UE SMART CITIES

Dynamic Models for Building Energy Management

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Introduction

The building sector is an energy consuming sector. It will then be very important to succeed in reducing the energy consumed by the buildings and to do this there are several parameters to take into account such as the geometry of the building and the materials which constitute it. The study made during these three sessions consists in the estimation of the energy efficiency of a smart home by modeling and simulation.

The plan for the three sessions can be found below:

Session 1 - **Model**
Session 2 - **Implementation** (Python code + run/debug)
Session 3 - **Optimization of the code** (design/operation)

Hypothesis

The heat transfer is assumed to take place in a single direction, and the heat transfer is considered only through the walls, on which the temperature is uniform at all points (heat transfer with the floor and ceiling are neglected). Also, all heat exchanges are considered linearized.

I _ Building plan

This study concerns a studio of approximately 28 m² composed of two rooms including a bathroom/toilet and a main room. The main room is assumed to consist of a kitchen, a dining room as well as a small room separated from the kitchen by a small wall which acts as a separation.

The studio has two windows with views to the outside, one in the bedroom and one in the dining room. Note that the walls on the left and right are adjacent to other apartments and therefore heat exchange through these walls are not considered. The main door of the apartment opens onto a corridor common to the apartments of the building. We will model the heat exchanges between the apartment and the outside. The precise dimensions of the studio are shown in figure 1 below.
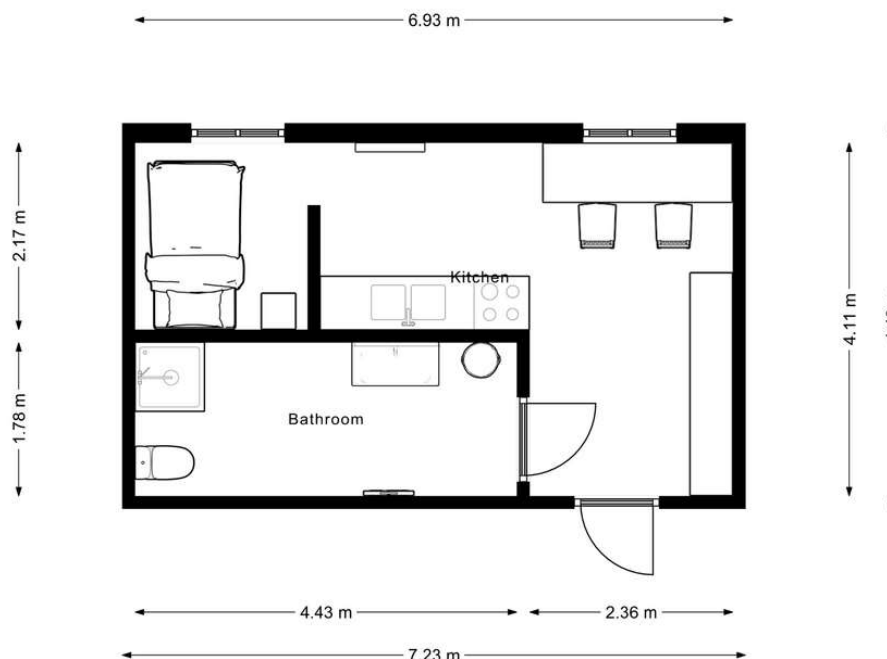
Figure 1. The studied building and its dimensions with an assumed ceiling height of 2,4 meters.

II _ Thermal and mathematical model

1.Composition of the walls

A heat flow could be described as a flow of energy due to variances in temperature between two positions, and it can occur in the form of conduction, convection, radiation as well as advection, similar to the case presented in figure 3.
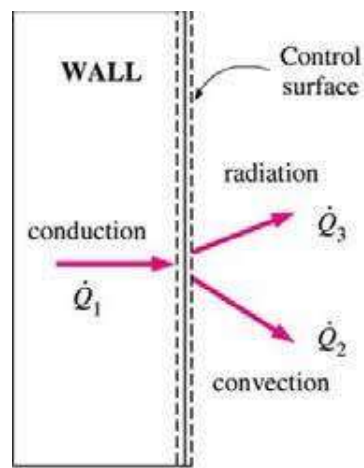

Figure 3. Heat exchange through a wall in the form of conduction, convection and radiation. Advection is not presented in the figure but can be described as a heat flow due to a mass transfer mechanism.

