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ASSIGNMENT 3

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Download all python codes from

https://github.com/ponnaboinakalpana12/ ASSIGNMENT3

and latex-tikz codes from

https://github.com/ponnaboinakalpana12/ ASSIGNMENT3

1 Question No 2.58

Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of 60°

2 Solution

Data from the given question:

	Symbols	Circle
Centre	0	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
Radius	r	5

Let PA and PB be tangents to circle with radius 5 cm which are inclined to each other at an angle 60°. We know a tangent is always perpendicular to the radius.

$$\therefore OA \perp AP, OB \perp BP. \tag{2.0.1}$$

$$\angle OAP = 90^{\circ}, \angle OBP = 90^{\circ}.$$
 (2.0.2)

We know that, line joining the centre and the external point bisect the angle between pair of tangents from that external point.

$$\angle APB = \theta = 60^{\circ}$$

OP bisect the angle $\theta = 60^{\circ}$ $In\triangle OAP$,

$$\sin\frac{\theta}{2} = \frac{OA}{OP}$$
 (2.0.3)
$$\frac{1}{2} = \frac{5}{OP}$$
 (2.0.4)

$$\frac{1}{2} = \frac{5}{OP} \tag{2.0.4}$$

$$\implies OP = 10 \tag{2.0.5}$$

$$\therefore \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \tag{2.0.6}$$

Now,

$$\angle AOB + \angle OBP + \angle APB + \angle PAO = 360^{\circ} \quad (2.0.7)$$

$$\implies \angle AOB + 90^{\circ} + 60^{\circ} + 90^{\circ} = 360^{\circ}$$
 (2.0.8)

$$\angle AOB = 360^{\circ} - 140^{\circ}$$
 (2.0.9)

$$\angle AOB = 120^{\circ}.$$
 (2.0.10)

$$\angle AOP = 60^{\circ} (2.0.12)$$

Lemma 2.1. The coordinates of A and B can be written as follows:

$$\mathbf{A} = O + r \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \tag{2.0.13}$$

$$\mathbf{B} = O + r \begin{pmatrix} \cos(360 - \theta) \\ \sin(360 - \theta) \end{pmatrix}$$
 (2.0.14)

$$\mathbf{B} = O + r \begin{pmatrix} \cos \theta \\ -\sin \theta \end{pmatrix} \tag{2.0.15}$$

Proof: Now substitute the values, we get For finding coordinate of A:-

$$\implies \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 5 \begin{pmatrix} \cos 60 \\ \sin 60 \end{pmatrix} \implies \mathbf{A} = \begin{pmatrix} 2.5 \\ 4.33 \end{pmatrix}$$
(2.0.16)

For finding the coordinate of B:

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 5 \begin{pmatrix} \cos 60 \\ -\sin 60 \end{pmatrix} \implies \mathbf{B} = \begin{pmatrix} 2.5 \\ -4.33 \end{pmatrix}$$
(2.0.17)

$$\therefore \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 2.5 \\ 4.33 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2.5 \\ -4.33 \end{pmatrix}.$$
(2.0.18)

Plot Tangents PA and PB:

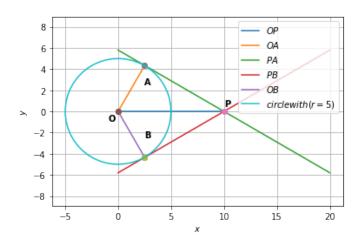


Fig. 2.1: Tangent lines to circle of radius 5 units.