

# ASSIGNMENT 3

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Download all python codes from

<https://github.com/ponnaboinakalpana12/ASSIGNMENT3>

and latex-tikz codes from

<https://github.com/ponnaboinakalpana12/ASSIGNMENT3>

## 1 QUESTION No 2.58

Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of  $60^\circ$

## 2 SOLUTION

Data from the given question :

	Symbols	Circle
Centre	<b>O</b>	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
Radius	$r$	5

The angle between the tangents from P given by  $\theta = 60^\circ$

**Lemma 2.1.** Given a circle of radius  $r$  and angle between the tangents, the intersection of the tangents and points of contact are :

$$P = de_1$$

$$\text{where, } e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

to the circle given by

$$x = \frac{r^2}{d}e_1 \pm r\sqrt{1 - \frac{r^2}{d^2}}e_2 \quad (2.0.1)$$

If  $x$  be a point of contact for tangents

$$PA \perp PB \quad (2.0.2)$$

$$\Rightarrow (O - x)^T(x - P) = 0 \quad (2.0.3)$$

$$\text{or, } P^T x \Rightarrow \|x\|^2 = r^2 \quad (2.0.4)$$

$$\Rightarrow e_1^T x = \frac{r^2}{d} \quad (2.0.5)$$

$$\text{where, } d = \frac{r}{\sin \frac{\theta}{2}} \quad (2.0.6)$$

Proof: From (2.0.6)

$$\sin \frac{\theta}{2} = \frac{r}{d} \quad (2.0.7)$$

Now substitute the values , we get

$$d = \frac{r}{\sin \frac{\theta}{2}} \quad (2.0.8)$$

$$\Rightarrow d = \frac{5}{\sin 30^\circ} \quad (2.0.9)$$

$$\Rightarrow d = 10 \quad (2.0.10)$$

and from (2.0.1)

$$x = \frac{r^2}{d} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \pm r\sqrt{1 - \frac{r^2}{d^2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.11)$$

$$= \frac{25}{10} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \pm 5\sqrt{1 - \frac{25}{100}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.12)$$

$$x = 2.5 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \pm 4.33 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow \mathbf{A} = \begin{pmatrix} 2.5 \\ 4.33 \end{pmatrix} \quad (2.0.14)$$

$$\Rightarrow \mathbf{B} = \begin{pmatrix} 2.5 \\ -4.33 \end{pmatrix} \quad (2.0.15)$$

from (2.1)

$$P = 10 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.16)$$

$$\Rightarrow P = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \quad (2.0.17)$$

The coordinates are:

$$\therefore \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 2.5 \\ 4.33 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2.5 \\ -4.33 \end{pmatrix}.$$

(2.0.18)

Plot Tangents PA and PB :

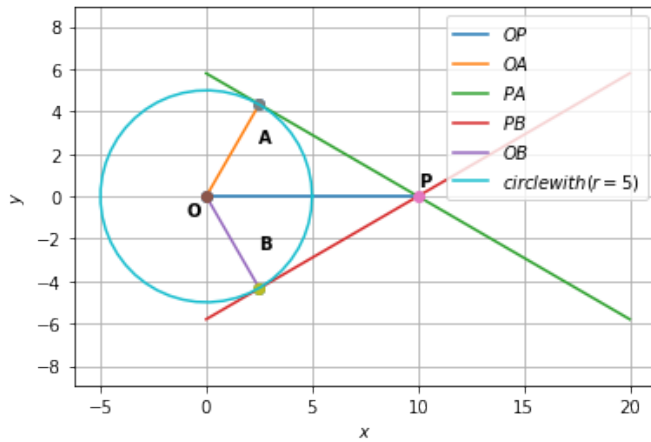


Fig. 2.1: Tangent lines to circle of radius 5 units.