In this studied case, the walls are considered to consist of several layers of materials influencing the heat flow and therefore the internal temperature in the building. For example, the exterior wall is considered to consist of both concrete and insulation as presented in figure 4.
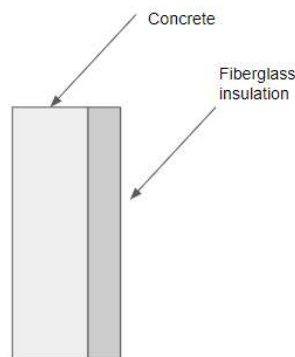

Figure 4. The exterior wall is considered to consist of concrete and fiberglass insulation.

The walls of this room aren't all the same :
- the wall of the main-room with the kitchen to the outside (wall 1)
  & to the corridor (wall 4) : concrete + insulation
- the wall of the main-room with the kitchen to the bathroom (wall 2) : concrete + wood
- the wall of the bathroom to the corridor (wall 3): concrete + wood

All the doors are in wood, and we consider the windows to be in glass.

2. Parameters

Table 1. Description of the materials used in the building model regarding thickness, heat transfer coefficient, density as well as specific heat capacity.

Material	Thickness [mm]	Lambda [W/m/K]	Density [kg/m^3]	Specific heat capacity [J/kgK]
Air	–	0,0244	1,208	1006
Concrete	130	1,8	2300	880
Insulation	100	0,0345	22	850
Glass	20	1,4	2210	730
Wood	20	0,15	532	2700

Table 2. Description of the surfaces regarding the building model components.

Part of building	Surface [m^2] We chose a height of 2,40m
Wall 1 (Exterior wall)	13,6
Wall 2 (Bathroom outer walls)	14,9
Wall 3 (Hallway to corridor)	5,7
Wall 4 (Bathroom to corridor)	10,6
Windows	3
Door	2,1

3.Thermal model of the room

Below, in figure 5, the thermal modeling of the room is presented. The model is based on the heat flows which take place in the building plan described in figure 1. Further, the model consists of different types of components including branches, nodes and external heat sources representing how the heat flows through, for example, walls and windows.
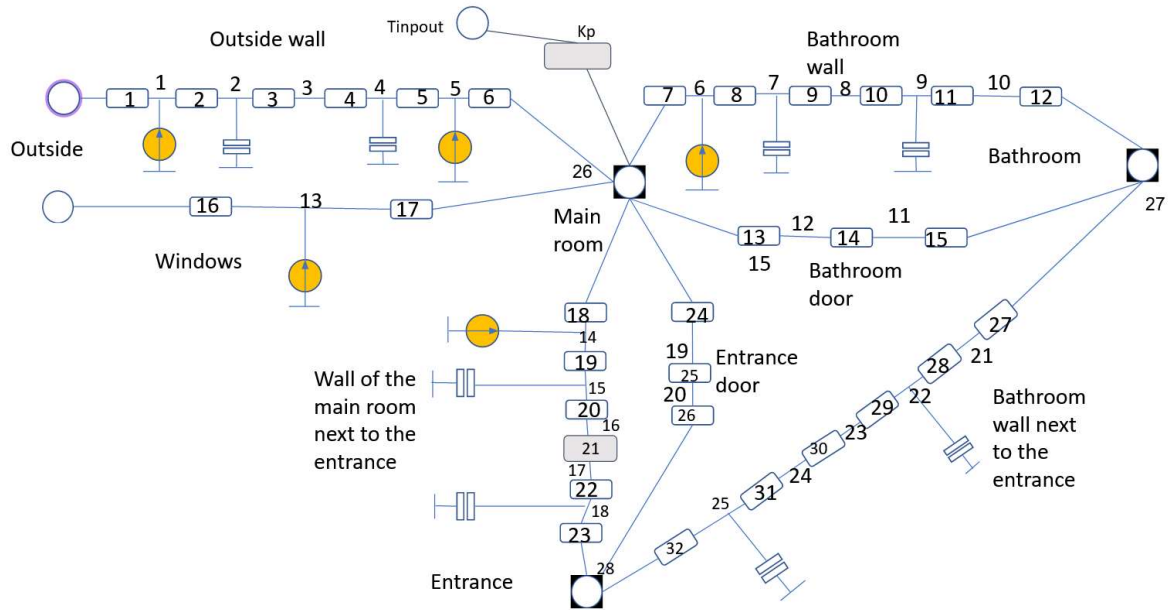

