

Unit 2

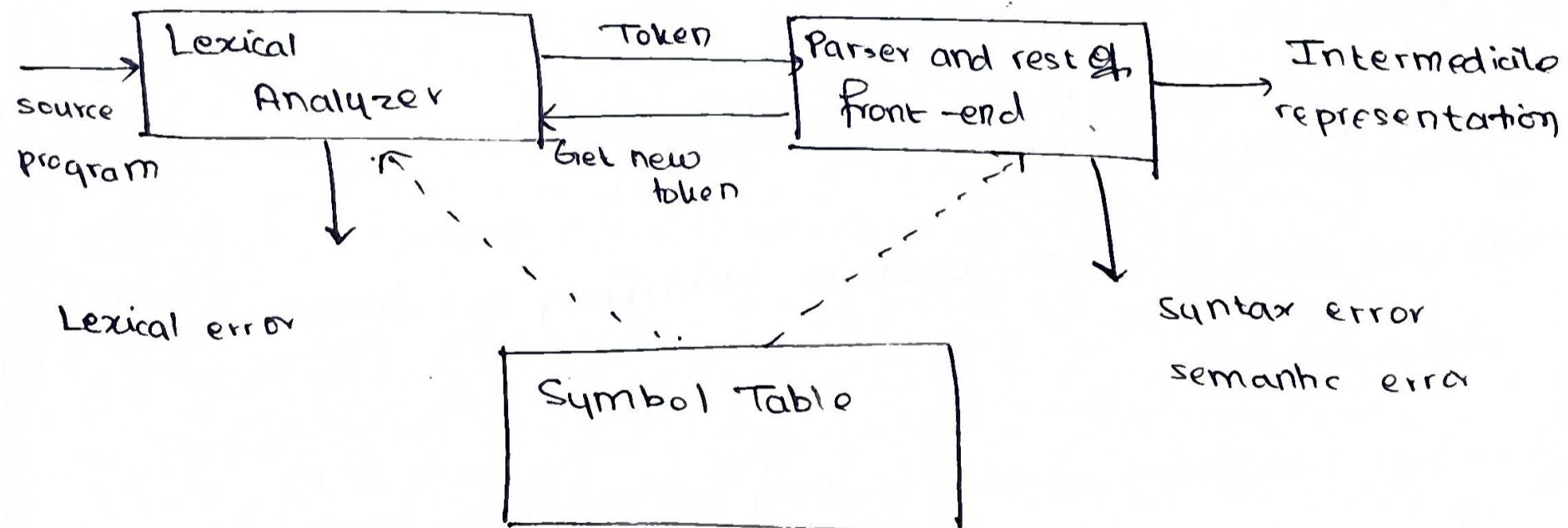
Syntax Analysis

Role of parser - writing grammars for language constructs - Types of grammar : Ambiguity - Deterministic & Recursive ; Top down parsers.

Recursive descent parser - predictive parser ; Bottom up parsers.

SLR Parser - CLR Parser - LALR Parser ; Error handling & recovery in syntax analyzer ; Syntax analyzer generator ; Structure of yacc program - creating yacc lexical analyzers with lex

* Role of Parser



* Syntax Analyzer (also called parser)

→ Creates the syntactic structure of the given source program

→ The syntactic structure is mostly a parse tree

→ The syntax of a programming lang. is described by a CFG

→ The syntax analyzer (parser) checks whether a given source program satisfies the rules implied by a CFG or not

- (i) If it satisfies → the parser creates the parse tree of that program
- (ii) Else, gives an error message.

* Kinds of Parsers

1. Top-Down Parser - the parse tree is created top to bottom, starting from the root
2. Bottom-Up Parser - the parse is created bottom to top, starting from the leaves.

→ Both parsers scan the input from the left to right (one symbol at a time).

* Error Handling

- A good compiler should assist in identifying and locating errors.
- Errors can be:

(i) Lexical Errors - mis spelling an identifier, keyword or operator

(ii) Syntax Errors - such as an arithmetic expression with unbalanced expression parentheses

(iii) Semantic Errors - such as an operator applied to an incompatible operand

(iv) Logical Errors: such as an indefinitely recursive call. (2)

* Error Recovery Strategies

- (i) Panic Mode - Discard input until a token in a set of designated synchronizing tokens is found
- (ii) Phase-Level Recovery - Perform local correction on the input to repair the error
- (iii) Error productions - Augment grammar with productions for erroneous constructs
- (iv) Global Correction - choose a minimal sequence of changes to obtain a global least-cost correction

* Grammars

→ Inherently recursive structures of a programming language are defined by a context-free grammar, as a 4-tuple

$$G = (N, T, P, S)$$

↑ ↑ ↑ ↑
non-terminals terminals] start symbol

products
 $\alpha \rightarrow \beta$
 $\alpha \in N$
 $\beta \in (N \cup T)^*$

Derivation → A sequence of replacements of non-terminal symbols is called a derivation

\Rightarrow : derived in one step

$\xrightarrow{*}$: derives in zero or more steps

$\xrightarrow{+}$: derived in one or more steps

(i) If we always choose the left-most non-terminal in each derivation step, this derivation is called as the left-most derivation

(ii) If we always choose the right-most non-terminal in each derivation step, this derivation is called the right-most derivation

Example : Consider the grammar:

$$E \rightarrow E+E$$

$$E \rightarrow E-E$$

$$E \rightarrow E * E$$

$$E \rightarrow E/E$$

$$E \rightarrow -E$$

$$E \rightarrow (E)$$

$$E \rightarrow id$$

Use LMD & RMD for -(id+id)

LMD

$$E \rightarrow -E$$

$$\xrightarrow{*} \rightarrow -(E)$$

$$\xrightarrow{*} \rightarrow -(E+E)$$

$$\xrightarrow{*} \rightarrow -(id+E)$$

$$\xrightarrow{*} \rightarrow -(id+id)$$

RMD

$$E \Rightarrow -E$$

$$\Rightarrow -(E)$$

$$\Rightarrow -(E+E)$$

$$\Rightarrow -(E+id)$$

$$\Rightarrow -(E+id)$$

* Constructing CFGs from Regular Expressions

Algorithm

For each state i of the NFA, create a non-terminal A_i

Begin

(i) If state i has a transition to j on symbol a ,
introduce production $A_i \rightarrow aA_j$

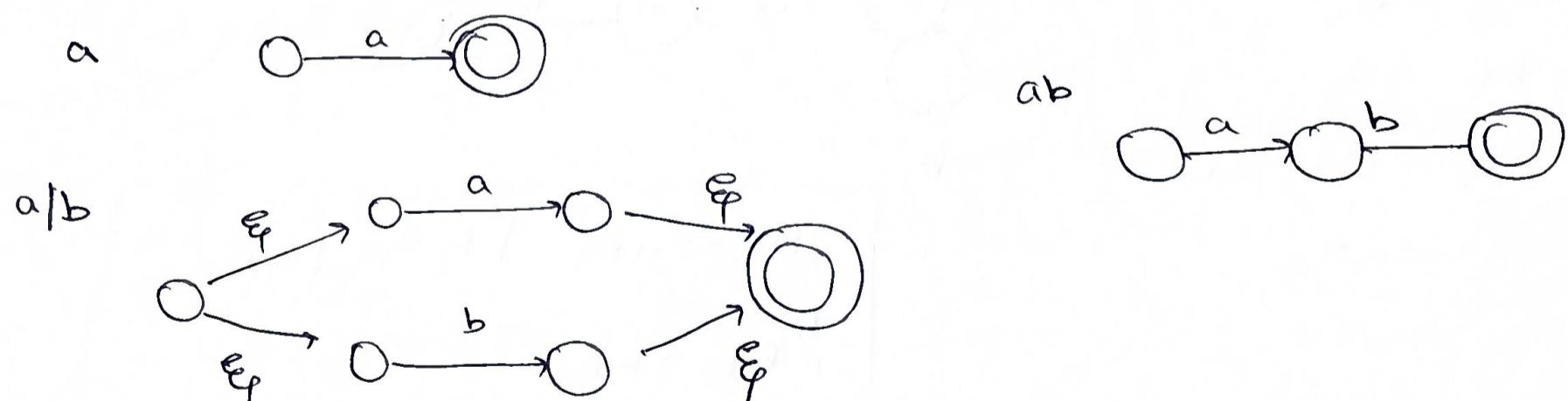
(ii) If state i goes to state j on ϵ ,
introduce production $A_i \rightarrow A_j$

End

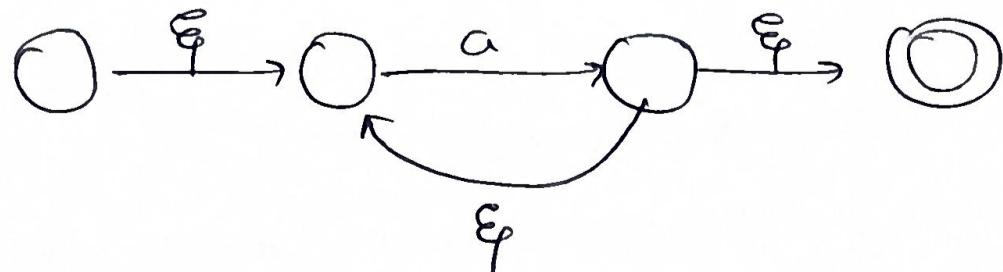
If i is an accepting state, introduce $A_i \rightarrow \epsilon$

If i is the start state, make A_i the start symbol for the grammar.

RE - NFA Rules from TAC (Thompson's Construction)

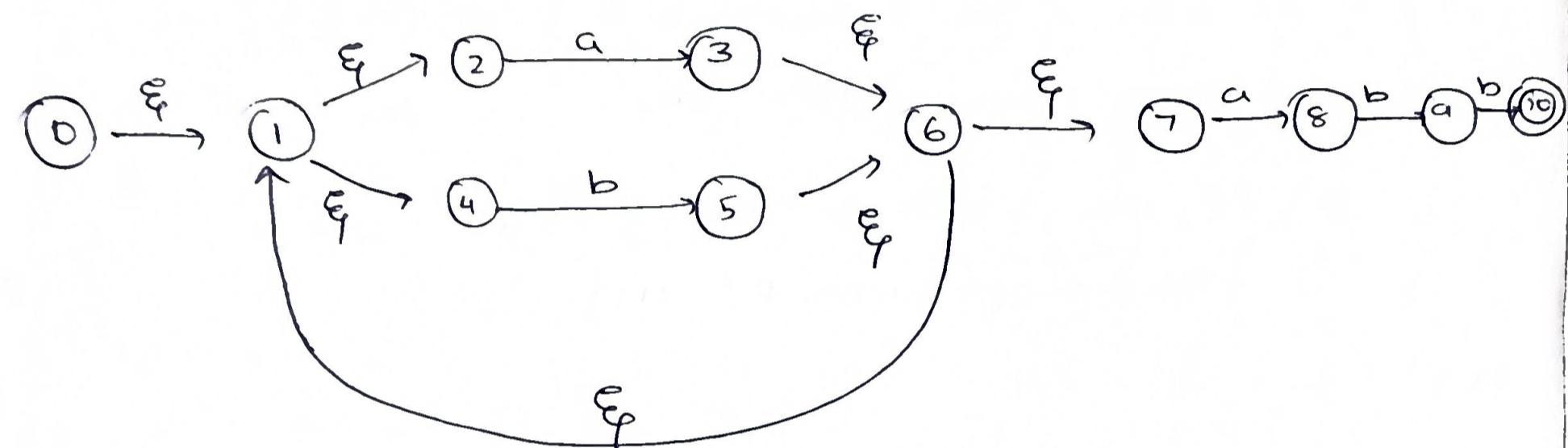


a*



Example Construct a grammar for the following regular expressions.

