Electrical Circuit Analysis

Basic circuit components

1. **Charge:** One coulomb of charge is the charge associated with 6.242×10^{18} electrons.

unit : coulomb (q or Q)

2. **Electrical potential:** The amount of work done to move a unit positive charge from one point to another.

$$\text{electric potential} = \frac{\text{work done}}{\text{charge}} = \frac{w}{q}$$

unit : Volt (V)

3. **Electric current** : Flow of electrons in a conductor is known as electric current. Direction of current is from the +ve terminal to the -ve terminal, though the actual direction of movement of electrons is from the -ve to the +ve terminal.

$$I = dq/dt$$

unit: ampere (A)

- no. of electrons crossing a fixed point per second : $q = ne$

$$n = \frac{q}{e} = \frac{1}{1.602 \times 10^{-19}} = 6.242 \times 10^{18} \text{ electrons/second.}$$

4. **Electric circuit** : The closed path in which current flows from a voltage or a current source.

5. **Circuit element**: an individual circuit component with two terminals, by which it may be connected to other electric components. A circuit element can be a resistor, ~~ts~~ capacitor, inductor, current or a voltage source.

Voltage and Current Sources : classified as independent and dependent sources

Independent sources

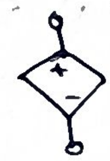
Independent voltage source : independent of the current flowing through the circuit.

~~Dependent voltage source :~~

Independent current source: independent of the voltage across it.

Dependent sources

(i) Voltage dependent voltage source : also called Voltage Controlled Voltage Source (VCVS), in which the source is dependent on the voltage at some other point in the circuit.



$V_c = KV_x$

(K is a dimensionless quantity)

(ii) Current dependent voltage source: also called Current Controlled Voltage Source (CCVS), in which the source voltage is dependent on the current flowing through any element in the circuit.

$V_s = \gamma I_x$

(γ is a scaling factor with unit V/A).

(iii) Voltage Dependent Current source: also called Voltage Controlled Current Source (VCCS), in which the source current is dependent on the voltage at some other location.

$I_s = gV_x$

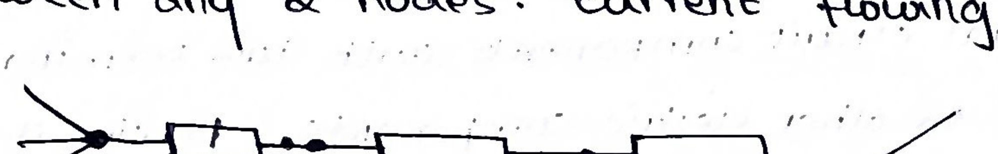
(g is a scaling factor with unit A/V)

(iv) Current Dependent Current Source: Also called Current controlled current source (CCCS), in which the source current is dependent on the current at some other location in the system.

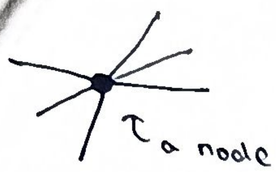
$I_s = KV_x$

(K is a dimensionless quantity)

Branch : 2 or more components connected in series between any 2 nodes. Current flowing through a ~~node~~ branch = branch current



Node or Junction : Terminal of any branch of a network, or a terminal common to two or more branches



Network : an interconnection of electric circuit elements or branches

Lumped Network: A network in which the individual circuit components cannot be electrically separated / individually isolated and represented as a whole.

Distributed Network: A network containing physically separate and individually isolated circuit components like resistors, inductors and capacitors. eg. a transmission line

Passive Network: a network with only passive circuit elements like resistors, capacitors and inductors

Active Network: a network consisting of active elements such as op-amps, transistors etc, along with other elements

Linear Element and Network: A circuit element is called linear if the relationship between the current and voltage involves a linear relationship with a constant coefficient.

For a resistor : $V = RI$.

~~capacitor~~ inductor : $v = \frac{L\,di}{dt}$

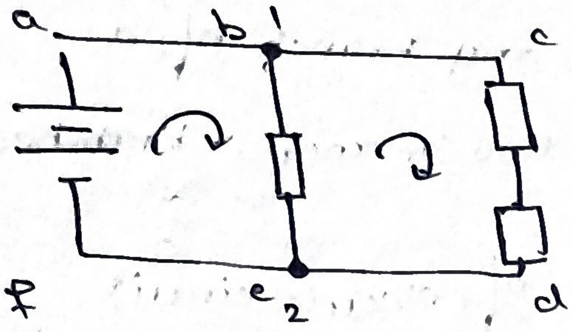
capacitor : $v = \frac{1}{C}\int I\,dt$

A network with linear elements is called a linear network.

Bilateral Network: a network is called bilateral if the relationship between current and voltage does not change in both directions.

Mesh: a closed path starting & ending at the same node, not passing any node or branch more than once.

Loop: a closed path in a circuit, in which no node is encountered more than once (CAN INCLUDE OTHER PATHS WITHIN IT)



abef and bcde are meshes

abef, bcde and acdf are loops.

Resistors connected in series

equivalent resistance = $R_1 + R_2 + R_3$

Limitations

1. Any discontinuity or break in any portion of the circuit would mean that current would stop flowing through the network
2. Since there is a division of voltage, not practical to use series circuits in homes and industries
3. Not practical to connect devices with different current rating in series

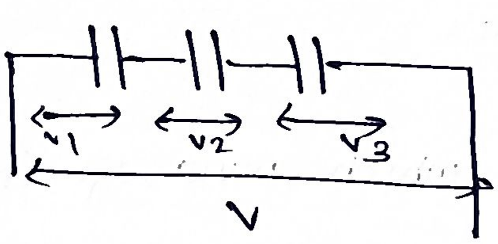
Resistors connected in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

Advantages

1. current would still flow through the circuit even if there was any discontinuity / breaks
2. Appliances with different power / current rating can be connected in parallel.
3. Parallel circuits for household and industrial wiring.

