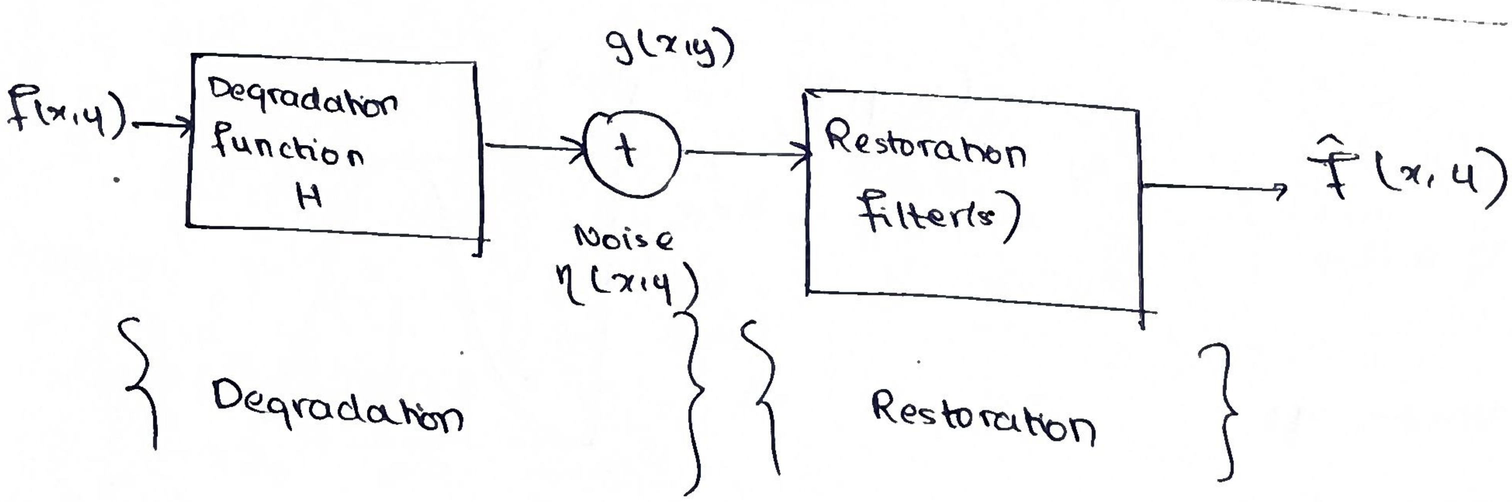
Image Processing and Analysis

Unit 3

Image Restoration

Image Restoration : Degradation model; noise models; restoration in the presence of noise using spatial filtering - mean filters - order statistics - adaptive filters; periodic noise reduction by frequency domain filters; Band reject filters - band pass filters - notch filters - optimum notch filtering; estimating the degradation function - inverse filtering; weiner filtering

* A model of the Image Degradation / Restoration Process



→ The degradation function is H.

→ The additive noise is $\eta(x,y)$

In the spatial domain, the degradation function H is given as:

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

In the frequency domain

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Restoration happens as:

$g(x,y) \rightarrow$ Restoration Filter $\rightarrow \hat{f}(x,y)$

* Noise Models

A. Sources of noise : (i) Image acquisition

(ii) digitization

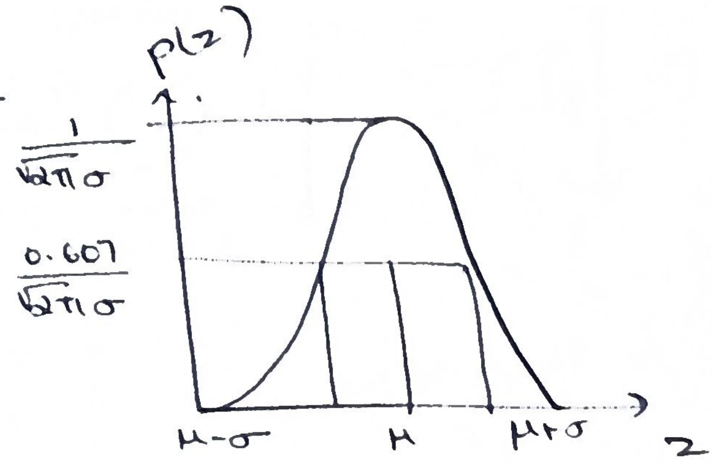
(iii) transmission

B. White noise

C. Random noise - affects both the dark and light areas of the image.

D. Gaussian noise - is given by the PDF of the Gaussian random variable z,

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(z-\mu)^2}{2\sigma^2}}$$



here mean : μ

standard deviation : σ

variance : σ^2

(may be due to electronic circuit noise, due to poor illumination / high temperature)

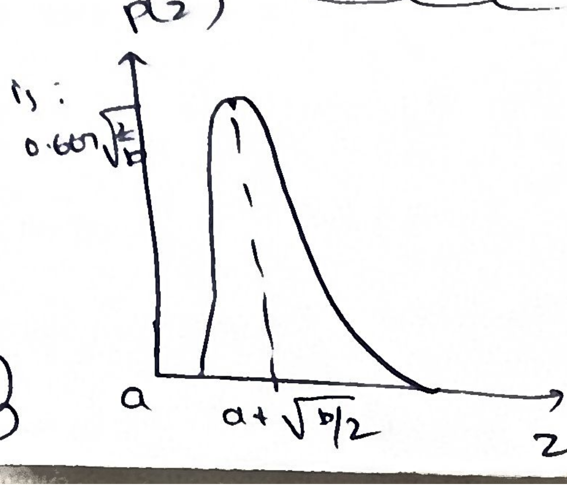
E. Rayleigh noise : The PDF of Rayleigh noise is:

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{\frac{-(z-a)^2}{b}} & , z \geq a \\ 0 & , z < a \end{cases}$$

mean : $\mu = a + \sqrt{\pi b/4}$

variance : $\sigma^2 = \frac{b(4-\pi)}{4}$

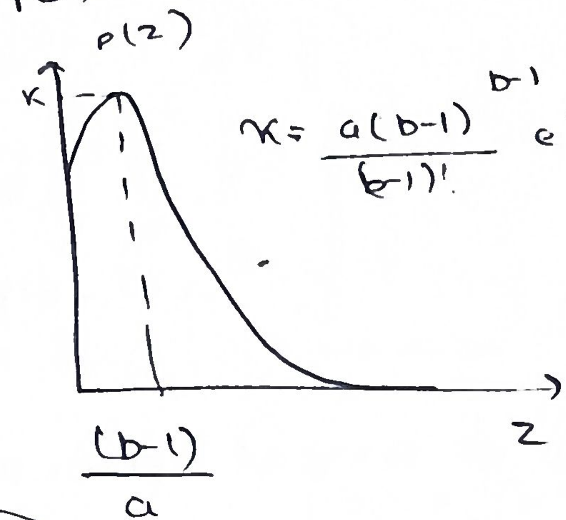
(happens w/ range imaging)



F. Erlang (Gamma Noise)

The PDF for Erlang noise:

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & , z \geq 0 \\ 0 & , z < 0 \end{cases}$$



$K = \dfrac{a(b-1)^{b-1}}{(b-1)!} e^{-(b-1)}$

mean: $\mu = b/a$

variance: $\sigma^2 = b/a^2$

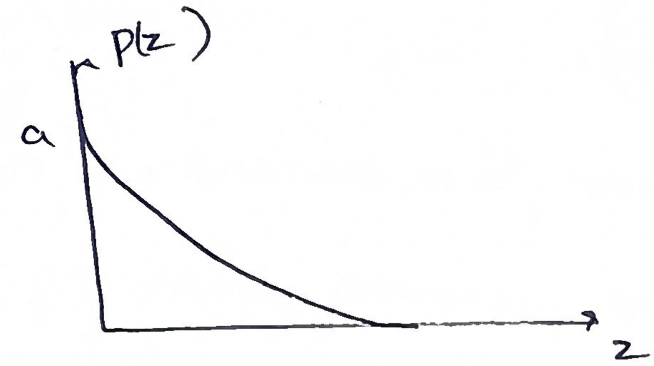
exponential & gamma noise happen in laser imaging?