Figure 5. The thermal model of the studied system based on the presented model in figure 1. The nodes and resistances were numbered appropriately and utilized to determine the matrices for the model to be adopted.

Inner nodes of each wall material and each room has the following **heat capacity**:

$$C = \rho. \, c. \, e. \, S.$$

where:

- ρ is the density of the different materials (kg/m3)
- c is the specific heat mass (J/kg/K)
- S is the surface of the wall (m2)
- e is the thickness of the wall (m)

For the **convective resistances**, they could be described by the formula

$$R = 1/(h.S)$$

where:

- h is the thermal convection coefficient (W/m2/K)

The values of **conduction resistance** are **R =e/(λ.S)**

where:
 * λ is the thermal conductivity of the wall part (W/m/K)
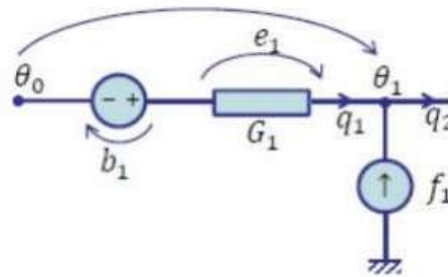
4. Code implementation

The construction of the thermal model requires the different principles of thermodynamics:

Heat transfer (or heat) is thermal energy in transit due to temperature difference

0th **principle** : temperature scales $(e_1 = \theta_0 - \theta_1 + b)$
1st **principle** : energy conservation $(q_1 - q_2 = -f)$
2nd **principle** and **constitutive laws:** direction / value of heat $(q_1 = G_1 e_1)$

We have the following three equations:

$$e = -A\theta + b$$
$$q = Ge$$
$$C\frac{d\theta}{dt} = A^T q + f$$

where e is the vector of temperature differences between two nodes, and where q is the vector of heat flow through each resistor.

The three equations above combined lead us to the following differential equation, which governs the system :

$$C\dot{\theta} = -A^T GA\theta + A^T Gb + f$$

5. Matrix A, C and G

For our model, we have 28 temperature nodes and 33 heat flows (we consider the controller Kp in it) . Therefore, the matrices and the vectors that will be created will depend on this number of nodes and flows.

    We have then:
- the matrix **A** (33x28) which relates the temperatures between the nodes (composed of 1 for incoming flows to the node, -1 for an outgoing flow, and the rest is made of 0).
- the vector **b** (33x1) which relates the temperature sources
- the vector **f** (28x1), the vector of the flux sources
- the matrix **C** (28x28) for the thermal capacities
- the **G** matrix (33x33) for the conductances
- finally, the vector **Ɵ** (28x1) which concerns the temperatures at each node

About the matrix A :
(The matrix A is very big so here is a reconstruction of what we obtained with the code)
A = zeros (33,28)

About the G matrix :
You can see in **Annexe 2** the exact calculations, so the exact components of the matrix wrote down :

We used : $G_{conduction} = \frac{\lambda * S}{w}$, $G_{radiation} = G_{convection} = h * S$

Where λ is the conductivity of the material (as defined in Table 1), S is the surface through which the heat flux is transmitted, w is the thickness of the wall layer (or slice of it), h is the convection coefficient or the equivalent radiative coefficient

