ASSIGNMENT 3

P.Kalpana

Download all python codes from

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https://github.com/ponnaboinakalpana12/
    ASSIGNMENT3
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and latex-tikz codes from

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1 Question No 2.58

Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of 60°

2 Solution

Data from the given question :

	Symbols	Circle
Centre	$\mathbf{O}$	$\begin{pmatrix}0\\0\end{pmatrix}$
Radius	r	5

Let PA and PB be tangents to circle with radius 5 cm which are inclined to each other at an angle 60°. We know a tangent is always perpendicular to the radius .

$$\therefore OA \perp AP \tag{2.0.1}$$

We know that, line joining the centre and the external point bisect the angle between pair of tangents from that external point.
$\angle APB = 60°$
In $\triangle OAP$,

$$\sin 30° = \frac{OA}{OP} \tag{2.0.2}$$

$$\frac{1}{2} = \frac{5}{OP} \tag{2.0.3}$$

$$\implies OP = 10 \tag{2.0.4}$$

$$\therefore \mathbf{O} = \begin{pmatrix}0\\0\end{pmatrix}, \mathbf{P} = \begin{pmatrix}10\\0\end{pmatrix} \tag{2.0.5}$$

$$(\mathbf{O}-\mathbf{A})^T(\mathbf{A}-\mathbf{P}) = 0 \quad (\because OA \perp AP) \tag{2.0.6}$$

$$\mathbf{A}^T(\mathbf{A}-\mathbf{P}) = 0 \quad \left(\because \mathbf{O} = \begin{pmatrix}0\\0\end{pmatrix}\right) \tag{2.0.7}$$

$$\mathbf{A}^T\mathbf{A} - \mathbf{A}^T\mathbf{P} = 0 \tag{2.0.8}$$

$$\|\mathbf{A}\|^2 = \mathbf{A}^T\mathbf{P} \tag{2.0.9}$$

$$\|\mathbf{A}\|^2 = \mathbf{P}^T\mathbf{A} \quad \left(\because \mathbf{A}^T\mathbf{P} = \mathbf{P}^T\mathbf{A}\right) \tag{2.0.10}$$

$$\mathbf{P}^T\mathbf{A} = 25 \quad \left(\because \|\mathbf{A}\|^2 = 25\right) \tag{2.0.11}$$

$$\begin{pmatrix}10 & 0\end{pmatrix}\mathbf{A} = 25 \quad \left(\because \mathbf{P} = \begin{pmatrix}10\\0\end{pmatrix}\right) \tag{2.0.12}$$

$$\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{A} = \frac{5}{2} \tag{2.0.13}$$

$$\mathbf{A} = \begin{pmatrix}\frac{5}{2}\\0\end{pmatrix} + \lambda\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.14}$$

$$\mathbf{A} = \mathbf{a} + \lambda\mathbf{m} \tag{2.0.15}$$

$$\mathbf{a} = \begin{pmatrix}\frac{5}{2}\\0\end{pmatrix}, \mathbf{m} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.16}$$

We know,

$$\|\mathbf{a} + \lambda\mathbf{m}\|^2 = 25 \tag{2.0.17}$$

$$(\mathbf{a} + \lambda\mathbf{m})^T(\mathbf{a} + \lambda\mathbf{m}) = r^2 \tag{2.0.18}$$

$$\lambda^2 = \frac{r^2 - \|\mathbf{a}\|^2}{\|\mathbf{m}\|^2} \tag{2.0.19}$$

$$\lambda = \pm 4.33 \tag{2.0.20}$$

Substitute λ value in (2.0.14) we get

$$\mathbf{A} = \begin{pmatrix}\frac{5}{2}\\4.33\end{pmatrix}, \mathbf{B} = \begin{pmatrix}\frac{5}{2}\\-4.33\end{pmatrix} \tag{2.0.21}$$

Plot of Tangents PA and PB :

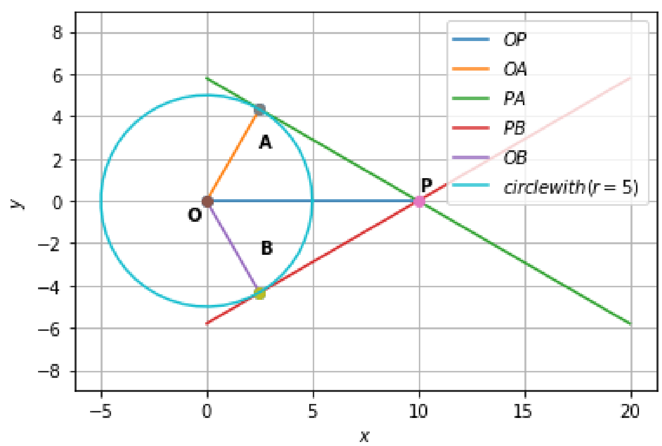

Fig. 2.1: Tangent lines to circle of radius 5 units.