① $(a|b)^* abb$



$$A_0 \rightarrow A_1$$

$$A_1 \rightarrow A_2 \mid A_4$$

$$A_2 \rightarrow aA_3$$

$$A_4 \rightarrow bA_5$$

$$A_3 \rightarrow A_6$$

$$A_5 \rightarrow A_6$$

$$A_6 \rightarrow A_7 \mid A_1$$

$$A_7 \rightarrow aA_8$$

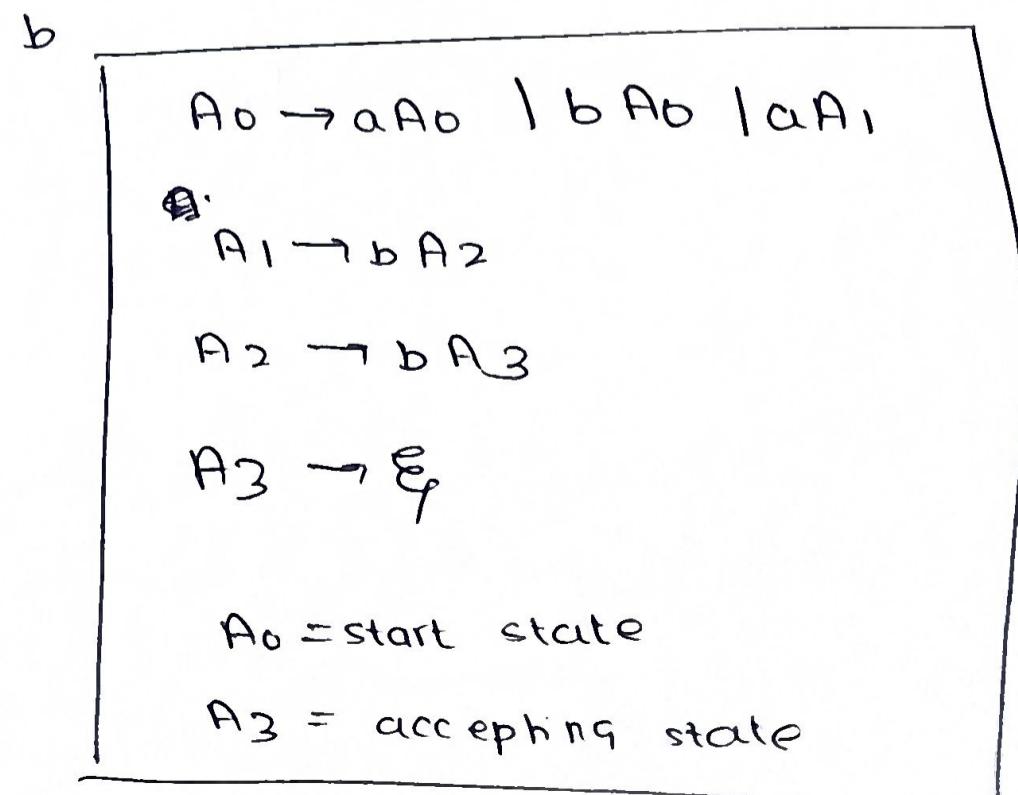
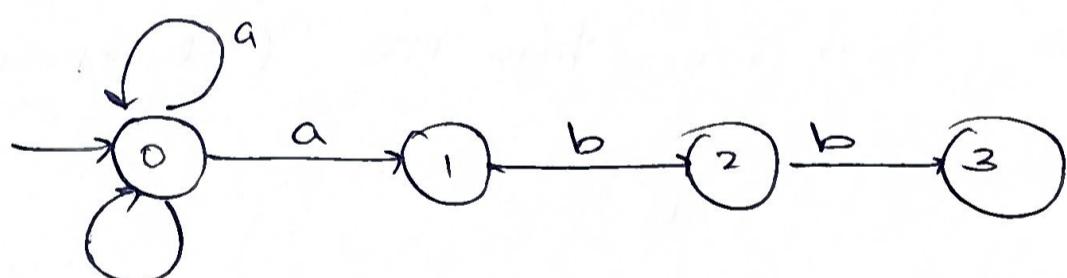
$$A_8 \rightarrow bA_9$$

$$A_9 \rightarrow bA_{10}$$

$$A_{10} \rightarrow \epsilon$$

But for simplicity

DO THIS



* Ambiguity

(7)

If more than one parse tree can be constructed for a sentence, then the grammar is called ambiguous.

Example: Check if the following grammar is ambiguous.

$$E \rightarrow E+E$$

$$E \rightarrow E-E$$

$$E \rightarrow E * E$$

$$E \rightarrow E/E$$

$$E \rightarrow -E$$

$$E \rightarrow (E)$$

$$E \rightarrow id$$

using the sentence

$$id + id * id$$

$$E \rightarrow E+E$$

$$\Rightarrow E + E * E$$

$$\Rightarrow id + E * E$$

$$\Rightarrow id + id * E$$

$$\Rightarrow id + id * id$$

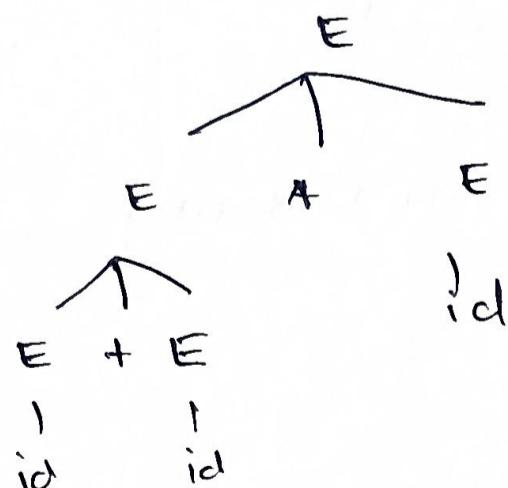
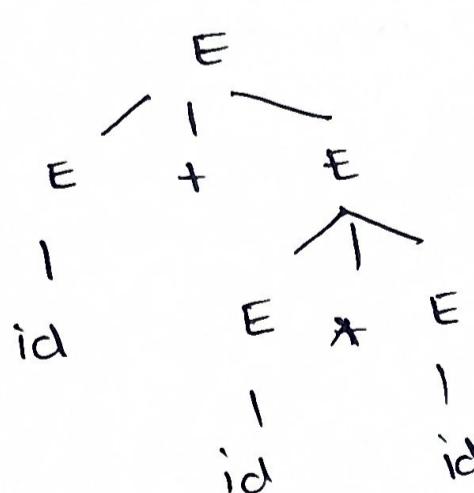
$$E \Rightarrow E * E$$

$$\Rightarrow E + E * E$$

$$\Rightarrow id + E * E$$

$$\Rightarrow id + id * E$$

$$\Rightarrow id + id * id$$



Two different parse trees \Rightarrow ambiguous

- For most parsers, the grammar must be unambiguously such that there is a unique selection of the parse tree for a sentence.
- Much prefer one of the parse trees of a sentence to disambiguate the grammar, and restrict to this choice.

Example Consider the following CFG.

$$S \rightarrow aABe$$

$$A \rightarrow Abc \mid b$$

$$B \rightarrow d$$

Parse abbcde using LMD & RMD

LMD

$$S \rightarrow aABe$$

$$\Rightarrow \cancel{a} ABe \Rightarrow aAde$$

$$\Rightarrow \cancel{aa}bcBe \Rightarrow abcde$$

RMD

$$S \rightarrow 'a ABe$$

$$\Rightarrow aAbcBe$$

$$\Rightarrow abbc\cancel{B}e$$

$$\Rightarrow abbcde$$

* Left Recursion and Left Factoring

* Types of Grammar

1. Ambiguous Grammar
2. Unambiguous Grammar
3. Recursive Grammar
4. Non-deterministic grammar
5. Deterministic Grammar

* Left Recursion

(9)

- A grammar is left-recursive if it has a non-terminal A such that there is a derivation
- $A \Rightarrow A\alpha$
- Top-down parsing techniques cannot handle left-recursive grammars
- The left-recursion may appear in a single step of the derivation (immediate left-recursion), or may appear in more than one step of the derivation

Handling Left Recursion (Immediate)

$$A \rightarrow A\alpha_1 | \dots | A\alpha_m | B_1 | B_2 | \dots | B_n$$

Then

(B_i s do not start with A)

$$A \rightarrow B_1 A' | B_2 A' | \dots | B_n A'$$

$$A' \rightarrow \alpha_1 A' | \dots | \alpha_m A' | \epsilon$$

Example 1 Eliminate left-recursion

$$E \rightarrow E + T | T$$

$$T \rightarrow T * F | F$$

$$F \rightarrow id | (E)$$

Ans LR in $E \rightarrow E + T$ and $T \rightarrow T * F$

The grammar becomes:

$$E \rightarrow^A E_1 T \mid^B T \quad \& \quad T \rightarrow T \& F \mid F$$

Anc w/o LR :

$$E \rightarrow T E'$$

$$E' \rightarrow + T E' \mid \emptyset$$

$$T \rightarrow F T'$$

$$T' \rightarrow * F T' \mid \emptyset$$

$$F \rightarrow \text{id} \mid (E)$$

* Removing left Recursion that is not immediate

→ A grammar may not be immediately left recursive, but it can still be left - recursive

$$\text{eq. } S \rightarrow Aa \mid b$$

$$A \rightarrow Sd \mid d$$

Not immediately left-recursive,

but $S \Rightarrow Aa \Rightarrow Sd a \Rightarrow$ left recursion

A ∵ Need to eliminate all left recursion from the grammar

Algorithm

Input: Grammar G_1 with no cycles or \emptyset -productions

Arrange the non-terminals in some order A_1, A_2, \dots, A_n

for $i = 1 \dots n$ do

 for $j = 1 \dots n$ do

 replace each

$$A_i \rightarrow A_i \emptyset$$

with

$$A_i \rightarrow S_1 S_2 S_3 \dots S_k$$

where

$$A_j \rightarrow S_1 S_2 \dots S_k$$

end do

eliminate the immediate left recursion in A_i .