G = diag (g)

```
>>> g
array([340.8       ,  377.50153846, 377.50153846,    9.40608    ,
         9.40608    , 109.056     , 119.232     , 149.04      ,
       149.04      , 412.72615385, 412.72615385, 119.232     ,
        16.8       ,  21.        ,  16.8       ,  75.        ,
        24.        ,  45.312     ,   3.90816   ,   3.90816   ,
       156.84923077, 156.84923077,  45.312     ,  16.8       ,
        21.        ,  16.8       ,  85.056     ,   7.33608   ,
         7.33608   , 294.42461538, 294.42461538,  85.056     ,
         1.        ])
```

with g :

```
>>> G
array([[340.8       ,    0.        ,    0.        , ...,    0.        ,
          0.        ,    0.        ],
       [  0.        ,  377.50153846,    0.        , ...,    0.        ,
          0.        ,    0.        ],
       [  0.        ,    0.        ,  377.50153846, ...,    0.        ,
          0.        ,    0.        ],
       ...,
       [  0.        ,    0.        ,    0.        , ...,  294.42461538,
          0.        ,    0.        ],
       [  0.        ,    0.        ,    0.        , ...,    0.        ,
         85.056     ,    0.        ],
       [  0.        ,    0.        ,    0.        , ...,    0.        ,
          0.        ,    1.        ]])
```

About the matrix C :

You can see in **Annexe 3** the exact calculations, so the exact components of the matrix wrote down :

As said before, we used the formula : **C = ρ. c. e. S.**

The C matrix is very big.

So here is only the c matrix, with C =diag(c)

```
>>> C
array([    0.        ,  2807.10144  ,     0.        ,     0.        ,     0.        ,
          0.        ,  3069.03168  ,     0.        ,     0.        ,     0.        ,
          0.        ,     0.        ,     0.        ,     0.        ,     0.        ,
          0.        ,     0.        ,  1166.33088  ,     0.        ,     0.        ,
          0.        ,     0.        ,     0.        ,     0.        ,  2189.34144  ,
          0.        ,     0.        ,     0.        ])
```

For the simulation, we erased all the low conductances.

6. Vectors b and f in time

The vector of temperature sources is b. It has the size of the number of branches and if its element is corresponding to a branch without a source is zero. If the flow in a source is from low potential to high potential (i.e. from - to +), the source is positive

Here we have constraints on the model which are temperatures sources :

```
>>> b
array([ 5.,    0.,    0.,    0.,    0.,    0.,    0.,    0.,    0.,    0.,    0.,    0.,    0.,
        0.,    0.,   20.,    0.,    0.,    0.,    0.,    0.,    0.,    0.,    0.,    0.,    0.,
        0.,    0.,    0.,    0.,    0.,    0.,   20.])
```

The vector **f** (28x1), the vector of the flux sources :

f[1,1] = 1 (corresponds to the **outdoor** radiative flow rate)
f[5,1] = 1 (corresponds to the **indoor** radiative flow rate)
f[6,1] = 1 (corresponds to the **indoor** radiative flow rate)
f[13,1] = 1 (corresponds to the **absorbed** radiative flow rate)
f[14,1] = 1 (corresponds to the **indoor** radiative flow rate)

Regarding the radiation : ϕ_{rad} = hr (Ts - Te)

We will use : 1000 W/m2 K and 500 W/m2 K values for our simulation.

```
>>> f
array([1000.,      0.,      0.,      0., 1000.,    500.,      0.,      0.,      0.,
          0.,      0.,      0.,    500., 1000.,      0.,      0.,      0.,      0.,
          0.,      0.,      0.,      0.,      0.,      0.,      0.,      0.,      0.,
          0.])
```

III_Simulation

1.Steady-state

We will use : h_{out} = air convection coefficient = 25 & h_{in} = 8

For all the simulations we perform, it is necessary to go through a state representation of the system. The transition from thermal matrix modelling to the state representation corresponds to the transition between this equation :

$$C\dot{\theta} = -A^T G A \theta + A^T G b + f$$

and the state space model :
$$\begin{cases} \dot{x} = A_s x + B_s u \\ y = C_s x + D_s u \end{cases}$$

where:

x – the state vector: temperature nodes with capacities;
y – output vector: temperatures in the nodes of interest;
u - input vector: temperature and flow-rate sources;
A_s – state matrix;
B_s – input matrix;
C_s - output matrix;
D_s - feedthrough matrix.

$$u = \begin{bmatrix} b \\ f \end{bmatrix}$$

The input vector u is :

This transformation is achieved with the fTC2SS function by coding as follows :
[As,Bs,Cs,Ds] = fTC2SS(A,G,b,C,f,y)

For this simulation we consider an initial temperature of 20°C in the main room, an outside temperature of 5°C. In addition, we consider that there is no heating system in operation and that the solar radiant flux is zero.