Capacitors in series



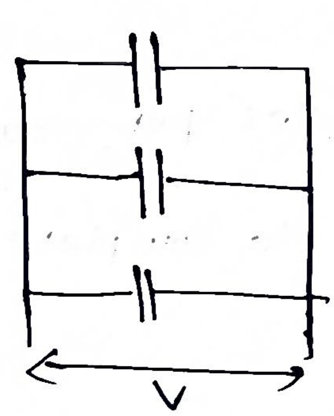
$V = V_1 + V_2 + V_3$

$C = \frac{Q}{V}$ or $V = \frac{Q}{C}$

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

i.e $\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$

Capacitors in parallel



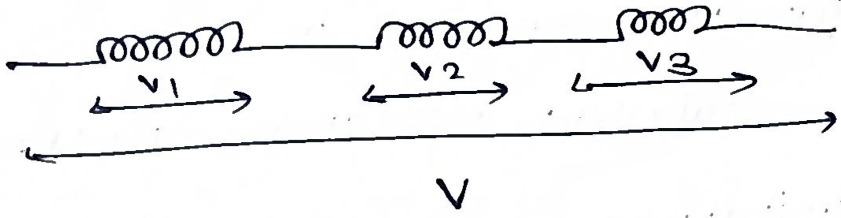
$Q = Q_1 + Q_2 + Q_3$

$C = \frac{Q}{V}$

$CV = C_1V + C_2V + C_3V$

$$\boxed{C = C_1 + C_2 + C_3}$$

Inductors in series



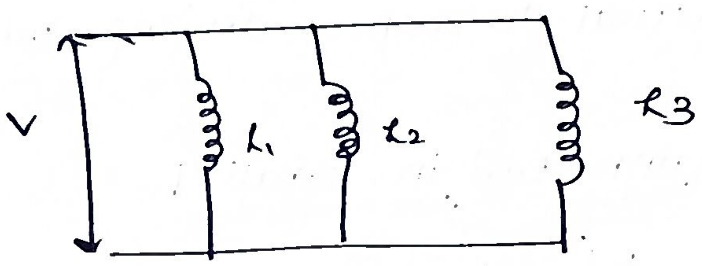
$V = V_1 + V_2 + V_3$

$V = L\frac{di}{dt}$

$L\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt}$

$$\boxed{L = L_1 + L_2 + L_3}$$

Inductors in parallel



$\frac{dI}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt}$

$\frac{V}{L} = \frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3}$

$$\boxed{\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

AC Quantity : a quantity whose magnitude and direction changes with time.

Waveform : graphical representation of the instantaneous value of any quantity plotted against time.

Alternating Current : A current which reverses its direction at periodic intervals.

Amplitude : The max +ve or −ve ~~quanti~~ value of an AC quantity is called its amplitude (magnitude)

Frequency: no. of cycles per second, unit : Hertz

Period : Time taken to complete 1 cycle, reciprocal of frequency

RMS Value / Virtual Value of AC

It is defined as that value of DC current which when flowing through a given circuit in a given time, produces the same amount of heat as would be produced by an AC current flowing in the circuit for the same time.

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0 \qquad (I_0 = \text{peak current})$$

|||ly $$V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0$$

Average / Mean value of AC : It is defined as that value of direct current that sends the same charge in a circuit in the same time as sent by the given alternating current in the half time period.

$$I_{av} = \frac{2}{\pi} I_0 = 0.637\, I_0$$

|||ly $$V_{av} = \frac{2}{\pi} V_0 = 0.637\, V_0$$

Form Factor : defined as the ~~RM~~ ratio of the RMS value of current to the average value

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

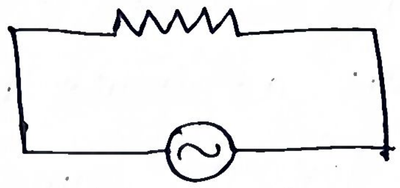
$$\text{Form Factor} = \frac{\text{RMS value}}{\text{Average value}} = \frac{0.707\, I_0}{0.637\, I_0} = 1.11$$

Peak Factor / cost factor : defined as the ratio of the peak value to the RMS value.

$$\text{peak factor} = \frac{\text{peak value}}{\text{RMS value}} = \frac{I_0}{\frac{I_0}{\sqrt{2}}} = \sqrt{2} = 1.414$$

Impedance: The net opposition to the flow of current offered by a resistor, inductor and capacitor.

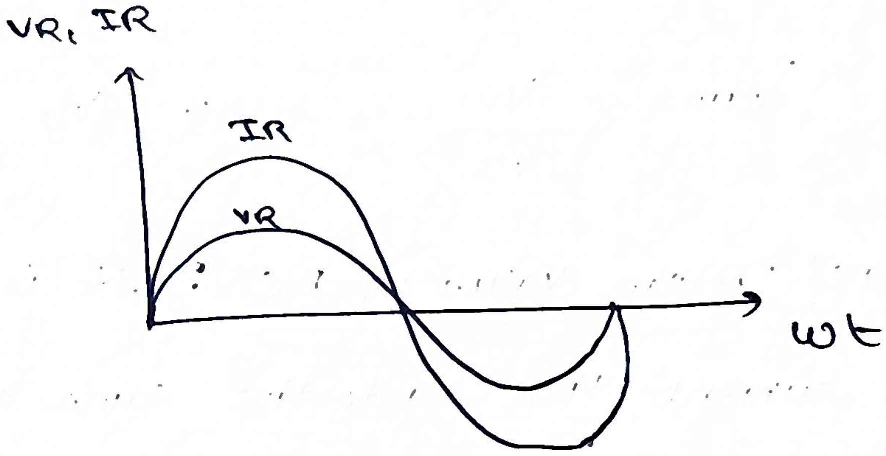
Impedance of a purely resistive circuit



$V_0 \sin\omega t$

$V = V_0 \sin\omega t$ — (1)

If I is the current through the circuit, and R is the resistance, then potential drop $= IR$



$V_0 \sin\omega t = IR$

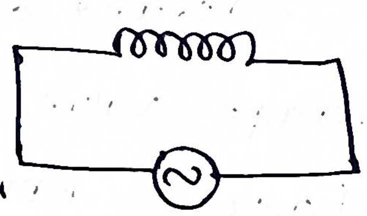
$I = \frac{V_0}{R} \sin\omega t$

$I = I_0 \sin\omega t$ — (2)

From (1), (2): voltage and current are in-phase.

Impedance: $Z = \frac{V}{I} = \frac{V_0 \sin\omega t}{I_0 \sin\omega t} = \frac{V_0 \sin\omega t}{(V_0/R)\sin\omega t} = \underline{\underline{R}}$

Impedance of a purely inductive circuit



$V_0 \sin\omega t$

$V = V_0 \sin\omega t$ — (1)

As an AC current flows through the inductor a back emf $= -L\frac{di}{dt}$ is set up.

Net emf $= V - L\frac{di}{dt}$

$V - L\frac{di}{dt} = 0$ (no resistance in circuit)

$V = L\frac{di}{dt}$ — (2)

Equating (1), (2)

$$L\frac{di}{dt} = V_0 \sin\omega t$$

$$di = \frac{V_0}{L} \sin\omega t \, dt$$

$$\int di = \int \frac{V_0}{L} \sin\omega t$$

(avg. current must be 0, $\cos\omega t$ is time dependent

$\Rightarrow$ avg. value is zero

$\Rightarrow$ value of const of integration is also 0.