end do

Example 1 Remove left recursion in.

$$S \rightarrow Aa \mid b$$

$$A \rightarrow Ac \mid Sd \mid F$$

Assume that the ordering of non-terminals is S, A

1. $S \rightarrow Aa \mid b \rightarrow \text{no LR}$

2. $A \rightarrow Ac \mid Sd \mid F$

↓

$$S \rightarrow Aa \mid b$$

$\beta_1 \quad \beta_2$

$$A \rightarrow Ac \mid Aa \beta_1 \mid bd \mid F$$

↓

Ans

$S \rightarrow Aa \mid b$ $A \rightarrow bdA' \mid FA'$ $A' \rightarrow cA' \mid adA' \mid \epsilon$
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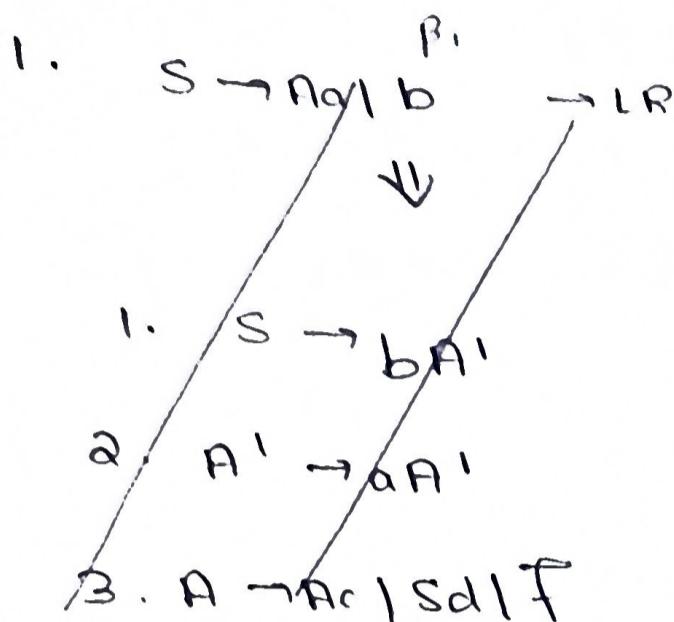
Example 2

$$S \rightarrow Aa|b$$

$$A \rightarrow Ac|Sa|f$$

Assume that the ordering of non-terminals is A, S

Ans



Ans 1. $S \rightarrow Aa|b$

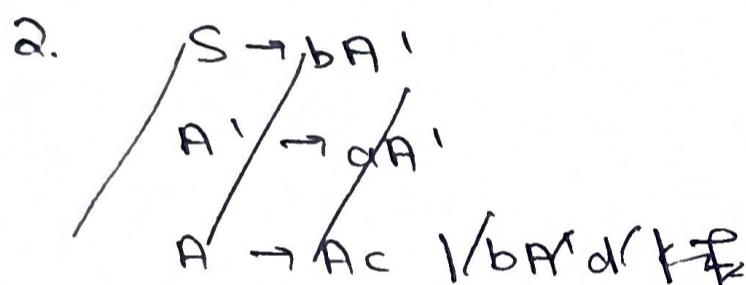
2. $A \rightarrow Ac|Sa|f$ \xrightarrow{LR}

P₂A

1. $S \rightarrow Aa|b$

$A \rightarrow SaA' | fA'$

$A' \rightarrow cA' | \epsilon$



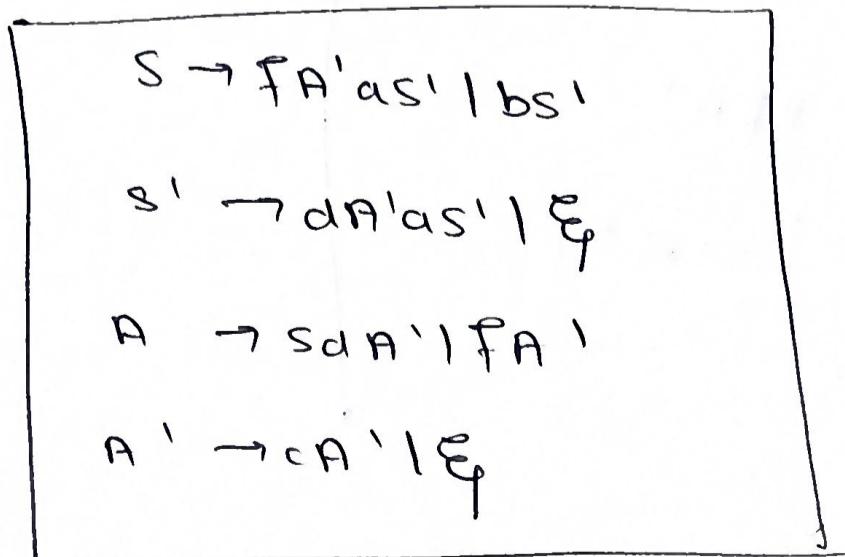
P₂S

$P_1 \quad P_2$

$$S \rightarrow SaA'a | fA'b | b$$
 \xrightarrow{LR}

$$\left. \begin{array}{l} S \rightarrow fA'as' | bs' \\ s' \rightarrow dA'as' | \epsilon \end{array} \right\}$$

\therefore The final grammar becomes:



Example 3

Remove LR

$$A \rightarrow ABDIAIA | a$$

$$B \rightarrow Belb$$

1. $A \rightarrow aA'$
2. $A' \rightarrow bA' | aA' | \epsilon$
3. $B \rightarrow bB'$
4. $B' \rightarrow eB' | \epsilon$

Example 4 Remove LR

$$S \rightarrow (L) | a$$

$$L \rightarrow LS | S$$

$$1. \quad S \rightarrow (L) | a$$

$$L \rightarrow SL'$$

$$L' \rightarrow , SL' | \epsilon$$

* Non-Deterministic Grammar

→ Grammar with a common prefix between at least two different productions coming from the same LHS.

$$\text{eg. } S \rightarrow \underline{a} \underline{s} \underline{b} \underline{l} \underline{a} A \underline{l} b$$

$$\text{eg. } S \rightarrow \underline{a} \underline{b} \underline{l} \underline{a} \underline{b} A$$

$$A \rightarrow \underline{bab} \underline{l} \underline{bab} c$$

Disadvantage: During parsing - needs a lot of back tracking
(time consuming)

Deterministic Grammar \rightarrow Grammar without any common prefix in any of the different productions from the same LHS

★ ★ ★ To make the grammar suitable for predictive or top-down parsing, we need to convert the non-deterministic grammar into deterministic grammar using Left Factoring

* Left Factoring?

\rightarrow A predictive parser (a top-down parser without backtracking) insists that the grammar must be left-factored.

If

stmt \rightarrow if expr then stmt . }
 | expr the stmt else stmt

When we see if, we would not know which production rule to choose.

To left factor :

$A \rightarrow \alpha\beta_1 | \alpha\beta_2$ (which α to choose?)

rewrite as

β_1, β_2 are different

$A \rightarrow \alpha A'$

$A' \rightarrow \beta_1 | \beta_2$

Algorithm

$A \rightarrow \alpha\beta_1 | \dots | \alpha\beta_n | \gamma_1 | \gamma_2 | \dots | \gamma_m$

convert to $A \rightarrow \alpha A' | \gamma_1 | \gamma_2 | \dots | \gamma_m$ 2 $A' \rightarrow \beta_1 | \dots | \beta_n$

Example 1 Left - factor :

$$A \rightarrow abB \mid aB \mid cdg \mid cdeB \mid cdfB$$

Ans $A \rightarrow aA'$

$$A' \rightarrow bB \mid B$$

$$A \rightarrow cdA''$$

$$A'' \rightarrow g \mid eB \mid fB$$

Example 2 Left - factor :

$$A \rightarrow \underline{aa} \mid \underline{a} \mid \underline{ab} \mid \underline{abc} \mid b$$

Ans $A \rightarrow aA' \mid b$

$$A' \rightarrow d \mid \epsilon \mid b \mid bc$$

⇓

$$A \rightarrow aA' \mid b$$

$$A' \rightarrow d \mid \epsilon \mid b \mid A''$$

$$A'' \rightarrow \epsilon \mid c$$

Example 3 Left factor : $S \rightarrow iETs \mid CETseS \mid a$
 $E \rightarrow b$

Ans $S \rightarrow iETsS' \mid a$

$$S' \rightarrow \epsilon \mid eS$$

$$E \rightarrow b$$

Example 4 : left factor :

$S \rightarrow \underline{a} S S b S \mid \underline{a} S a S b \mid \underline{a} b b \mid b$

Ans

$S \rightarrow a S' \mid b$

$S' \rightarrow \underline{S} S b S \mid \underline{S} a S b \mid b b$



$S \rightarrow a S' \mid b$

$S' \rightarrow \underline{s} b \underline{s} c \mid s s'' \mid b b$

$S'' \rightarrow S b S \mid a S b$

* Top - Down Parser → The parse tree is created top to bottom
 → can be : (i) recursive descent parsing
 (ii) predictive parsing

Difficulties with Top Down Parsing

(i) Left Recursion

(ii) Backtracking

(iii) Selection of Alternatives

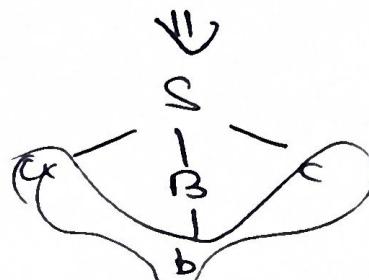
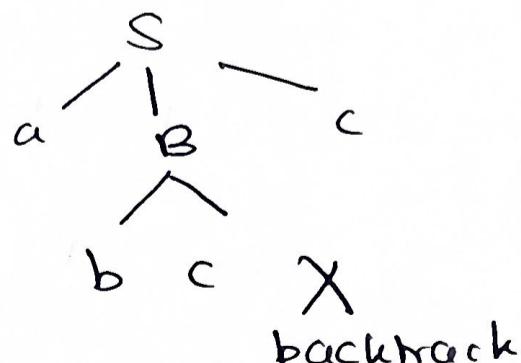
(iv) Error Reporting

Consider the grammar:

$S \rightarrow a B c$

$B \rightarrow b c \mid b$

For abc



A. | Recursive Descent Parsing |

- Backtracking is needed - if choice of production rule doesn't work, backtrack to other alternatives
- A general parsing technique, but not used widely
- Not efficient

B. | Predictive Parsing |

- no backtracking needed
- efficient
- needs a special form of grammar called LL(1) grammar
- Recursive Predictive Parsing is like A but no backtracking, also called the LL(1) parser.