G. Exponential Noise

Exponential noise occurs due to illumination changes

The PDF of exponential noise is:

$$p(z) = \begin{cases} a e^{-az} & , \text{for } z \geq 0 \\ 0 & , z < 0 \end{cases}$$



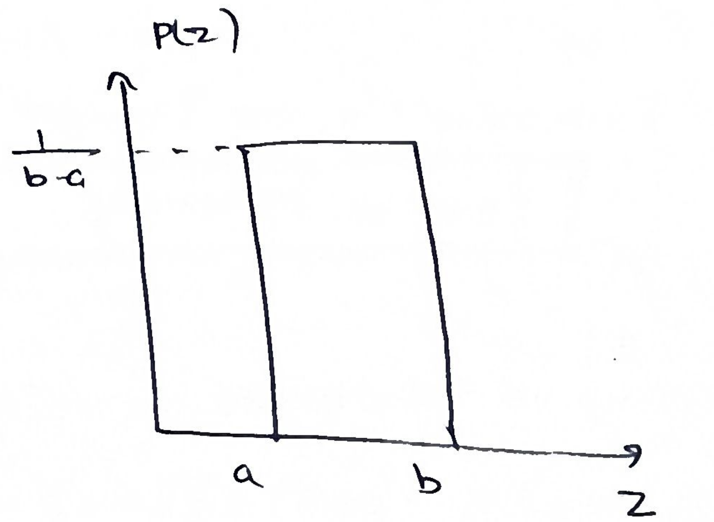
mean: $\mu = 1/a$

variance: $\sigma^2 = 1/a^2$

H. Uniform Noise

The PDF for uniform noise,

$$p(z) = \begin{cases} \dfrac{1}{b-a} & , a \leq z \leq b \\ 0 & , \text{otherwise} \end{cases}$$



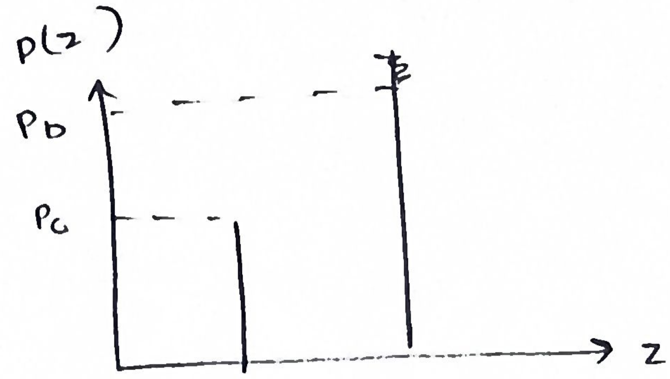
mean: $\mu = \dfrac{a+b}{2}$

variance: $\sigma^2 = \dfrac{(b-a)^2}{12}$

I. Impulse Noise

→ called salt and pepper noise

The PDF of bipolar impulse noise is:



$$p(z) = \begin{cases} P_a & , \text{for } z=a \\ P_b & , \text{for } z=b \\ 0 & , \text{otherwise} \end{cases}$$

quick transients, like faulty switching

b>a ⇒ b would appear as a light dot, a would be a dark dot

Unipolar: either P_a or P_b is zero.

For an 8 bit image - $a=0$
$b=255$

* Periodic Noise

→ arises from electrical or electromechanical interference during the image capturing process

→ It can be reduced by frequency domain filtering

→ An image affected by periodic noise will look like a repeating pattern has been added on top of the original image.