$$I = -\frac{V_0}{L\omega} \cos\omega t + \text{constant}$$

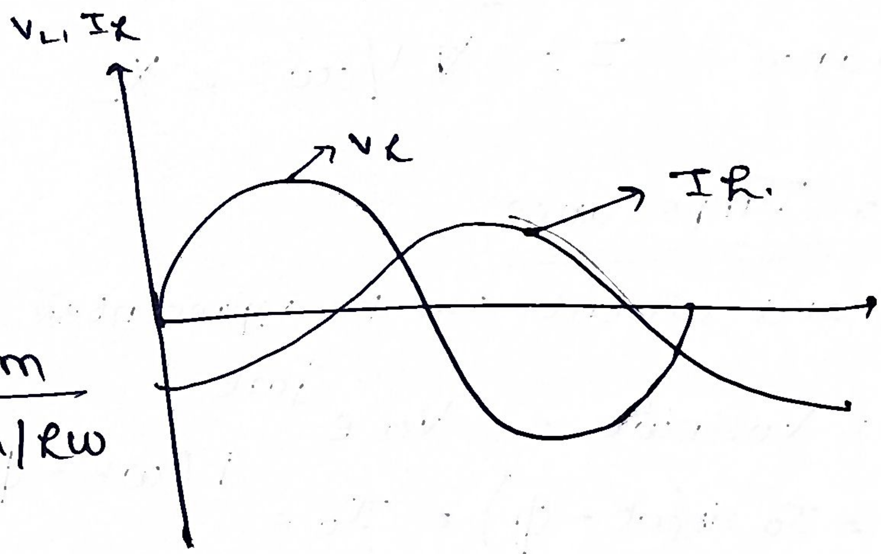
$$I = -\frac{V_0}{L\omega} \cos\omega t$$

$$I = -\frac{V_0}{L\omega} \sin(\pi/2 - \omega t)$$

$$I = \frac{V_0}{L\omega} \sin(\omega t - \pi/2)$$

$$I = I_0 \sin(\omega t - \pi/2)$$

$V = V_0 \sin\omega t$



Current lags by $\pi/2$

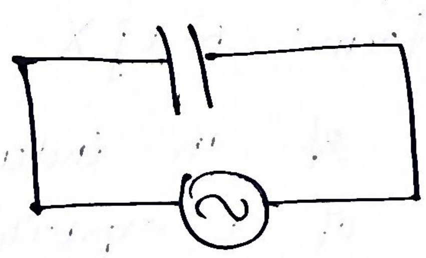
$$I_0 = \frac{V_0}{L\omega}$$

$$\text{Impedance} = \frac{\cancel{V_0}}{\cancel{I_0}} \quad \frac{V_m}{I_m} = \frac{V_m}{V_m/L\omega}$$

$$\boxed{Z = L\omega} = X_L$$

Impedance of a purely capacitive circuit

$V = V_0 \sin\omega t$ = potential difference across the plates of the capacitor



$V = V_0 \sin\omega t$

$$V = \frac{Q}{C}$$

$$Q = CV$$

$$Q = C V_0 \sin\omega t$$

$$I = \frac{dQ}{dt}$$

$$I = \frac{d}{dt}\left(CV_0 \sin\omega t\right)$$

$$I = CV_0 \cos\omega t \cdot \omega$$

$$I = C\omega \cdot V_0 \cdot \cos\omega t$$

$$I = \frac{V_0}{1/C\omega} \cos\omega t$$

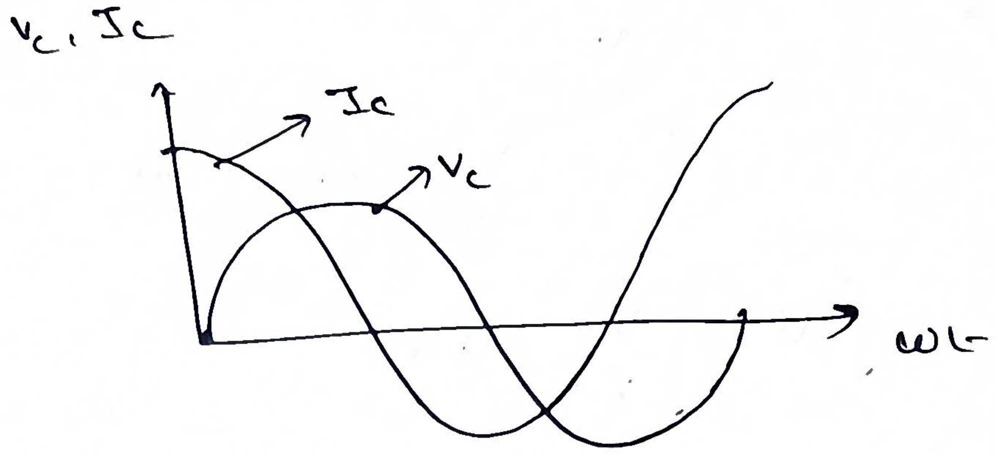
$$I = \frac{V_0}{1/C\omega} \cdot \sin\left(\omega t + \pi/2\right)$$

$$I = I_0 \sin\left(\omega t + \pi/2\right)$$

$$I_0 = \frac{V_0}{1/C\omega}$$

current leads by $\pi/2$

Impedance : $Z = 1/C\omega = X_C$



Complex Impedance

Voltage and current can be represented in a complex Euler form as:

$$V = V_0 \sin\omega t = V_0 e^{j\omega t}$$

$$I = I_0 \sin(\omega t - \phi) = I_0 e^{j(\omega t - \phi)}$$

where $e^{j\omega t} = \cos\omega t + j\sin\omega t$

General form: $R + jX$, X = reactance

impedance of an inductor $= j\omega L$

of a capacitor $= \frac{-j}{\omega C}$

Apparent power (S): product of applied voltage V and current I

$V_0 = \sqrt{2}\, V_{rms}$.

$$S = VI$$

$$S = \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} = \frac{V_0 I_0}{2}$$

$$= V_{rms} I_{rms}$$

where I_0, V_0 = max current & max voltage

VA

Average / Active / Real power (P): It is the product of the apparent power and the cosine of the angle between voltage and current.

$$P = S \cdot \cos\phi$$

$$= V_{rms} I_{rms} \cos\phi$$

$$= \frac{V_0 I_0}{2} \cdot \cos\phi$$

(W)

Reactive power (Q): product of apparent power and the sine of the angle between voltage and current

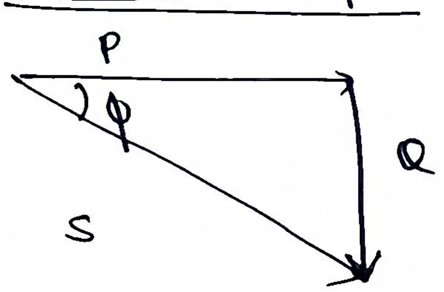
$$Q = S\sin\phi$$

$$Q = V_{rms} I_{rms} \sin\phi$$

$$= \frac{V_0 I_0}{2} \cdot \sin\phi$$

unit: volt-ampere reactive (VAR)

Power Triangle



P = average / real power

Q = reactive power

S = apparent power.

* Energy stored in a capacitor = $\frac{1}{2} CV^2$

* Energy stored in an inductor = $\frac{1}{2} LI^2$

Superposition Theorem: For a linear bilateral, lumped circuit the response of a network (voltage or current through an element) with several independent sources can be obtained as the sum of the responses to sources, taken one at a time.