Example : Consider the following grammar:

$$\begin{aligned}
 E &\rightarrow TE' \\
 E' &\rightarrow +TE' \mid \epsilon \\
 T &\rightarrow FT' \\
 T' &\rightarrow *FT' \mid \epsilon \\
 F &\rightarrow (E) \mid id
 \end{aligned}$$

Construct functions for a recursive descent parser and parse the following:

- (i) id + id * id
- (ii) id - id

Ans

① Procedure E()

Begin

TL()

EPrime()

End

② Procedure EPrime()

If input symbol = '+' then

Begin

Advance()

TL()

EPrime()

End

③ Procedure F()

If input symbol = 'id' then

Advance()

else if input symbol = '(' then

begin

Advance()

EL()

if input symbol = ')' then

Advance()

else

Error()

end

else

error();

④ Procedure TPrime()

If input symbol = '*' then

Begin

Advance()

FL()

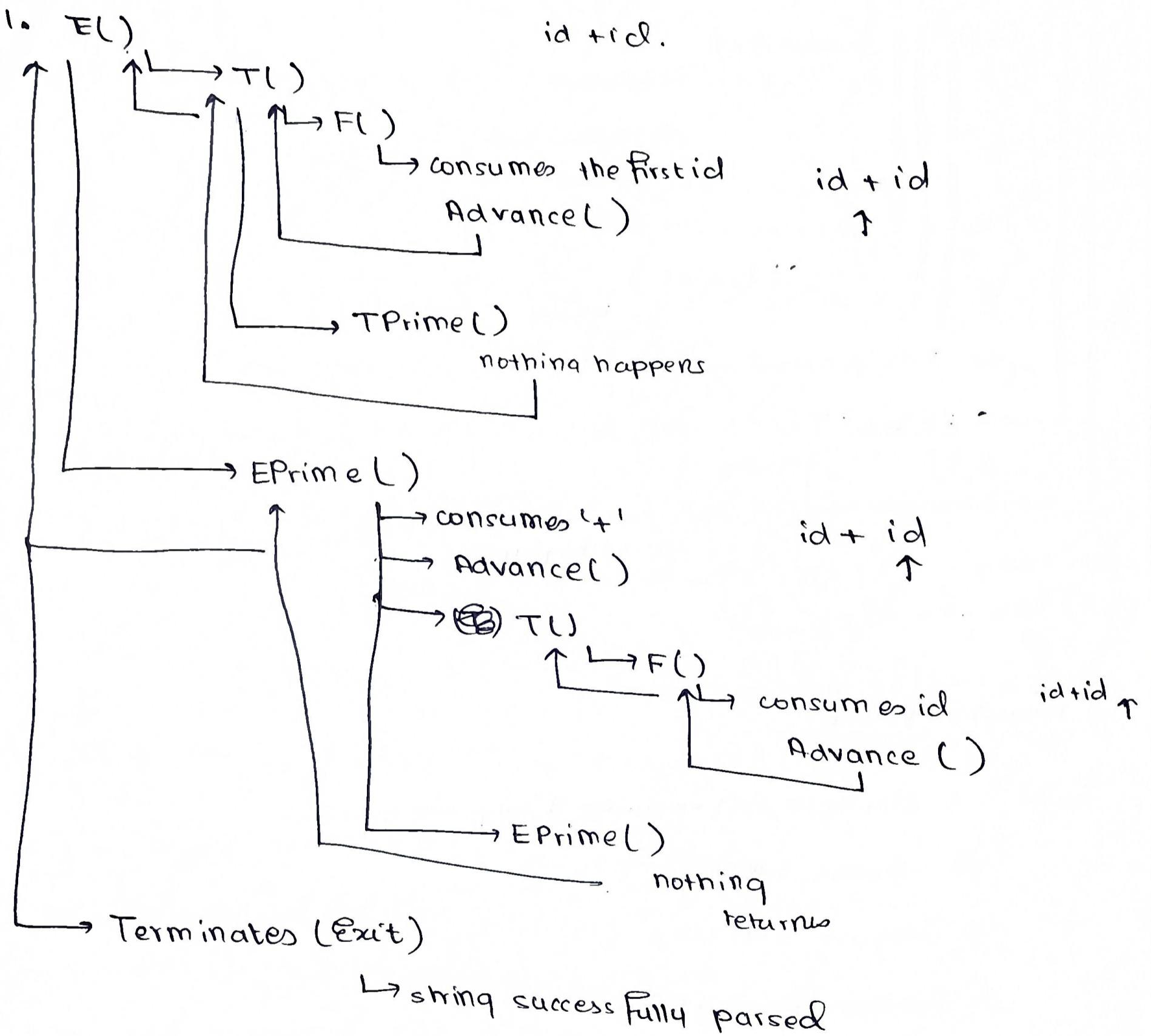
TPrime()

End

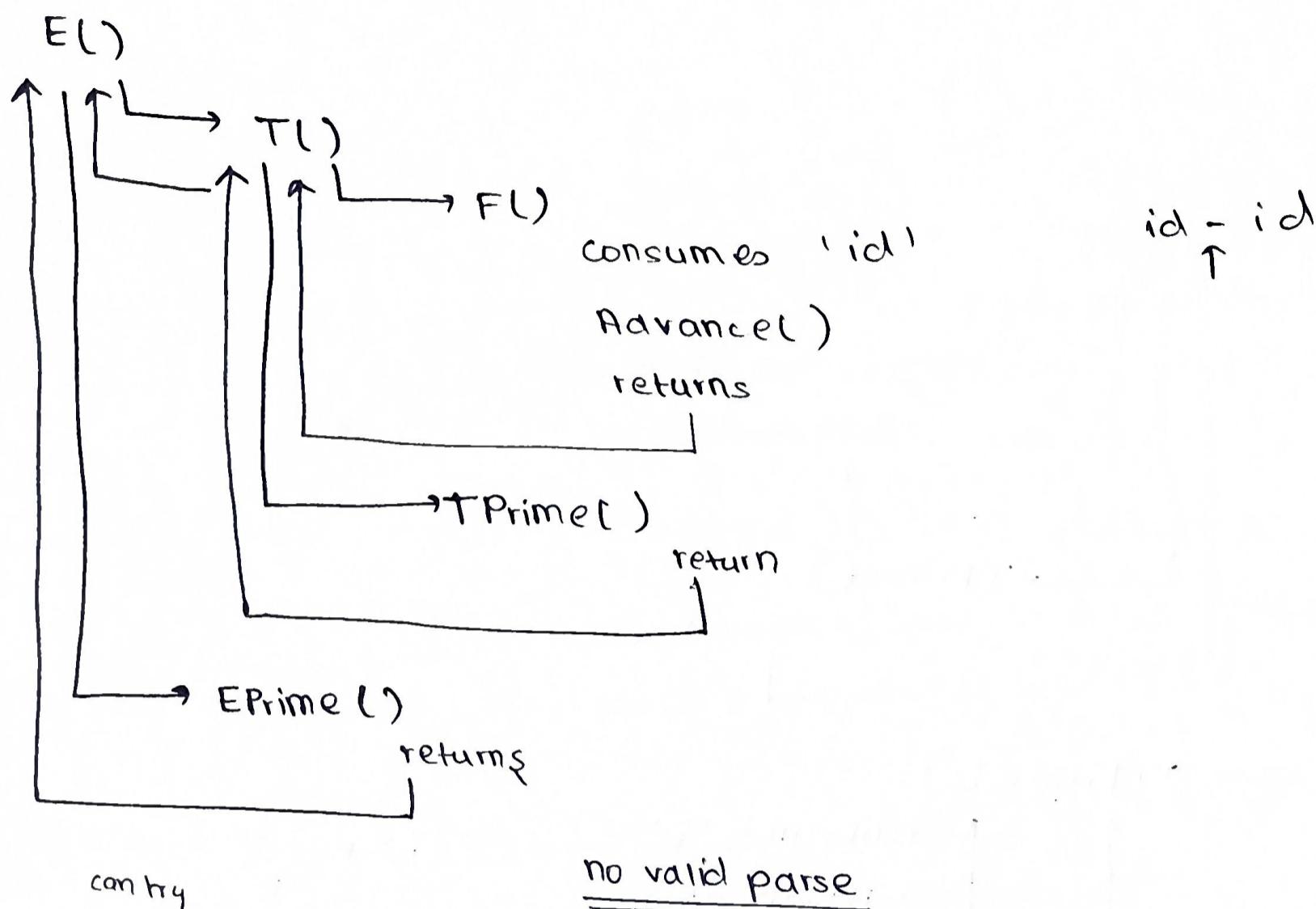
(19)

(i) id + id

Function call



(ii) id - id



(iii) id + id * id

* Predictive Parser

Steps

1. Eliminate left-recursion if any
2. Compute FIRST
3. Compute FOLLOW
4. Construct parsing table
5. Use parsing algo to check if syntactically correct or not

COMPUTE FIRST

Start from bottom

(i) X is terminal - $\text{FIRST}(X) = X$

(ii) X is a NT, $X \rightarrow \epsilon$ $\text{FIRST}(X) = \epsilon$

(iii) X is a NT, $X \rightarrow ABCD$ $\text{FIRST}(X) = \text{FIRST}(A)$

(see first symbol)

if $\text{FIRST}(A) = \epsilon$,

find $\text{FIRST}(B)$ and so on

If all $\text{FIRST}(Y_i) = \epsilon$ $\text{FIRST}(X) = \epsilon$

COMPUTE FOLLOW

→ For non-terminals (start from top) start symbol

(i) If start symbol, x $\text{FOLLOW}(X)$ has the symbol $\$$ too.

(ii) If $A \rightarrow \alpha \underline{B} \beta$ (B is squished between 2 characters)

then $\text{FOLLOW}(B) = \text{FIRST}(\beta)$

(no- ϵ)

If ϵ , then add $\text{FOLLOW}(A)$

(iii) If $A \rightarrow \alpha \underline{B}$ $\text{FOLLOW}(B) = \text{FOLLOW}(A)$

CONSTRUCT PARSE TREE

Rows = NT

columns = T + \$

For each production rule $A \rightarrow \alpha$

For each terminal a in $\text{FIRST}(\alpha)$ Add $A \rightarrow \alpha$ to $M[A, a]$

If ϵ in $\text{FIRST}(\alpha)$, for each terminal a in $\text{FOLLOW}(A)$

Add $A \rightarrow \alpha$ to $M[A, a]$

If ϵ in $\text{FIRST}(\alpha)$ and $\$$ in $\text{FOLLOW}(A)$

Add $A \rightarrow \alpha$ to $M[A, \$]$

Example 1 : Construct a predict parser for the following grammar.