* Image Restoration in the Presence of Noise Only

$\boxed{\text{Spatial Filtering}}$

Degradation in the spatial domain is given by:

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

A. Mean Filters

(i) Arithmetic mean Filter - removes Gaussian noise / uniform noise

- a low pass Filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

(ii) Geometric mean Filter - removes gaussian noise / uniform noise.

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

(iii) Harmonic mean Filter - works well for salt noise, but fails for pepper noise

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

(iv) Contraharmonic mean filter - best suited for impulse noise

$Q > 0 \Rightarrow$ eliminates pepper noise

$Q < 0 \rightarrow$ eliminates salt noise

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

B. Order Statistics Filters

(i) Median Filters - effective in the presence of both bipolar and unipolar impulse noise

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}} \{g(s,t)\}$$

(ii) Max and min Filters

$\boxed{\text{Max:}}$ reduces pepper noise

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

$\boxed{\text{min}}$: reduces salt noise

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

(iii) Midpoint Filter - works best for randomly distributed noise, like Gaussian or uniform noise.

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}} \{g(s,t)\} + \min_{(s,t)\in S_{xy}} \{g(s,t)\}\right]$$

(iv) Alpha-trimmed mean filter: delete the $d/2$ lowest & highest gray-level values

→ useful in situations involving multiple types of noise, such as a combination of salt and pepper & Gaussian noise

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

c. Adaptive Filters.

(i) Local Noise Reduction Filter

Given by:

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma^2_\eta}{\sigma^2_L}\left[g(x,y) - m_L\right]$$

→ if σ^2_η = zero - then simply return the value of $g(x,y)$

→ If $\sigma^2_\eta < \sigma^2_L$ - return a value close to $g(x,y)$.

→ If $\sigma^2_\eta = \sigma^2_L$, return the arithmetic mean value m_L

(ii) Adaptive Median Filter

—helps remove salt & pepper (impulse) noise

- provides smoothing
- reduces distortion, such as excessive thinning or thickening of object boundaries

Consider the following variables

(i) z_{min} = minimum gray level value in S_{xy}

(ii) z_{max} = maximum gray level value in S_{xy}

(iii) z_{med} = median of gray levels in S_{xy}

(iv) z_{xy} = gray level at the coordinates (x,y).

(v) S_{max} = maximum allowed size of S_{xy}.

Algorithm

Level A: $A_1 = z_{med} - z_{min}$

$A_2 = z_{med} - z_{max}$

If $A_1 > 0$ and $A_2 < 0$, go to level B

else: increase the window size

If window size $\leq S_{max}$ — repeat level A

else output z_{med}.