Voltage sources are short-circuited

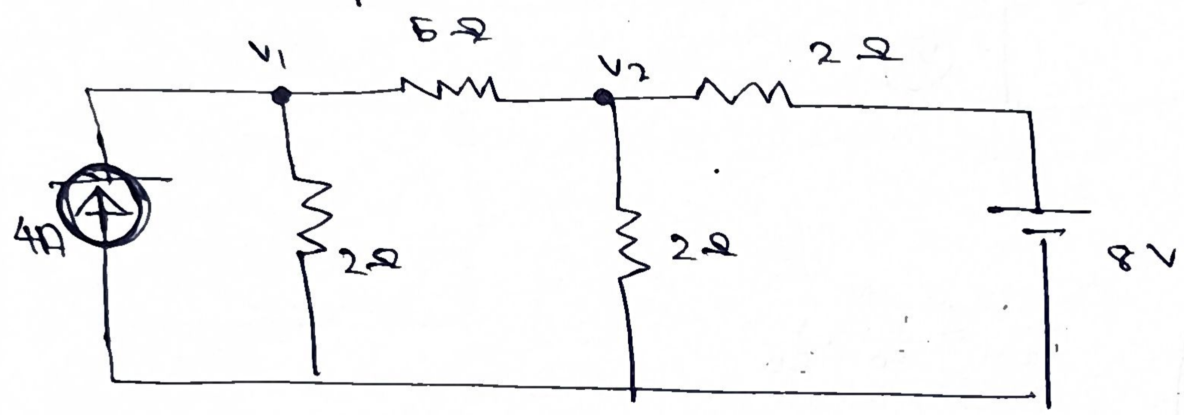
Open sources are open-circuited.

Note: CDR

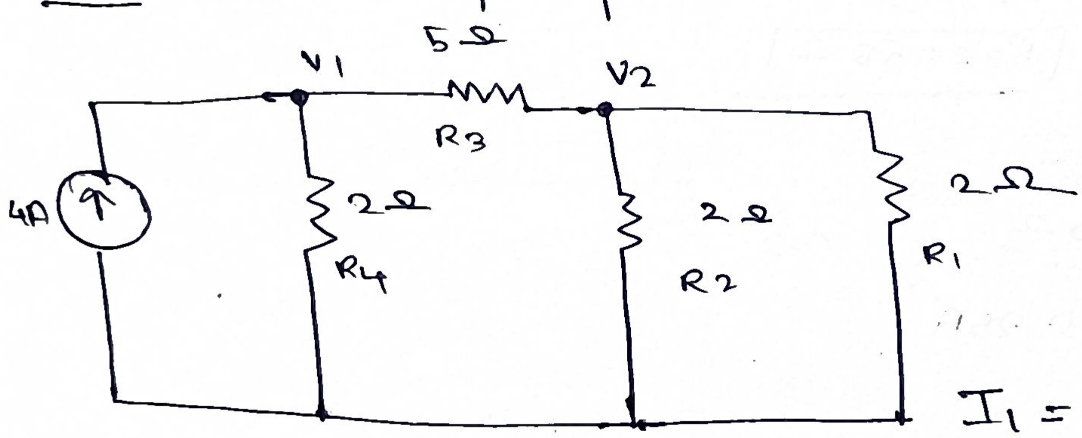
$= I_{total} \times \left(\frac{R_{opp}}{R_{opp} + R_{current\ branch}} \right)$

$VDR = V_{total} \times \left(\frac{R_{opp}}{R_{opp} + R_{current\ branch}} \right)$

Ex1 In the circuit, find the node voltages V_1 and V_2 using the superposition theorem.



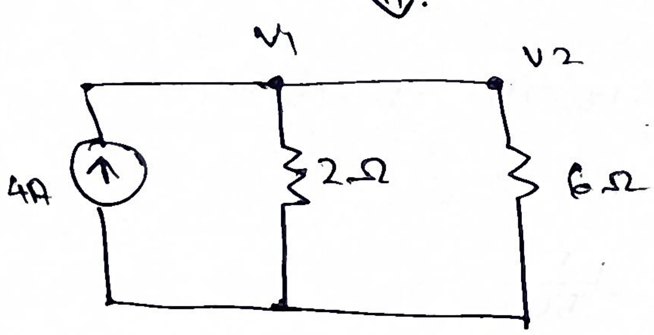
Case1 : considering only the current source.



$\frac{1}{R_{12}} = \frac{1}{2} + \frac{1}{2} = 1$

$R_{12} = 1\Omega$

$R_{23} = 6\Omega$.