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' \mid \epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' \mid \epsilon$$

$$F \rightarrow (E) \mid id$$

①

$$\text{FIRST}(E) = \text{FIRST}(T) = \text{FIRST}(F) = \{ (, id \}$$

$$\text{FIRST}(E') = \{ +, \epsilon \}$$

$$\text{FIRST}(T') = \{ *, \epsilon \}$$

②

$$\text{FOLLOW}(E) = \{ \$,) \}$$

$$\text{FOLLOW}(T) = \{ +, \$,) \}$$

$$\text{FOLLOW}(F) = \{ *, +, \$,) \}$$

$$\text{FOLLOW}(E') = \{ \$,) \}$$

$$\text{FOLLOW}(T') = \{ +, \$,) \}$$

③ Construction of Parsing Table

$E \leftarrow E + T$

$Z \leftarrow Z, T$

$i \leftarrow i + 1$

$E \leftarrow$

$, T, E \leftarrow$

$\$ P_1 +$

$P_1, T, E \leftarrow$

$\$ P_1 \rightarrow \bar{P}_1$

$\bar{P}_1, T, E \leftarrow$

$\$ P_1 + P_1$

$\bar{P}_1, T, E \leftarrow$

$\$ P_1 + P_1 *$

$\bar{P}_1, T, E \leftarrow$

$T \leftarrow T * \leftarrow , T$

$\$ P_1 + P_1 *$

$, T, E \leftarrow$

$\$ P_1 + P_1 + \bar{P}_1$

$\bar{P}_1, T, E \leftarrow$

$P_1 \leftarrow F$

$\$ P_1 + P_1 * P_1$

$\bar{P}_1, T, E \leftarrow$

$T \leftarrow T - F$

$\$ P_1 + P_1 * P_1$

$\bar{P}_1, T, E \leftarrow$

$E \leftarrow T E$

$\$ P_1 + P_1 * P_1$

$\bar{P}_1, E \leftarrow$

OUTPUT

INPUT

STACK

pop out
input
Top stack \Rightarrow match
 \Rightarrow see table
Input
Not on top of stack \Rightarrow

$P_1 + P_1 * P_1 = m$

④ Parse input string

	$P_1 \leftarrow F$	$Z \leftarrow , T$	$F \leftarrow (E)$	$Z \leftarrow T, T \leftarrow E T$	$Z \leftarrow , T$	F
$Z \leftarrow , T$						
$T \leftarrow T - F$						
$E \leftarrow , E + T E$						
E	P_1	$($	$)$	$*$	$+$	N

$\$ E' T^+$	$\star \underline{id} \$$	
$\$ E' T$	$id \$$	$T \rightarrow F T'$
$\$ E' T' F$	$id \$$	$F \rightarrow id.$
$\$ E' T' id$	$\underline{id} \$$	
$\$ E' T'$	$\$$	$T' \rightarrow \epsilon$
$\$ E'$	$\$$	$E' \rightarrow \epsilon$
$\underline{\$}$	$\underline{\$}$	

Stack & input are empty \rightarrow parse is valid

Example 2 : Compute FIRST & FOLLOW for the following grammars

$$\begin{aligned} S &\rightarrow A \\ A &\rightarrow aB \mid Ad \\ B &\rightarrow b \\ C &\rightarrow g \end{aligned}$$

Ans Remove LR

$$\begin{aligned} S &\rightarrow A \\ A &\rightarrow aBA' \\ A' &\rightarrow aA' \mid \epsilon \\ B &\rightarrow b \\ C &\rightarrow g \end{aligned}$$

FIRST

$$\text{FIRST}(S) = \{a\}$$

$$\text{FIRST}(B) = \{b\}$$

Find the NTs

$$\text{FIRST}(A') = \{d, \emptyset\}$$

$$\text{FIRST}(BA) = \{a\}$$

$$\text{FIRST}(S) = \text{FIRST}(A) = \{a\}$$

FOLLOW

~~$\text{Follow}(S) =$~~

$$\text{FOLLOW}(S) = \{\$\}$$

$$\text{FOLLOW}(A) = \{\$\}$$

$$\text{FOLLOW}(A') = \{\$\}$$

$$\text{FOLLOW}(A')$$

$$\text{FOLLOW}(B) = \text{FIRST}(A') = \{d, \emptyset\}$$

$$\text{FOLLOW}(C) = \{\}$$

Example 3 : Construct a predictive parser for the following grammar.

$$S \rightarrow (L) \mid a$$

$$L \rightarrow L, S \mid S$$

Parse the string : (a, a)

Ans Eliminate Left recursion

$$S \rightarrow (L) \mid a$$

$$L \rightarrow SL^1$$

$$L^1 \rightarrow , SL^1 \mid \emptyset$$

The grammar is:

$$S \rightarrow (L) | a$$

$$\textcircled{5} \quad L \rightarrow SL^1$$

$$L^1 \rightarrow , SL^1 | \epsilon$$

$$\text{FIRST}(S) = \{ (, a \}$$

$$\text{FIRST}(L) = \text{FIRST}(S) = \{ (, a \}$$

$$\text{FIRST}(L^1) = \{ , , \epsilon \}$$

~~FIRST(S)~~

~~$$\text{FOLLOW}(S) = S, (, a \}$$~~

~~FOLLOW FIRST(L^1), FOLLOW(L)~~

~~$$\text{FOLLOW}(S) = \{ , , \$ \}$$~~

~~FOLLOW(L), FOLLOW(L^1)~~

~~$$\text{FOLLOW}(S) = \{ \$, , , \}$$~~

$$\text{FOLLOW}(L) = \{) \}$$

$$\text{FOLLOW}(L^1) = \{) \}$$

see first symbol of each production
and find FIRST() if \$\in\$ follow

NT	()	a	,	\$
S	$s \rightarrow (L)$		$s \rightarrow a$		
L	$L \rightarrow SL^1$		$L \rightarrow SL^1$		
L^1		$L^1 \rightarrow \epsilon$		$L^1 \rightarrow , SL^1$	

Parsing string (a,a)

Stack

\$ S\$) L (\$) L\$) L' S\$) L' a\$) L'\$) L' S ,\$) L' S\$) L' a\$) L'\$)\$

Input

(a,a) \$(a,a) \$

Output

 $s \rightarrow (L)$ $L \rightarrow SL'$ $s \rightarrow a$ $L' \rightarrow , SL'$ $s \rightarrow a$ $L' \rightarrow \epsilon$

String parsed successfully

* / \ / Grammar and Error Recovery Techniques

Example 4 : Construct a predictive parser for the following grammar.

$$S \rightarrow i E t s \underline{s} s' | a$$

$$S' \rightarrow e S | \epsilon$$

$$E \rightarrow b$$

Ans $\text{First}(E) = b$

$$\text{First}(S') = \{e, \epsilon\}$$

$$\text{FIRST}(S) = \{i, a\}$$

$$\text{FOLLOW}(S) = \{ \$, e \} \quad \text{FOLLOW}(S), \text{ FOLLOW}(S')$$

$$\text{FOLLOW}(S') = \begin{matrix} \text{FOLLOW}(S) \\ \{\$, e\} \end{matrix}$$

$$\text{FOLLOW}(E) = \{t\}$$

	i	t	a	e	b	\$
S	$s \rightarrow i E t s s'$		$s \rightarrow a$			
S'				$s' \rightarrow e S$ $s' \rightarrow \epsilon$		$s' \rightarrow \epsilon$
E					$E \rightarrow b$	

* LL(1) Grammar

→ A grammar whose parsing table has no multiply-defined entries is said to be an LL(1) Grammar.

LL(1)

↓ ↗ left most derivation

input scanned
from L → R

* What to do if parse table has multiply defined entries?

1. Eliminate left recursion
2. Left factor grammar
3. If the new grammar's parsing table still contains multiply defined entries, that grammar is ambiguous or it is inherently not a LL(1) grammar.

* Properties of LL(1) Grammar

→ A grammar G_1 is LL(1) iff the following conditions hold for two distinctive production rules

$$A \rightarrow \alpha \text{ and } A \rightarrow \beta$$

- Both $\alpha \neq \beta$ cannot derive symbols starting w/ the same terminals
- At most one of $\alpha \neq \beta$ can derive to ϵ

* Error Recovery in Predictive Parsing

An error may occur in predictive parsing if:

- (i) the terminal symbol on top of the stack doesn't match with the current input symbol
- (ii) If the top of the stack is a non-terminal A, the current input symbol is a, and the parsing table entry $M[A, a]$ is empty

* What should the parser do in case of an error?

1. An error msg. as meaningful as possible
2. Be able to recover from that error case and continue w/ parsing with the rest of the input.

* Error Recovery Techniques

1. | Panic Mode Error Recovery |

→ Skipping the input symbols until a synchronizing token is found

2. | Phrase-level Error Recovery |

→ Each empty entry in the parsing table is filled w/ a pointer to a specific error routine to take care of that error case.

3. | Error Productions |

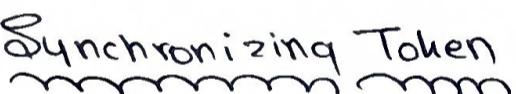
→ Augment the grammar with productions that generate erroneous constructs

- When an error production is used by the parser, generate appropriate error diagnostics.
- Since it is almost impossible to know all the errors that can be made, not practical

4. Global Collection

- Ideally, we want the compiler to make as few changes as possible in processing incorrect inputs.
- Have to globally analyze the input to find the error.
- Expensive method, not used in practice.

* Panic - Mode Error Recovery → pro - can be automated
 con - error msgs needed

- Skip all the input tokens until a synchronizing token is found.
- Synchronizing Token  - All the terminal symbols in the follow set of a non-terminal can be used as a synchronizing token set for that NT

∴ In a simple system:

- ① All the empty entries are marked as sync
- ② Parser will skip all the input symbols until a symbol in the follow set of the non-terminal A is on top of the stack
- ③ Then pop that NT-A from the stack
- ④ Parsing continues from that state.

⑤ To handle unmatched terminal symbols, the parser pops that unmatched terminal symbol from the stack & issues an error msg. about the unmatched terminal symbol.