We obtain the following values :

```
[ 5.194   5.367    5.54   12.475 19.411 20.008 20.007 20.007 20.007 20.007
  20.004 20.002   8.646 20.021 20.014 20.006 20.006 20.006 20.002 20.003
  20.006 20.006 20.005 20.005 20.005 20.     20.006 20.005]
[ 5.194   5.367    5.54   12.475 19.411 20.008 20.007 20.007 20.007 20.007
  20.004 20.002   8.646 20.021 20.014 20.006 20.006 20.006 20.002 20.003
  20.006 20.006 20.005 20.005 20.005 20.     20.006 20.005]
Max error in steady-state between thermal circuit and state-space: 6.39e-14
```

2. Steady -state : step response

We simulate a step response using the following vector:
b[[0, 15, 32]] = 0 + np.array([1, 1, 20]

It means that the outside temperature is 1°C, the initial temperature of the main room is 20°C and we use a controller to have the temperature of the main room to 1°C.

The temperature shown is from the main room. It goes from 20°C to 1°C.

3 _ Dynamic model

In dynamic simulation, the inputs vary in time. Let's indicate the sources in the circuit by ones.
Let's consider that the output of the circuit is the indoor temperature.
The state-space representation is obtained from the differential-algebraic equations of the thermal circuit :

[As, Bs, Cs, Ds] = dm4bem.tc2ss(A, G, b, C, f, y)

The maximum time step for numerical stability of Euler explicit integration in time is given by the minimum eigenvalue λ of the state matrix A_s :

$$\Delta t \leq \min(-2/\lambda_i) = \min T_i/2$$

where T_i is the time constants, $T_i = -\frac{1}{\lambda_i}$

The maximum time step for numerical stability of Euler explicit integration in time depends on the value of the P-controller gain and if the capacities of the indoor air and window glass are taken into account.

We obtain a maximum time step of 30,61 sec.

Let's look at the step response :
For the input vector u, the outdoor temperatures will be T_o = 1, the indoor set-point temperature will be T_{sp} = 0, and the heat flow sources will be zero.

The vectors for the evolution of temperatures (states) in time for Euler explicit and implicit are:

temp_exp = np.zeros([n_tC, t.shape[0]])
temp_imp = np.zeros([n_tC, t.shape[0]])

By integrating the state space model :

$$\begin{cases} \dot{\theta} = A_s\theta + B_s u \\ y = C_s\theta + D_s u \end{cases}$$

by using Euler forward (or explicit) :