Level B $B_1 = z_{xy} - z_{min}$

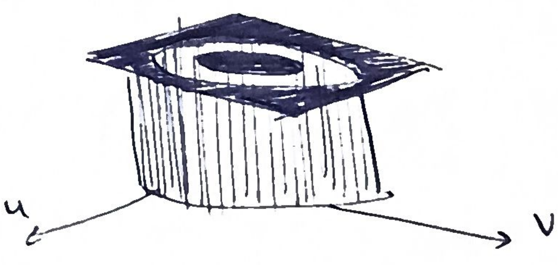
$B_2 = z_{xy} - z_{max}$

If $B_1 > 0$ and $B_2 < 0$, output z_{xy}

else output z_{med}

* Periodic Noise Reduction by Frequency Domain Filtering

A. Band Reject Filters



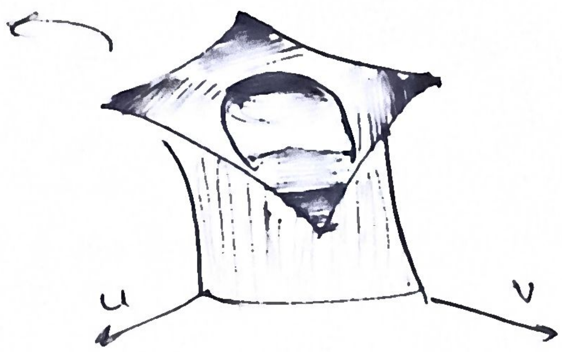
(i) Ideal Bandreject filter

$$H(u,v) = \begin{cases} 1, & \text{if } D(u,v) < D_0 - \frac{W}{2} \\ 0, & \text{if } D_0 - \frac{W}{2} \leq D(u,v) \leq D_0 + \frac{W}{2} \\ 1, & \text{if } D(u,v) > D_0 + \frac{W}{2} \end{cases}$$

$$D(u,v) = \left[(u - M/2)^2 + (v - N/2)^2\right]^{1/2}$$

(ii) Butterworth band reject filter

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)\,W}{D^2(u,v) - D_0^2}\right]^{2n}}$$



(iii) Gaussian bandreject filter

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v) - D_0^2}{D(u,v)\,W}\right]^2}$$

B. Band Pass Filters

$$H_{bp}(u,v) = 1 - H_{br}(u,v)$$

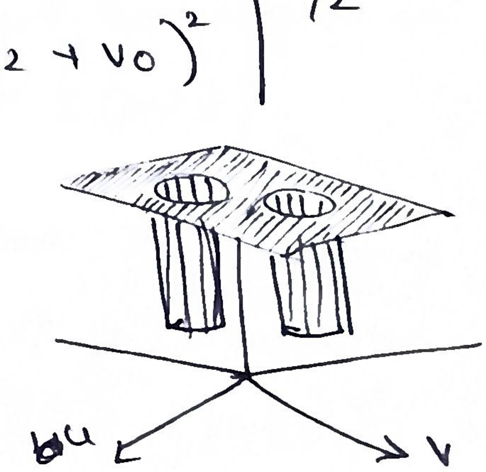
C. Notch Filters → a special form of a band-reject filter – instead of removing the entire range of frequencies, it removes only selective components

(i) Ideal Notch Filters

$$H(u,v) = \begin{cases} 0, & \text{if } D_1(u,v) \le D_0 \text{ or } D_2(u,v) \le D_0 \\ 1, & \text{otherwise} \end{cases}$$

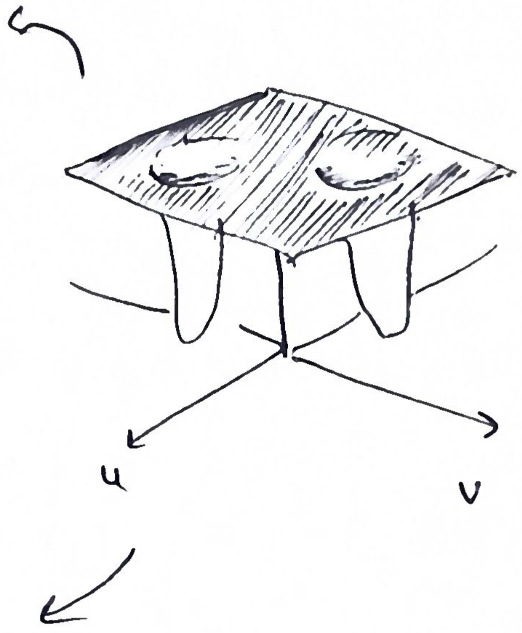
here $D_1(u,v) = \left[(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2\right]^{1/2}$

$$D_2(u,v) = \left[(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2\right]^{1/2}$$



(ii) Butterworth Notch Reject Filter

$$H(u,v) = \frac{1}{1 + \left[\frac{D_0^2}{D_1(u,v)\,D_2(u,v)}\right]^n}$$



(iii) Gaussian Notch Reject Filter.

$$H(u,v) = 1 - e^{1/2}\left[\frac{D_1(u,v)\,D_2(u,v)}{D_0^2}\right]$$

D. Notch Pass Filter

$$H_{np}(u,v) = 1 - H_{nr}(u,v)$$

E. Optimum Notch Filtering

→ This method is used when too much image info should not be accidently removed.