Example : Use predictive parsing to construct a parse tree for the following grammar. Parse the strings aab & ceadb and use panic-mode error recovery.

$$S \rightarrow A \cup S \mid \epsilon$$

$$A \rightarrow a \mid c A d$$

$$\text{FIRST}(A) = \{a, c\}$$

$$\text{Ans} \quad \text{FIRST}(S) = \{a, c, e, \epsilon\}$$

$$\text{FOLLOW}(A) = \{b, d\}$$

$$\text{FOLLOW}(S) = \{\$\}$$

NT	a	b	e	c	d	\$
S	$S \rightarrow A \cup S$	$S \rightarrow a$ sync	$S \rightarrow \epsilon$	$S \rightarrow A \cup S$	sync	$S \rightarrow \epsilon$
A	$A \rightarrow a$	sync	sync	$A \rightarrow c A d$	sync	sync

STACK

for aab

INPUT

OUTPUT

\$ Saab\$ $S \rightarrow a b S$ \$ Sbaaab\$\$ Sbab \$

no match - error

since $M[S, b] = \text{sync}$ 

In this case there
is ~~NOT~~-~~NOT~~
mismatch $\overset{b}{\text{pop}}$ from
stack , and
ignore that anything
happened

\$ Sbaab\$ $S \rightarrow A b S$ Take off the b\$ Sab\$ $A \rightarrow a$ \$ SbAab\$\$ Sbaab\$\$ Sbb\$\$ S\$ $S \rightarrow \epsilon$ \$accept

for ceadb

Stack

\$ S

Input

ceadb\$

Output

$S \rightarrow A B S$

\$ S b A

ceadb\$

~~S~~ A B S

$A \rightarrow c A d$

\$ S b d A C

ceadb\$

\$ S b d A

e a d b \$

error
take off A

keep taking of input until you get

(since here a NT is on top of the stack)

\$ S b d

db\$

$\text{FOLLOW}(A) = \{ b, d \}$

$= \{ b, d \}$

\$ S b

b\$

\$ S

\$

$S \rightarrow \epsilon$

\$

\$

accepted

* Phrase Level Error Recovery

→ pro: can be automated
→ con: recovery not intuitive

75

→ Each empty entry in the parse table is filled with a pointer to a special error routine which will take care of that error case.

→ These error routines may

- change / insert / delete i/p symbols
- issue appropriate error msgs
- pop items from the stack

→ Must be careful while designing these error routines as we may put the parser into an ∞ loop

* Error Productions

→ augment / ignore and parse

pro : powerful recovery method

con : cannot be automated

e.g. add error production

$$T^1 \rightarrow FT^1$$

to ignore missing * , eq id id

* YACC - Yet Another Compiler - Compiler

→ YACC is a tool designed to generate a parser based on a grammar you define.

→ used for LALR(1) grammars

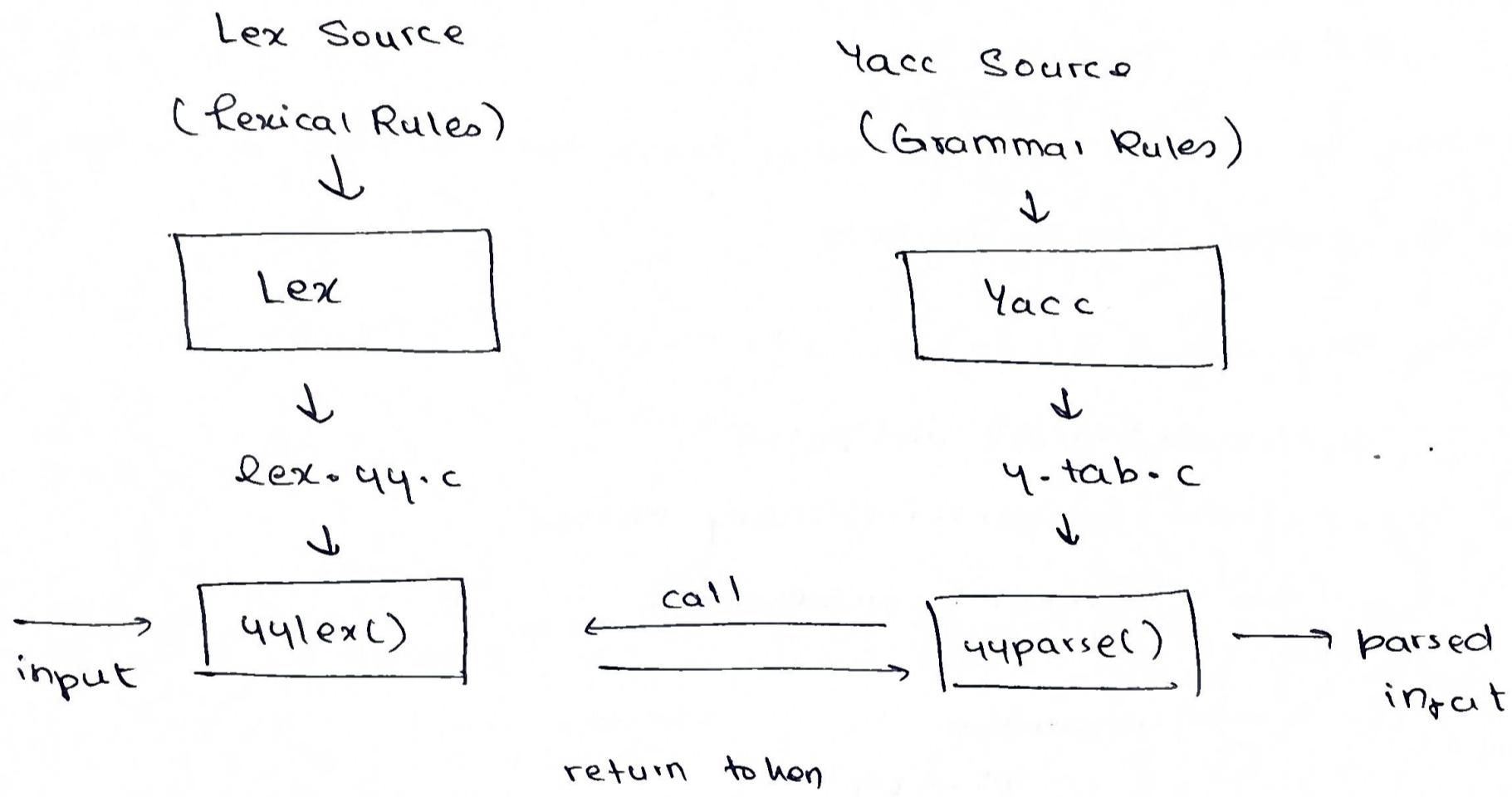
→ produces the source code for the parser (syntactic analyzer) of a language

* How YACC works with flex

1. Flex → for lexical analysis - scans the input and produces tokens
2. Yacc uses the tokens to perform syntactic analysis & checks if the tokens follow the grammar rules.

yylex() → flex fn. to return tokens

yyparse() → Yacc function to parse ilp



* YACC File Format

% S

C Declarations

% }

Yacc declarations

/*.

Grammar rules

/*.

Additional code

* Example of Grammar Rules

expr : expr '+' term { \$\$ = \$1 + \$3 }

| term

;

term : term '*' factor { }

| Factor

;

factor : '(' expr ')' { }

| ID

| NUM

;

→ Each production rule has an action enclosed in {}

\$\$ = value of current non-terminal

\$1, \$2, \$3 values of elements on the RHS of the prod rule.

* Special Yacc Declarations

%. start → start symbol

%. token → token symbols

%. type → data type for non-terminal symbols

%. right, %. left & %. nonassoc → associativity & precedence for operators

* Bottom-up Parsing

- Builds the parse-tree from the leaves up to the root
- This approach tries to find the right-most derivation of the input in reverse order.
- Starts with the input string & attempts to reduce it step-by-step to the start symbol

of Two - Types

- shift-reduce parsing
- operator-precedence parsing

Efficient Methods

- LR methods (left-to-right, rightmost derivation in reverse)
- SLR, Canonical LR, LALR

* Shift - Reduce Parsing

- reduce given input string into the start symbol
- Two main operations:
 - (i) Shift: Move the next input symbol onto a stack
 - (ii) Reduce: Replace a sequence of symbols on the stack, which matches the RHS of a production rule, with the corresponding non-terminal on the LHS

There are four actions the parser can take:

1. Shift: Push the next input symbol onto the stack
2. Reduce: Replace the top symbols on the stack with the LHS of a production rule.
3. Accept: If the input has been parsed successfully, the parser halts.
4. Error: If the parser detects an error, it calls an error recovery routine.

Example Parse $a b b c d e$ using shift-reduce method. The grammar is:

$$S \rightarrow a A B e$$

$$A \rightarrow A b c \mid b$$

$$B \rightarrow d$$

Input

Stack

$a b b c d e$

$\$$

$b b c d e$

$\$ a$

$b c d e$

$\$ a b$

$c d e$

$\$ a b b$

$d e$

$\$ a b b c$

$d e$

$\$ a \overset{A}{b} b c$

Operation/
Production Rule

shift

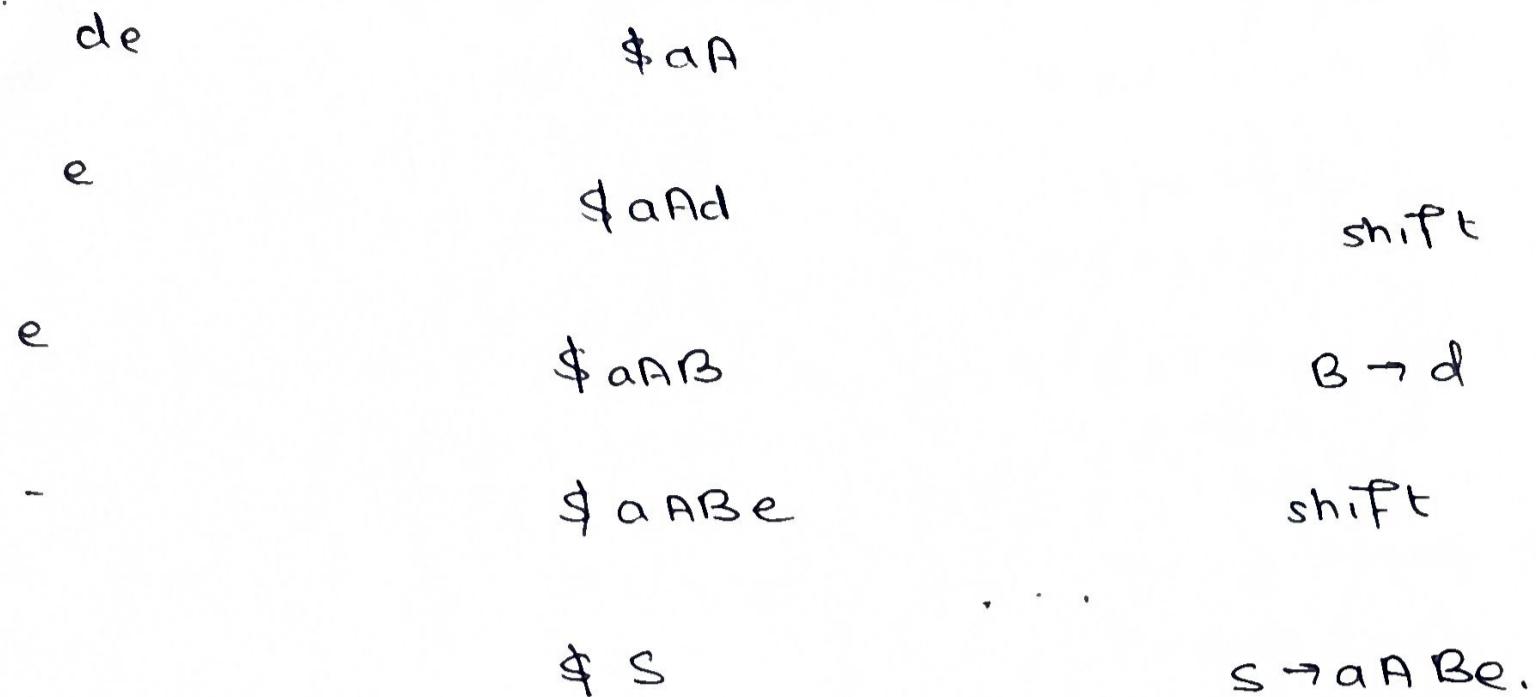
shift

shift

shift

$A \rightarrow A b c$

$A \rightarrow b$



* Handle

- A handle of a string is a substring that matches the RHS of a production
- Unambiguous forms of the grammar would have exactly one handle for every right sentential form.
- Handle pruning is the process of reducing handles repeatedly in a right-most derivation in reverse order.