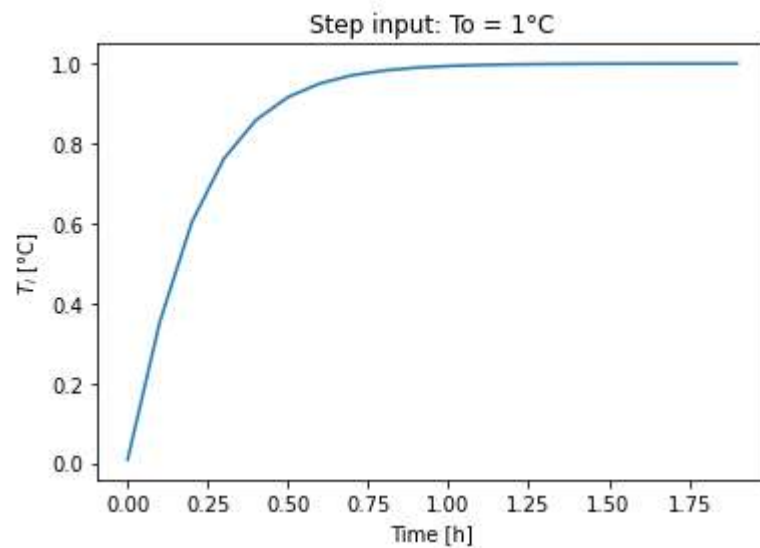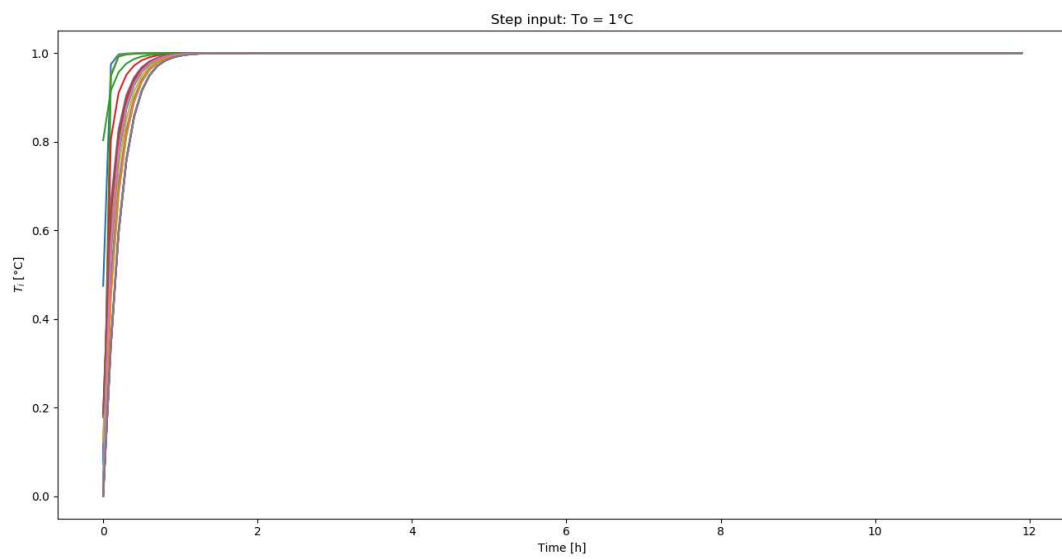$$\theta_{k+1} = (I + \Delta t A)\theta_k + \Delta t B u_k$$

and Euler backward (or implicit) integration :

$$\theta_{k+1} = (I - \Delta t A)^{-1}(\theta_k + \Delta t B u_k)$$

we obtain the outputs.

The obtained curves are presented at the following page:

Step input: To = 1°C

Step input: To = 1°C

Steady-state indoor temperature obtained with:
- DAE model: 1.0000 °C
- response to step input: 1.0000000000000022 °C

4.Simulation with weather data

We didn't succeed at simulating this part.
But we could have seen the impact of radiation on the indoor temperature.

Conclusion

It is crucial to analyse the heat flow in a building to be able to enable energy management and optimise the energy consumption of the building. By creating a model of a building and considering heat flows in the form of conduction, convection and radiation in both steady state and dynamic conditions, this can be accomplished. In this project, we were able to develop a coherent thermal model of a building and verify the relevance of our modelling choices through simulations. The obtained results are consistent for both the static and the dynamic model, but it can be seen that the obtained time step of about 30 seconds regarding the dynamic model, is relatively small. This implies that the simulation possibly could be improved.

Further, a controller was implemented which modelised a HVAC model capable of heating and cooling. Regarding this, it could be seen that the time step depends on Kp affecting the performance of the controller. When Kp goes towards the infinity, the controller becomes more idealised and when the value of Kp goes towards zero, the controller becomes less efficient.

