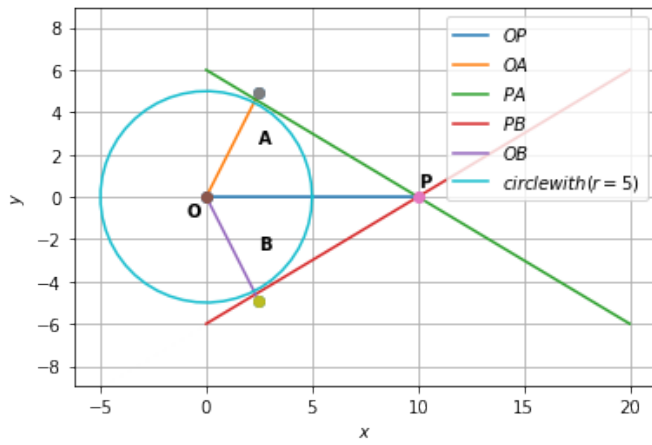
Plot of Tangents PA and PB :

Fig. 2.1: Tangent lines to circle of radius 5 units.