Dot and Cross products

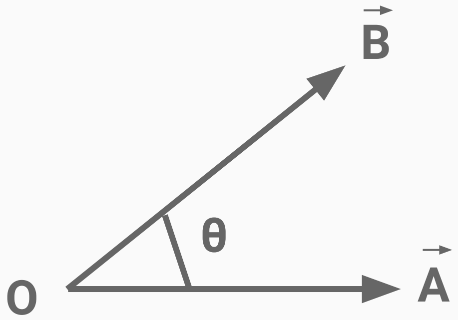
Dot product

The dot product of two vectors A and B is defined as:

$$\vec{A} \cdot \vec{B} = |\vec{A}|\ |\vec{B}| \cos \theta$$

$$\mathrm{dot}(\vec{A}, \vec{B}) = \mathrm{length}(\vec{A}) * \mathrm{length}(\vec{B}) * \cos(\theta)$$

$$\mathrm{dot}(\vec{A}, \vec{B}) = A_x * B_x + A_y * B_y + A_z * B_z$$



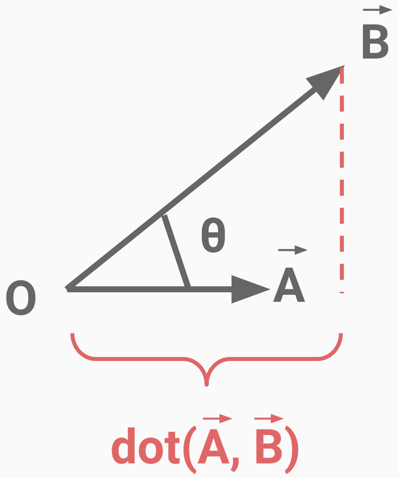
Dot product

The result of the dot product is a **scalar value**.

It denotes the degree of projection of one vector onto the other.

If $\vec{A}$ is normalized (i.e. $|\vec{A}| == 1$), $dot(\vec{A}, \vec{B})$ gives us the amount of $\vec{B}$ projected onto $\vec{A}$.

$$\mathbf{dot(\vec{A}, \vec{B}) = A_x * B_x + A_y * B_y + A_z * B_z}$$

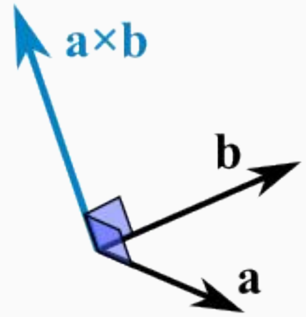


Cross product

The cross product of two vectors $\vec{A}$ and $\vec{B}$ is defined as:

$$\vec{A} \times \vec{B} = |\vec{A}|\ |\vec{B}|\ \sin\theta\ \vec{N}$$

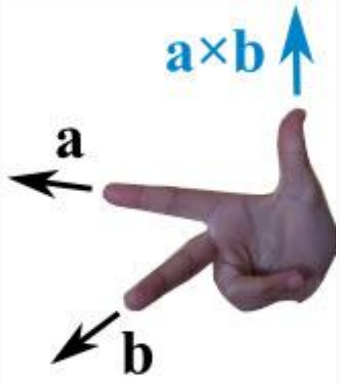
$$\text{cross}(\vec{A}, \vec{B}) = \text{length}(\vec{A}) * \text{length}(\vec{B}) * \cos(\theta) * \vec{N}$$

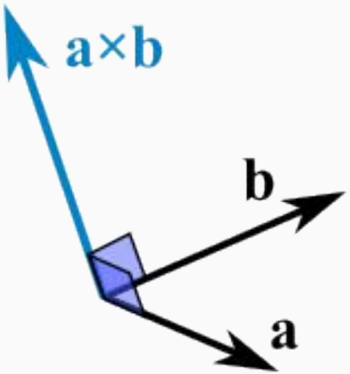


Cross product

It gives us a **vector** which is **perpendicular** to both $\vec{A}$ and $\vec{B}$.