$$S = \gamma_0 \xrightarrow{rm} \gamma_1 \xrightarrow{rm} \gamma_2 \xrightarrow{rm} \dots \xrightarrow{rm} \gamma_{n-1} \xrightarrow{} \gamma_n = w$$

Example for $S \rightarrow aABe$

$$\begin{aligned} A &\rightarrow Abc \mid b \\ B &\rightarrow d \end{aligned}$$

abbcde
a Abc de
a Adee
a ABe

underlined terms
are handles

however

aAbcde

is not a handle → result is
not in sentential
form
(cannot be derived from S)

Example Use a shift-reduce parser for the string $\text{id} + \text{id} * \text{id}$ for the grammar:

$$E \rightarrow E + T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (\text{E}) \mid \text{id}$$

Ans Right-most Derivation.

$$E \rightarrow E + T$$

$$\Rightarrow E + T * F$$

$$\Rightarrow E + T * \text{id}$$

$$\Rightarrow E + F * \text{id}$$

$$\Rightarrow E + \text{id} * \text{id}$$

$$\Rightarrow T + \text{id} * \text{id}$$

$$\Rightarrow F + \text{id} * \text{id}$$

$$\Rightarrow \text{id} + \text{id} * \text{id}$$

Input

Stack

Operation /

Production Rule

$\text{id} + \text{id} * \text{id}$

\$

-

$+ \text{id} * \text{id}$

\$ id

shift

$+ \text{id} * \text{id}$

\$ F

reduce

$$F \rightarrow \text{id}$$

$+ \text{id} * \text{id}$

\$ T

~~$F \rightarrow \text{id}$~~ reduce
 $T \rightarrow E$

$+ \text{id} * \text{id}$

\$ E

$$E \rightarrow T$$

$\text{id} * \text{id}$

\$ E +

shift

$* \text{id}$

\$ E + id

shift

$* \text{id}$

\$ E + F

$$\text{reduce}$$

 $F \rightarrow \text{id}$

* id

 $\$ E + T$

reduce

 $T \rightarrow F$

id

 $\$ E + T *$

shift

 $\$$ $\$ E + T * id$

shift

 $\$$ $\$ E + T * F$

reduce

 $F \rightarrow id$ $\$$ $\$ E + T * F$ $T \rightarrow T * F$ $\$$ $\$ E$ $E \rightarrow E + T$

↓

start symbol

* Conflicts during Shift-Reduce Parsing?

- There are CFGs for which shift-reduce parsers cannot be used.
- Stack contents and the next input symbol may not decide:

(i) shift/reduce conflict - whether to make a shift operation or a reduction

(ii) reduce/reduce conflict - The parser cannot decide which of several reductions to make.

Shift-Reduce Conflicts

$$\begin{aligned} S &\rightarrow \text{if } E \text{ then } S \mid \\ &\quad \text{if } E \text{ then } S \text{ else } S \mid \\ &\quad \text{other} \end{aligned}$$

Stack	Input	Action
\$. . F E then S	else ... \$	shift or reduce ?

Soln: resolve in favor of shift, so else matches closest if

Reduce - Reduce Conflicts

Grammar:

$$C \rightarrow AB$$

$$A \rightarrow a$$

$$B \rightarrow a$$

Stack

\$

\$ a

Input

aa \$

a \$

Action

shift

reduce to $A \rightarrow a$ or $B \rightarrow a$?

* Simple LR Parser

↳ A kind of shift-reduce parser that can handle a large class of context-free grammars.

L: left-to-right scanning of input

R: rightmost derivation in reverse

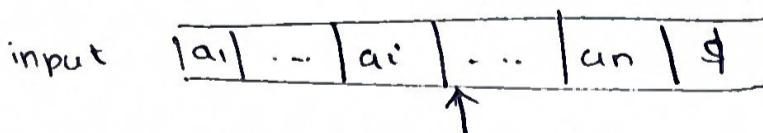
Types

→ SLR

→ CLR

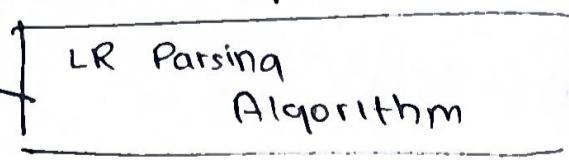
→ LALR

(lookahead)

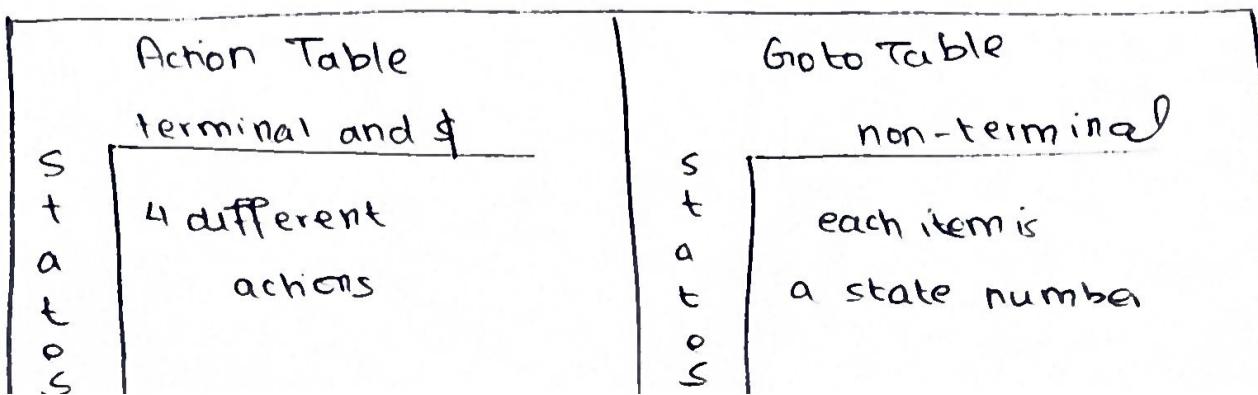


stack

s_m
x_m
s_{m-1}
x_{m-1}
\vdots
s_1
x_1
s_0



→ output



Steps
mn

1. Construct an augmented grammar G' for any grammar G
2. Construct LR(0) collection of items
3. Construct PT Δ
4. Find FOLLOW for the non-terminals
5. Construct the parsing table

Example Construct an SLR parser for the following grammar.

$$E \rightarrow E + T$$

$$E \rightarrow T$$

$$T \rightarrow T * F$$

$$T \rightarrow F$$

$$F \rightarrow (E)$$

$$F \rightarrow id$$

Parse the string id + id * id

Step 1: Construct the augmented grammar

$$E' \rightarrow E$$

$$E \rightarrow E + T$$

$$E \rightarrow T$$

$$T \rightarrow T * F$$

$$T \rightarrow F$$

$$F \rightarrow (E)$$

$$F \rightarrow id$$

Step 2 Construct the canonical LR(0) collection

Use the closure & GOTO operations

$I_0 : E' \rightarrow \cdot E$
 $E \rightarrow \cdot E + T$
 $E \rightarrow \cdot T$
 $T \rightarrow \cdot T * F$
 $F \xrightarrow{T \rightarrow F} \cdot (E)$
 $F \rightarrow \cdot id$

$I_1 : \text{GOTO}(I_0, E)$
 $E' \rightarrow E \cdot$

$I_2 : \text{GOTO}(I_0, T)$
 $E \rightarrow T \cdot$
 $T \rightarrow T \cdot * F$

$I_3 : \text{GOTO}(I_0, F)$
 $T \rightarrow F \cdot$

$I_4 : \text{GOTO}(I_0, ())$
 $F \rightarrow (\cdot E)$

$\text{2. } E \rightarrow \cdot E + T$
 $E \rightarrow \cdot T$
 $T \rightarrow \cdot T * F$
 $T \rightarrow \cdot F$
 $F \rightarrow \cdot (E)$
 $F \rightarrow \cdot id$

$I_5 : \text{GOTO}(I_0, id)$
 $F \rightarrow id \cdot$

$I_6 : \text{GOTO}(I_1, +)$
 $E \rightarrow E + \cdot T$
 $T \rightarrow \cdot T * F$
 $T \rightarrow \cdot F$
 $F \rightarrow \cdot (E)$
 $F \rightarrow \cdot id$

$I_7 : \text{GOTO}(I_2, *)$
 $T \rightarrow T * \cdot F$
 $F \rightarrow \cdot (E)$
 $F \rightarrow \cdot id$

$I_8 : \text{GOTO}(I_4, E)$
 $F \rightarrow (E \cdot)$
 $E \rightarrow E \cdot + T$

$I_9 : \text{GOTO}(I_6, +)$
 $E \rightarrow E + T \cdot$
 $T \rightarrow T \cdot * F$

$I_{10} : \text{GOTO}(I_7, F)$
 $T \rightarrow T * F \cdot$

$I_{11} : \text{GOTO}(I_8, ())$
 $F \rightarrow (E) \cdot$

Grammar :

- ① $E' \rightarrow E$
- ② $E \rightarrow E + T$
- ③ $E \rightarrow T$
- ④ $T \rightarrow T * F$
- ⑤ $T \rightarrow F$
- ⑥ $F \rightarrow (E)$
- ⑦ $F \rightarrow id$

Step 3 Construct the parse table

State	action							goto		
	-	*	()	id	\$	E	T	F	
0			s_4	$\$$	s_5		1	2	3	
1	s_6									
2	R_2	s_7		R_2		R_2				
3	R_4	R_4		R_4		R_4				
4			s_4	$\\$	s_5		8	2	3	
5	R_6	R_6		R_6		R_6				
6			s_4		s_5		8	9	3	
7			s_4		s_5				10	
8	s_6			s_{11}						
9	R_1	s_7		R_1		R_1				
10	R_3	R_3		R_3		R_3				
11	R_5	R_5		R_5		R_5				

$$\text{FOLLOW}(E) = \{\$, +,)\}$$

Grammar

$$\text{FOLLOW}(T) = \{\$, +,), *\}$$

$$1. E \rightarrow E + T$$

$$\text{FOLLOW}(F) = \{\$, +,), *, *\}$$

$$2. E \rightarrow T$$

$$3. T \rightarrow T * F$$

$$4. T \rightarrow F$$

$$5. F \rightarrow (E)$$

$$6. F \rightarrow \text{id}$$

For action table: look for $\cdot T$

write the no, where it becomes T .