The steps are as follows:

1. The Fourier tranform of the interference noise pattern is given by:

$$N(u,v) = H(u,v)\,G(u,v)$$

2. The interference noise pattern in the spatial domain would be:

$$\eta(x,y) = \mathcal{F}^{-1}\left(H(u,v)\,G(u,v)\right)$$

3. Subtract a weighted portion of $\eta(x,y)$ from $g(x,y)$ to obtain an estimate of $f(x,y)$

$$\hat{f}(x,y) = g(x,y) - w(x,y)\,\eta(x,y)$$

(4) Minimize the local variance of $\hat{f}(x,y)$.

(5) The result is:

$$w(x,y) = \frac{\overline{g\eta} - \bar{g}\,\bar{\eta}}{\overline{(\eta^2)} - (\bar{\eta})^2}$$

* Estimating the Degradation Function

A. Estimation by Image Observation

→ Suppose that one is given a degraded image without any knowledge about the degradation function H.

→ Gather info. about the image itself:

(i) look at small rectangular sections of the image containing sample structures

(ii) To reduce the effect of noise, look where the signal content is strong - i.e an area of high contrast

$$G(u,v) = H(u,v).F(u,v) + N(u,v)$$

assuming that noise is negligible:

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

B. Estimation by Experimentation

→ Images similar to the degraded image can be acquired with various system settings until they are degraded as closely as possible to the image one wishes to restore..

→ Obtain the impulse response of the degradation, by imaging an impulse (a small dot of light) using the same system settings.

Then: $H(u,v) = \frac{G(u,v)}{A}$

$G(u,v)$ = observed image

A = strength of the impulse

C. Estimation by Modeling

→ model proposed by Hufnagel and Stanley, based on the physical characteristics of atmospheric turbulence.

→ Given by:

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

k = constant – depends on the nature of the turbulence

→ has a fn. similar to the Gaussian low pass filter.

Inverse Filtering

Direct Inverse Filtering – The simplest approach to restoration is by direct inverse filtering, where an estimate $\hat{F}(u,v)$ of the transform of the original image is computed by dividing the transform of the degraded image $G(u,v)$ by the degradation transfer function:

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$ (No provision for handling noise)

* Minimum Mean Square Error (Weiner Filtering)

→ The weiner filtering methods considers images and noise as random variables, and the objective is to find an estimate $\hat{f}$ of the uncorrupted image f, such that the mean square error between them is minimized.

→ It is defined as: $e^2 = E\{(f - \hat{f})^2\}$

Consider the following terms:

$G(u,v)$ = Fourier transform of the degraded image

$H(u,v)$ = degradation transfer function

$H^*(u,v)$ = complex conjugate of $H(u,v)$.

$|H(u,v)|^2 = H^*(u,v)H(u,v)$

$S_\eta(u,v) = |N(u,v)|^2$ = power spectrum of the noise

$S_f(u,v) = |F(u,v)|^2$ = power spectrum of the undegraded image.

→ The minum of the error function is given by:

$$\hat{F}(u,v) = \left\{ \frac{H^*(u,v)\, S_f(u,v)}{S_f(u,v)\, |H(u,v)|^2 + S_\eta(u,v)} \right\} G(u,v)$$

% by $S_f(u,v)$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

$$H^*(u,v) = \frac{|H(u,v)|^2}{H(u,v)}$$

$$\hat{F}(u,v) = \left[\frac{|H(u,v)|^2}{H(u,v) \{ |H(u,v)|^2 + S_\eta(u,v) / S_f(u,v)} \right] G(u,v)$$

<u>For white noise</u>

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \quad \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] G(u,v)$$