$I_1 = 4 \times \frac{6}{6+2}$

$= \frac{4 \times 6}{8} = \underline{\underline{3A}}$

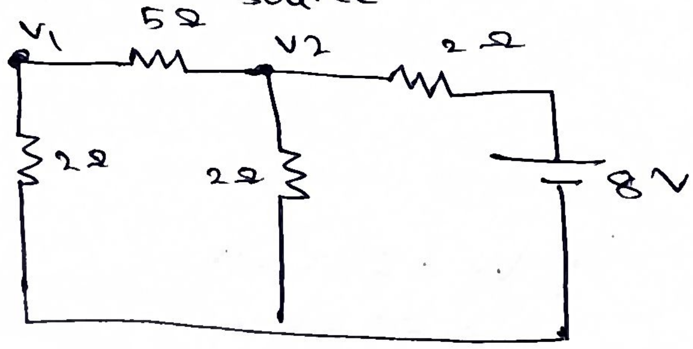
$V_1(i) = 3 \times 2 = 6\Omega$

$I_2 = 4 \times \frac{2}{2+6} = \frac{4 \times 2}{8} = 1A$

$V_2(i) = \cancel{6 \times 1 = 6}\ \underline{\underline{1 \times 1 = 1A}}$

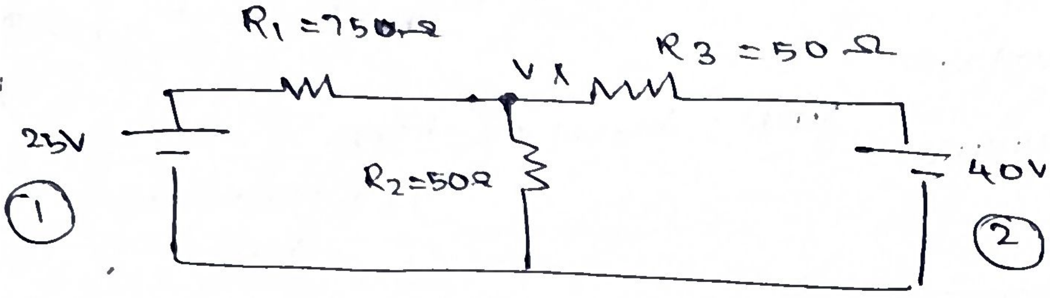
↳ consider only the 2 ll^{el} branches

Case 2 : with only the voltage source



$V_1(i) =$

Ex 2 Find the voltage V_x using superposition theorem

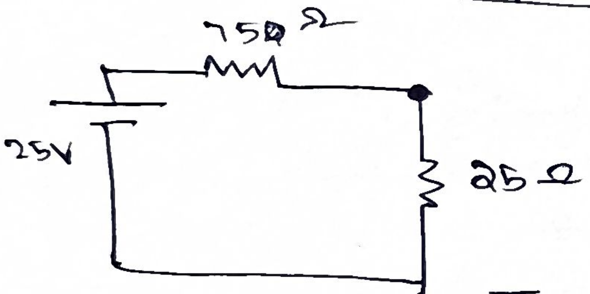


considering only V_1

$$\frac{1}{R_{23}} = \frac{1}{50} + \frac{1}{50}$$

$$\frac{1}{R_{23}} = \frac{2}{50}$$

$$\boxed{R_{23} = 25\ \Omega}$$



$$R_{eq} = 100\ \Omega$$

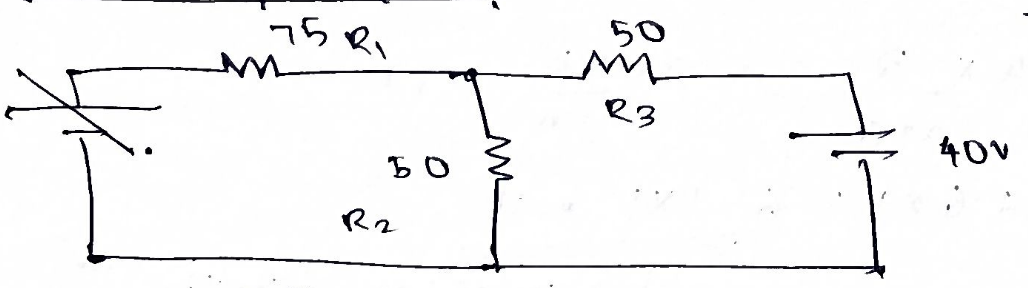
$$I_{total} = \frac{25}{100} = 0.25A$$

$V_1 = 0.25 \times 75 = 18.75$ V

$V_2 = 0.25 \times 25 = 6.25$ V

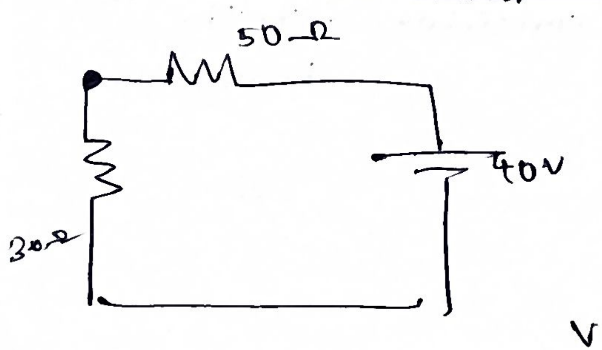
~~$V_1 - V_2 = 12$~~. $V_x = 25 - V_1 = \cancel{18.75}\ 25 - 18.75 = 6.25V$

considering only V 2



$$\frac{1}{R_{12}} = \frac{1}{75} + \frac{1}{50}$$

$R_{12} = 30\ \Omega$



~~V_3~~ $R_{eq} = 80\ \Omega$

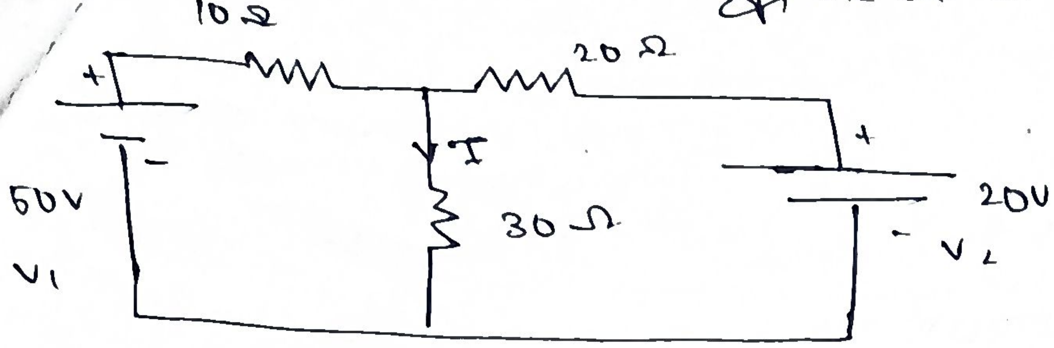
$$I_{total} = \frac{40}{80} = 0.5A$$

$V_3 = 0.5 \times 50 = 25$ V

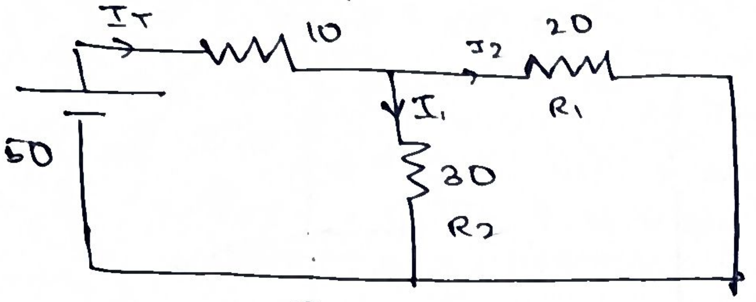
$V_x = 40 - 25 = 15$ V

$\therefore V_x = 6.25 + 15$
$= 21.25V$

3. Find the value of the current I with the superposition theorem.



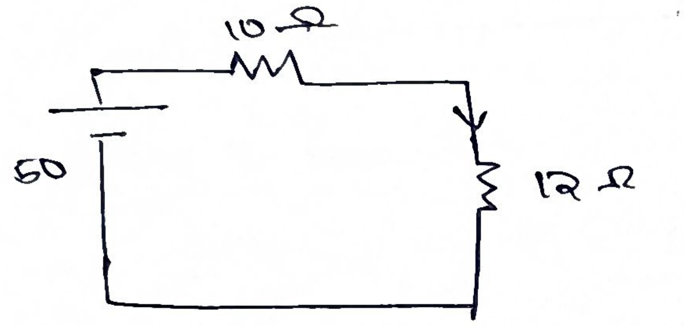
considering only V_1



$$\frac{1}{R_{12}} = \frac{1}{20} + \frac{1}{30}$$

$$\frac{1}{R_{12}} = \frac{3+2}{60}$$

$R_{12} = 12\,\Omega$

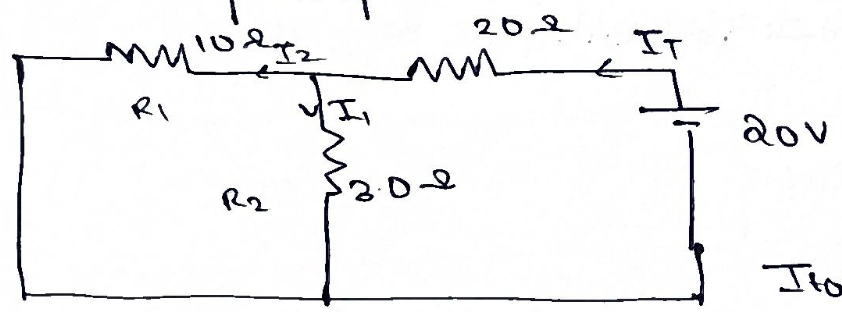


$$I_{total} = \frac{50}{22}$$

$I_T = 2.272\,A$

$$I_1 = 2.27 \times \frac{20}{50} = 0.909\,A$$

considering only V_2



$$\frac{1}{R_{12}} = \frac{1}{10} + \frac{1}{30}$$

$$\frac{1}{R_{12}} = \frac{4}{30}$$

$R_{12} = 7.5\,\Omega$.