The **Right Hand Rule** tells us the direction of the resulting vector.





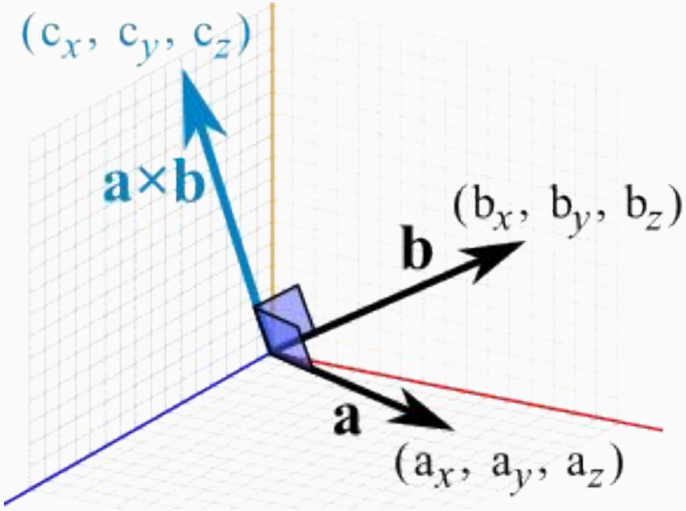
Cross product

The cross product of two vectors $\vec{A}$ and $\vec{B}$ can be computed as follows:

$$c_x = a_y b_z - a_z b_y$$

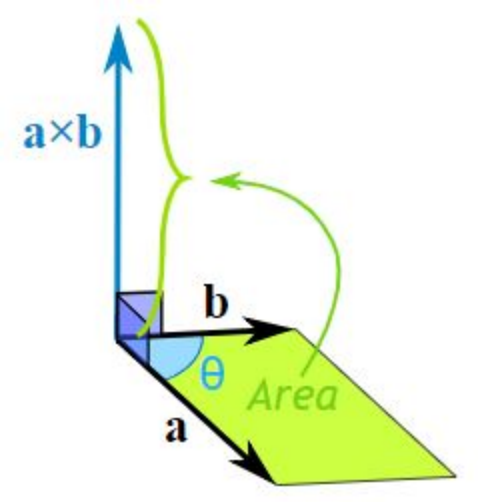
$$c_y = a_z b_x - a_x b_z$$

$$c_z = a_x b_y - a_y b_x$$



Cross product

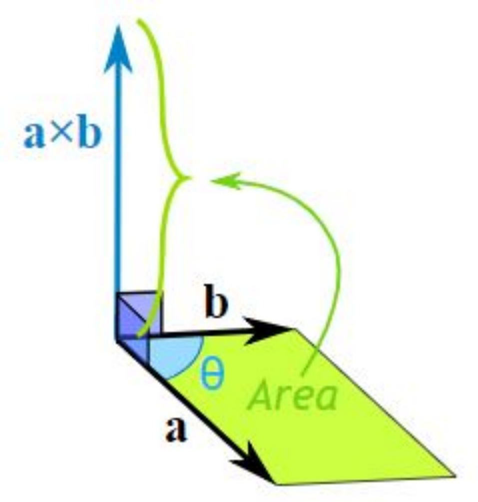
The magnitude of the resulting vector is the area of the parallelogram defined by A and B.



Cross product

The magnitude of the resulting vector is the area of the parallelogram defined by A and B.

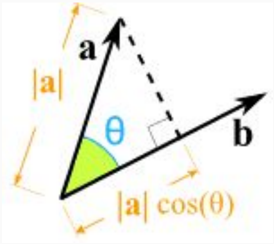
Most of the time we will be interested in the resulting vector direction, not magnitude, so **remember to normalize the result before using it**.



Dot product vs. Cross product

Dot product

- Result: A scalar value
- Projection of a onto b
    - Normalize b to find the projection measure in the current base coordinates.



Cross product

- Result: A third vector
- Perpendicular to a and b
- Area of the parallelogram
- Right Hand Rule