For GOTO table look for $\cdot E \rightarrow T$

write the no when it becomes NT .

For reduce operation, look for $NT \rightarrow$

write Prod no.

Example 2 : Construct a parse tree for the following grammar using SLR parsing

$$S \rightarrow L = R$$

$$S \rightarrow R$$

$$L \rightarrow * R$$

$$L \rightarrow id$$

$$R \rightarrow L$$

Ans Augmented Grammar

$$S' \rightarrow S$$

$$S \rightarrow L = R$$

$$S \rightarrow R$$

$$L \rightarrow * R$$

$$L \rightarrow id$$

$$R \rightarrow L$$

$$\text{FOLLOW}(S) = \{ \$ \}$$

$$\text{FOLLOW}(L) = \{ =, \$ \}$$

$$\text{FOLLOW}(R) = \{ \$, = \}$$

Canonical LR(0) Collection

I_0 : closure

$$S' \rightarrow \cdot S \quad \checkmark$$

$$S \rightarrow \cdot L = R \quad I_4 : \text{GOTO}(I_0, =)$$

$$S \rightarrow \cdot R \quad \checkmark$$

$$L \rightarrow \cdot * R \quad \checkmark$$

$$L \rightarrow \cdot id \quad \checkmark$$

$$R \rightarrow \cdot L \quad \checkmark$$

$$I_1 : \text{GOTO}(I_0, \cdot S)$$

$$S' \rightarrow S \cdot \quad \checkmark$$

$$I_2 : \text{GOTO}(I_0, L)$$

$$S \rightarrow L \cdot = R \quad \checkmark$$

$$R \rightarrow L \cdot$$

$$I_3 : \text{GOTO}(I_0, R)$$

$$S \rightarrow R \cdot \quad \checkmark$$

$$I_4 : \text{GOTO}(I_0, *)$$

$$L \rightarrow * \cdot R \quad \checkmark$$

$$R \rightarrow \cdot L$$

$$L \rightarrow \cdot id \quad \checkmark$$

$$L \rightarrow \cdot * R$$

$$I_5 : \text{GOTO}(I_0, id)$$

$$L \rightarrow id \cdot \quad \checkmark$$

$$I_6 : \text{GOTO}(I_2, =)$$

$$S \rightarrow L = \cdot R \quad \checkmark$$

$$R \rightarrow \cdot L$$

$$L \rightarrow \cdot * R \quad \checkmark$$

$$L \rightarrow \cdot id \quad \checkmark$$

$$I_7 : \text{GOTO}(I_4, R)$$

$$L \rightarrow * R \cdot \quad \checkmark$$

$$I_8 : \text{GOTO}(I_4, L)$$

$$R \rightarrow L \cdot$$

$$I_9 : \text{GOTO}(I_6, R)$$

$$S \rightarrow L = R \cdot \quad \checkmark$$

$$L \rightarrow * R \cdot$$

$$I_{10} : \text{GOTO}(I_6, *)$$

$$L \rightarrow * \cdot R \quad \checkmark$$

$$R \rightarrow \cdot L$$

$$L \rightarrow \cdot * R \quad \checkmark$$

$$L \rightarrow \cdot id \quad \checkmark$$

(redacted)

(redacted)

(redacted)

Parsing Table

	ACTION				GOTO			L
	=	*	id	\$	S	R	Q	L
0		S_4	S_5		-	2	3	
1	.			ACC	.			
2	S_6, R_5			R_5	.			
3	.			R_2	.			
4	.	S_4	S_5		.	7		8
5	R_4	.		R_4	.			
6	.	S_4	S_5		.	9		8
7	R_3	.		R_3	.			
8	R_5			R_5				
9	R_3			R_1, R_3				

GRAMMAR

- ① $S \rightarrow L = R$
- ② $S \rightarrow R$
- ③ $L \rightarrow * R$
- ④ $L \rightarrow id$
- ⑤ $R \rightarrow L$

Example 3 : Construct a parse table using an SLR parser for:

$$E \rightarrow E + T \mid T$$

$$T \rightarrow TF \mid F$$

$$F \rightarrow F * \text{ or } b$$

Ans Augmented Grammar

$$E' \rightarrow E$$

$$\textcircled{1} \quad E \rightarrow E + T$$

$$\textcircled{2} \quad E \rightarrow T$$

$$\textcircled{3} \quad T \rightarrow TF$$

$$\textcircled{4} \quad T \rightarrow F$$

$$\textcircled{5} \quad F \rightarrow F *$$

$$\textcircled{6} \quad F \rightarrow a$$

$$\textcircled{7} \quad F \rightarrow b$$

(Canonical LR(0) Collection)

Closure

$$I_0: \quad E' \rightarrow \cdot E$$

$$E \rightarrow \cdot E + T$$

$$E \rightarrow \cdot T$$

$$T \rightarrow \cdot TF$$

$$T \rightarrow \cdot F$$

$$F \rightarrow \cdot F *$$

$$F \rightarrow \cdot a$$

$$F \rightarrow \cdot b$$

$$I_1 = \text{GOTO}(I_0, E)$$

$$E' \rightarrow E \cdot$$

$$E \rightarrow E \cdot + T$$

$$I_2 = \text{GOTO}(I_0, T)$$

$$E \rightarrow T \cdot$$

$$T \rightarrow T \cdot F$$

$$F \rightarrow \cdot F *$$

$$I_3 = \text{GOTO}(I_0, F)$$

$$T \rightarrow F \cdot$$

$$F \rightarrow F \cdot *$$

$$I_4 = \text{GOTO}(I_0, a)$$

$$F \rightarrow a \cdot$$

$$I_5 = \text{GOTO}(I_0, b)$$

$$F \rightarrow b \cdot$$

$$I_6 : \text{GOTO}(I_1, T)$$

$$E \rightarrow E + \cdot T$$

$$T \rightarrow \cdot TF$$

$$T \rightarrow \cdot F$$

$$F \rightarrow \cdot F *$$

$$F \rightarrow \cdot a$$

$$F \rightarrow \cdot b$$

$$I_7 = \text{GOTO}(I_2, F)$$

$$T \rightarrow TF \cdot$$

$$F \rightarrow F \cdot *$$

$$I_8 = \text{GOTO}(I_3, *)$$

$$F \rightarrow F \cdot *$$

$$I_9 = \text{GOTO}(I_6, T)$$

$$E \rightarrow E + T \cdot$$

$$T \rightarrow T \cdot F$$

$$F \rightarrow \cdot F *$$

$$F \rightarrow \cdot a$$

$$F \rightarrow \cdot b$$

$$I_{10} = \text{GOTO}(I_6, F)$$

$$T \rightarrow F \cdot$$

$$F \rightarrow F \cdot *$$

$$\text{STOP}$$

$$I_{11} = \text{GOTO} f$$

State	ACTION						GOTO		
	+	*	a	b	\$	E	T	F	
0			S_4	S_5		1	2	3	
1	S_6				ACC				
2	R_2^*		S_4	S_5	R_2^*	.			7
3	R_4^*	S_8	R_4^*	R_4^*	R_4^*				
4	R_6	R_6	R_6	R_6	R_6				
5	R_7	R_7	R_7	R_7	R_7				
6	.		S_4	S_5		.	9	10	
7	R_3^*	S_8	R_3^*	R_3^*	R_3^*				
8	R_5	R_5	S_8 R_5	R_5	R_5				
9	R_1		S_4	S_5	R_1				7
10	R_4	S_8	R_4	R_4	R_4				

$$\text{FOLLOW}(E) = \{ \$, + \}$$

FIRST(F)

$$\text{FOLLOW}(T) = \{ \$, +, a, b \}$$

$$\text{FOLLOW}(F) = \{ *, a, b, \$, + \}$$

* Canonical LR Parser

Example 1 : Construct a CLR parser for the following grammar.

$$S \rightarrow C C$$

$$C \rightarrow c C$$

$$C \rightarrow d$$

Ans ① Augmented Grammar

$$1. S' \rightarrow S$$

$$2. S \rightarrow C C$$

$$3. C \rightarrow c C$$

$$4. C \rightarrow d$$

② Calculating LR(1) items

I_0 : closure

$$S' \rightarrow .S, \$$$

$$S \rightarrow .CC, \$$$

$$C \rightarrow .CC, c/d$$

$$C \rightarrow .d, c/d$$

$$I_1: \text{GOTO}(I_0, S)$$

$$S' \rightarrow S., \$$$

$$I_2: \text{GOTO}(I_0, C)$$

$$S \rightarrow C . C, \$$$

$$C \rightarrow .CC, \$$$

$$C \rightarrow .d, \$$$

$$I_3: \text{GOTO}(I_0, C)$$

$$C \rightarrow C . C, c/d$$

$$C \rightarrow .CC, c/d$$

$$C \rightarrow .d, c/d$$

$$I_4: \text{GOTO}(I_0, d)$$

$$C \rightarrow d . , c/d$$

$$I_5: \text{GOTO}(I_2, C)$$

$$S \rightarrow CC . , \$$$

$$I_6: \text{GOTO}(I_2, C)$$

$$C \rightarrow C . C, \$$$

$$C \rightarrow .CC, \$$$

$$C \rightarrow .d, \$$$

$$I_7: \text{GOTO}(I_2, d)$$

$$C \rightarrow d . , \$$$

$$I_8: \text{GOTO}(I_3, C)$$

$$C \rightarrow CC . , c/d$$

$$I_9: \text{GOTO}(I_3, C)$$

$$C \rightarrow C . C, c/d$$

$$C \rightarrow .d, c/d$$

$$C \rightarrow c . C, c/d$$

$$I_{10}: \text{GOTO}(I_3, d)$$

$$C \rightarrow d . , I_4$$

$$I_9: \text{GOTO}(I_6, C)$$

$$C \rightarrow CC . , \$$$

③ Constructing a Parse Tree Table

State	Action			GOTO		
	c	d	\$	s	s	c
0	s ₃	s ₄			1	2
1			acc			
2	s ₆	s ₇				5
3	s ₃	s ₄				8
4	R ₃	R ₃				
5			R ₄			
6	s ₆	s ₇				9
7			R ₃			
8	R ₂	R ₂				
9			R ₂			

No need to find follow for reduce.