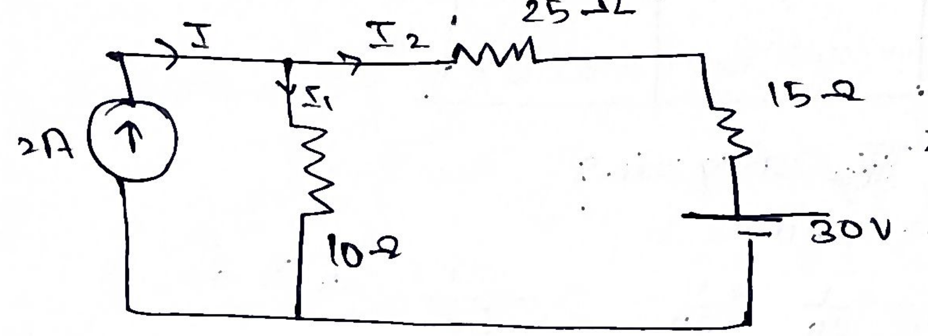
$$I_{total} = \frac{20}{27.5} = \cancel{2.667\,A}\ \cancel{0.741}$$

$= 0.727\,A$

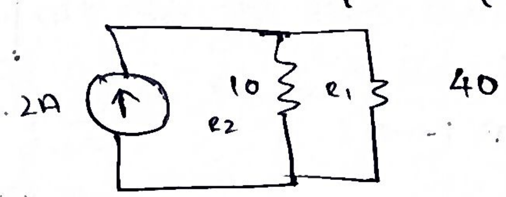
$$I_1 = \cancel{2.667}\ 0.727 \times \frac{10}{10+30} = \cancel{0.6667\,A}\ 0.181\,A$$

$\boxed{I = 1.09\,A}$

4. Find the value of the current I by means of the superposition theorem.



considering only the current source



$$\frac{1}{R_{eq}} = \frac{1}{40} + \frac{1}{10}$$

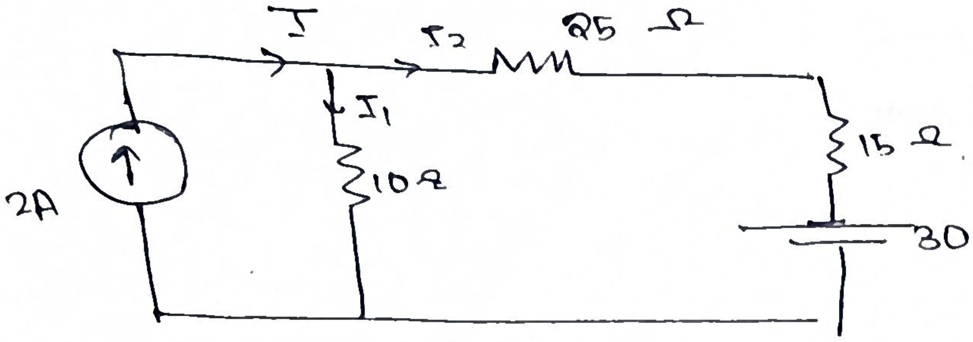
$$\frac{1}{R_{eq}} = \frac{2+8}{80}$$

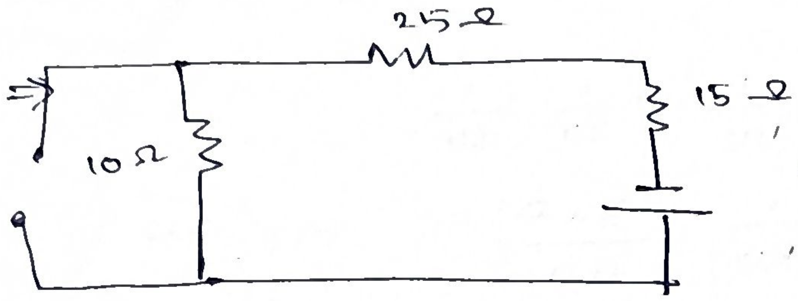
$R_{eq} = 8\,\Omega$

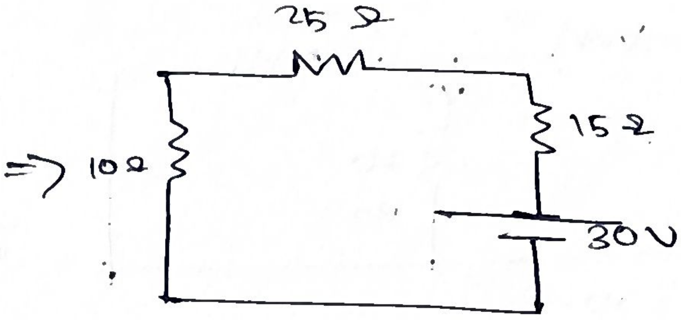
$$I_2 = 2 \times \frac{10}{10+40}$$

$$= 2 \times \frac{1}{5} = 0.4\,A \longrightarrow$$

considering only the voltage source







$R_{eq} = 50\,\Omega$

$I = \frac{30}{50} = 0.6A$

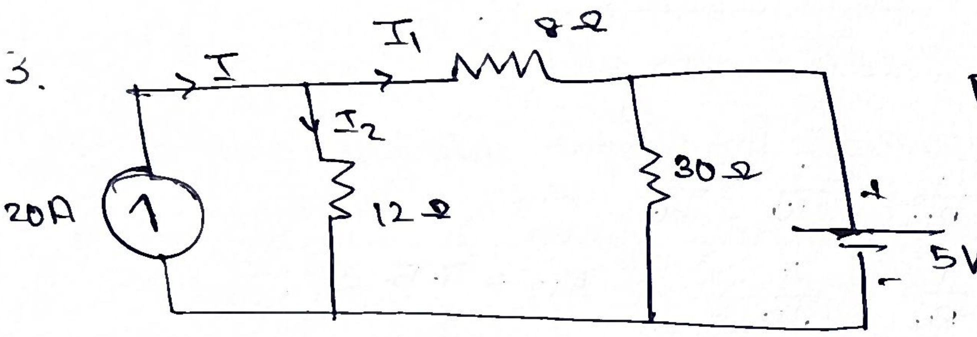
$I_T = 0.6$

$I_T = 0.6$

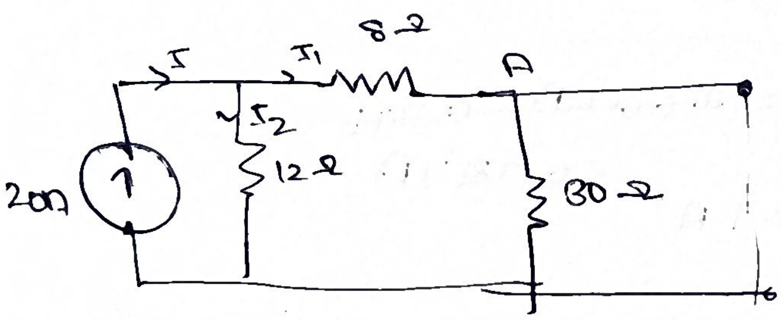
$\cancel{I_2 = \frac{0.6 \times 10}{10+40}}$

$\therefore$ Total $= 0.6 - 0.4 = \underline{0.2A}$

3.