References

L. Pérez-Lombard, J. Ortiz, C. Pout (2007). *A review on buildings energy consumption information.* Energy and Buildings, Vol. 40, Iss. 3, p. 394-398. Available at: https://www.sciencedirect.com/science/article/pii/S0378778807001016 [2022-04-22]

Annexe 1 : Precise table about the composition of the walls

Material	Thickness [mm] Width (w)	Lambda [W/mK] λ	Density [kg/m³] ρ	Specific heat capacity [J/kgK] c	Surface in m2 (height of 2,40 m)
Air	–	0.0244	1.208	1006	–
Concrete 1	130	1.8	2300	880	6.93*2.40-g=13.632
Concrete 2	130	1.8	2300	880	2.4*(4.43+1.78)=14.904
Concrete 3	130	1.8	2300	880	2.36*2.4=5.664
Concrete 4	130	1.8	2300	880	4.43*2.4=10.632
Insulation 1	100	0.0345	22	850	6.93*2.4-g=13.632
Insulation 3	100	0.0345	22	850	2.36*2.4=5.664
Insulation 4	100	0.0345	22	850	4.43*2.4=10.632
Glass	20	1.4	2210	730	2*1.5=3
Wood 2	20	0.15	532	2700	2.4*(4.43+1.78)=14.904
Wood door bathroom	20	0.15	532	2700	2.10*1=2.1
Wood door corridor	20	0.15	532	2700	2.10*1=2.1

About the construction of the matrix g

g=zeros(32,1)

Exterior wall :
g(1,1) = ho * S_concrete1
g(2,1) = 2*_concrete * S_concrete1 / w_concrete
g(3,1) = 2*_concrete * S_concrete1 / w_concrete
g(4,1) = 2*_insulation * S_insulation1 / w_insulation
g(5,1) = 2*_insulation * S_insulation1 / w_insulation
g(6,1) = hin * S_insulation1

Wall main door – bathroom :
g(7,1) = hin * S_wood2
g(8,1) = 2*_wood * S_wood2 / w_wood
g(9,1) = 2*_wood * S_wood2 / w_wood
g(10,1) = 2*_concrete * S_concrete2 / w_concrete
g(11,1) = 2*_concrete * S_concrete2 / w_concrete
g(12,1) = hin * S_concrete2

Door bathroom :
g(13,1) = hin * S_doorbathroom
g(14,1) = 2*_wood * S_doorbathroom / w_wood
g(15,1) = hin * S_doorbathroom

Windows :
g(16,1) = hout * S_windows
g(17,1) = hint * S_windows

Wall main room - corridor :
g(18,1) = hin * S_insulation3
g(19,1) = 2*_insulation * S_insulation3 / w_insulation
g(20,1) = 2*_insulation * S_insulation3 / w_insulation
g(21,1) = 2*_concrete * S_concrete3 / w_concrete
g(22,1) = 2*_concrete * S_concrete3 / w_concrete
g(23,1) = hin * S_concrete3

Door main room - corridor :
g(24,1) = hin * S_doorcorridor
g(25,1) = 2*_wood * S_doorcorridor / w_wood

g(26,1) = hin * S_doorcorridor

Wall bathroom - corridor :
g(27,1) = hin * S_insulation4
g(28,1) = 2*_insulation * S_insulation4 / w_insulation
g(29,1) = 2*_insulation * S_insulation4 / w_insulation
g(30,1) = 2*_concrete * S_concrete4 / w_concrete
g(31,1) = 2*_concrete * S_concrete4 / w_concrete
g(32,1) = hin * S_concrete4

Annexe 3 : Construction of the c matrix

About the construction of the matrix c
c = zeros(28,1) Exterior wall : (concrete and insulation) S_concrete1 =S_insulation1 c(2,1) = c_concrete * S_concrete1 * w_concrete * _concrete c(4,1) = c_insulation * S_insulation1 * w_insulation * _insulation Mur principal room to bathroom : S_concrete2 = S_wood2 c(7,1) = c_concrete * S_concrete2 * w_concrete * _concrete c(10,1) = c_wood * S_wood2 * w_wood * _wood Mur principal room to corridor : S_concrete3 =S_insulation3 c(15,1) = c_insulation * S_insulation3 * w_insulation * _insulation c(18,1) = c_concrete * S_concrete3 * w_concrete * _concrete Mur bathroom to corridor : S_concrete4 =S_insulation4 c(22,1) = c_insulation * S_insulation4 * w_insulation * _insulation c(25,1) = c_concrete * S_concrete4 * w_concrete * _concrete C = diag(c)