Find the current flowing through the 8Ω resistor.



The short-circuited branch takes all the current.

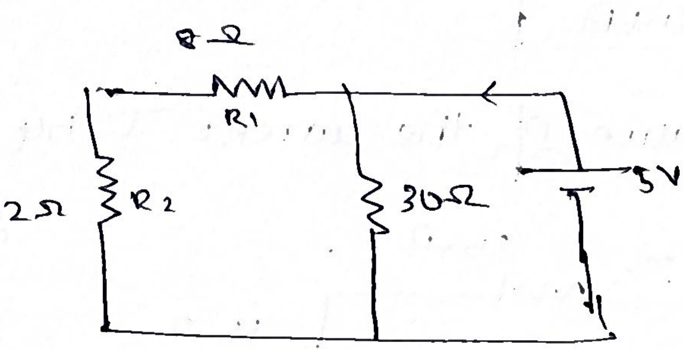
AB is a dummy branch.

$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{8}$ $R_{eq} = 4.8\,\Omega$

$I_T = 20A$

$I_1 = 20 \times \frac{12}{12+8}$

$I_1 = \frac{20 \times 12}{20} = \underline{12A} (\rightarrow)$



~~Req = 4.8~~

$R_{12} = 20\,\Omega$

$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{30} = \frac{3+2}{60} = R_{eq}\ 12\,\Omega$

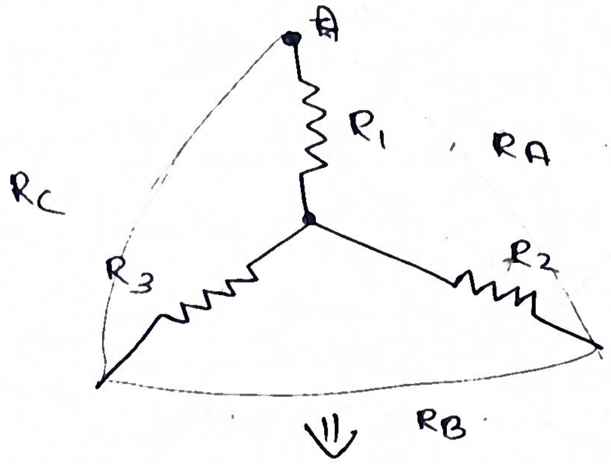
$I = 5/12$ $I_{Total} = 0.416A$

$I_1 = 0.416 \times \frac{30}{30+20} = 0.2496A$ $\boxed{\text{Total} = 11.75A}$

Star - Delta Conversion

Star to delta:

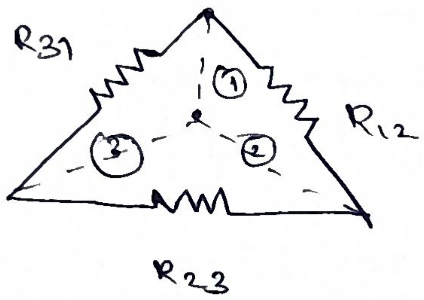
$$R_A = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3}$$



$$R_B = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}$$

$$R_C = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2}$$

Delta to star

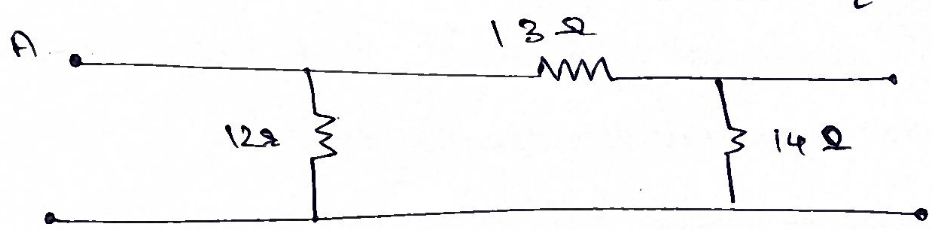


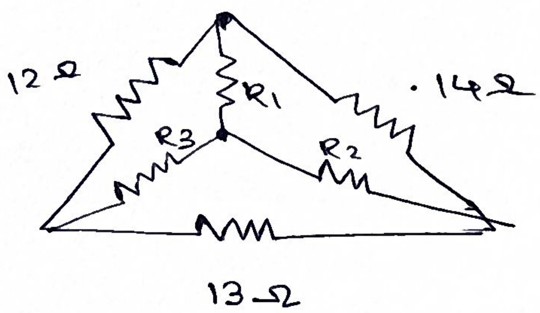
$$R_1 = \frac{R_{12} \times R_{13}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{23} \times R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{12} \times R_{23}}{R_{12} + R_{23} + R_{31}}$$

Ex1 Obtain the star connected equivalent for the delta connected circuit.



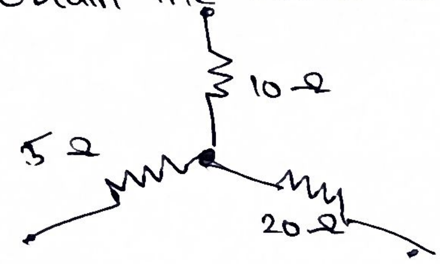


$$R_1 = \frac{14 \times 12}{12 + 13 + 14}$$

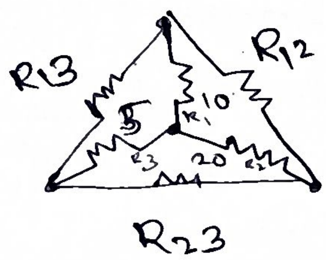
$$R_2 = \frac{13 \times 14}{12 + 13 + 14}$$

$$R_3 = \frac{12 \times 13}{12 + 13 + 14}$$

Ex2 Obtain the delta connected equivalent for the star-connected circuit shown.



⇒

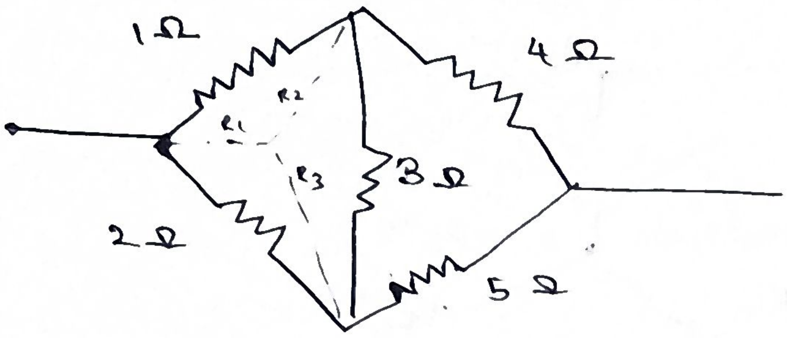


$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3}$$

$$R_{12} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{5}$$

IIIly $R_{23} = \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$, $R_{13} = \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$

Ex3 Find the equivalent resistance using a star-delta conversion

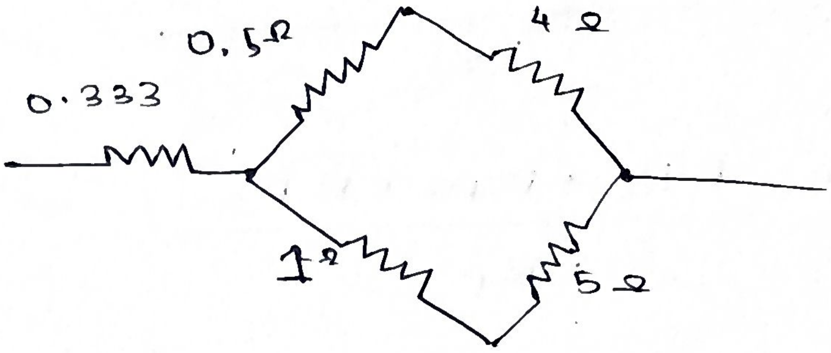


$R_1 = \frac{2}{6} = \frac{1}{3}\ \Omega = 0.333\ \Omega$

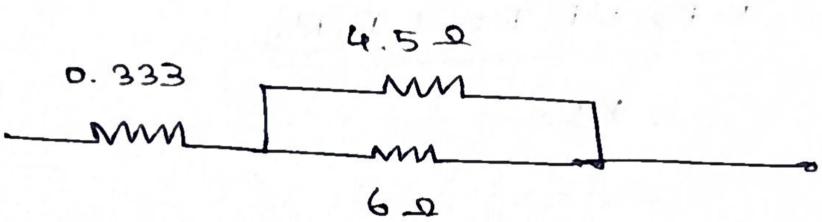
$R_2 = \frac{3}{6} = \frac{1}{2}\ \Omega = 0.5\ \Omega$

$R_3 = 1\ \Omega = 1\ \Omega$

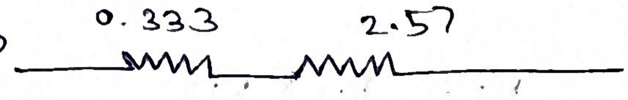
⇒



⇒

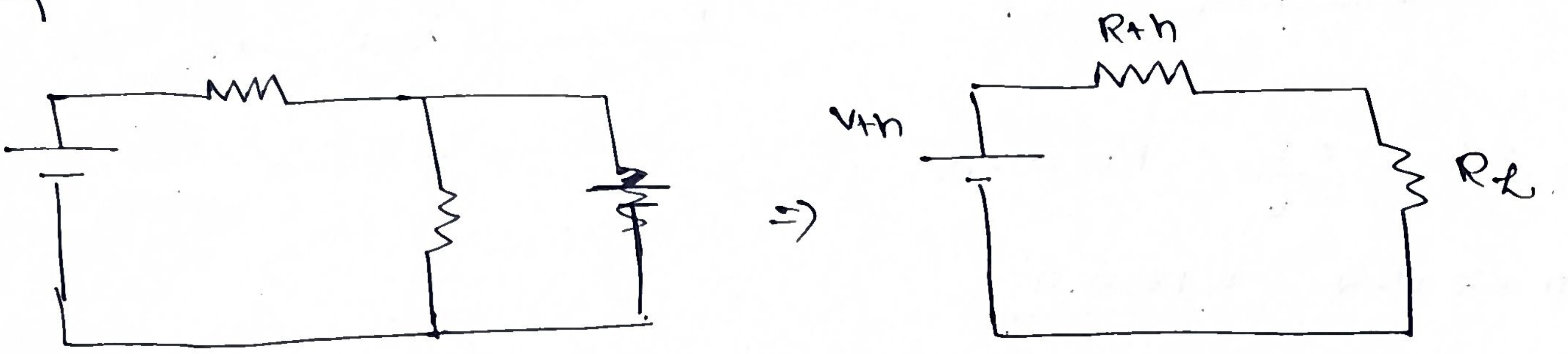


⇒



$= 2.904\ \Omega$

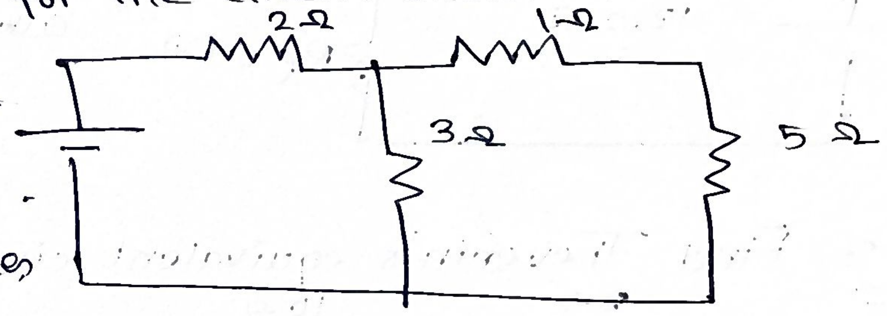
Thevenin's Theorem : Any linear bilateral network consisting of many sources and impedances can be replaced with an equivalent circuit consisting of a Thevenin's voltage source V_{th}, connected in series with a Thevenin's resistance R_{th} connected to a load impedance.



Procedure : (i) disconnect a load network from the circuit by substituting an open circuit, and find V across the load, which is equivalent to V_{TH}

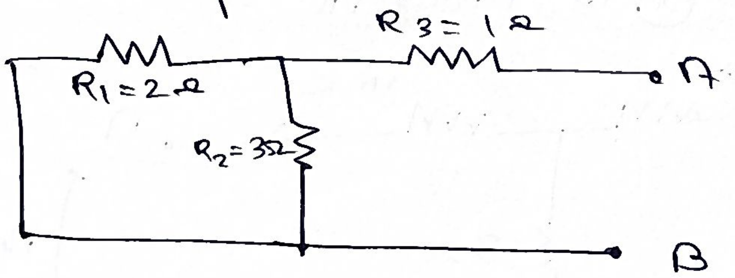
(ii) Find R_{th} by zeroing out every independent source (open current source, short voltage source)

(1) **For example**: Use Thevenin's theorem for the circuit shown to determine the current through 5 Ω.



Step 1: Find the Thevenin's resistance R_{TH}. Short circuit all voltage sources, current sources and the load resistance.

⇒



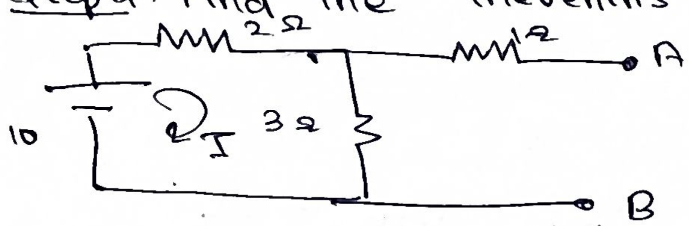
$$\frac{1}{R_{12}} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$$R_{12} = \frac{6}{5} = 1.2$$

$$R_{123} = R_{TH} = 1.2 + 1$$

$$\boxed{R_{TH} = 2.2\,\Omega}$$

Step 2: Find the Thevenin's Voltage: short circuit the load alone.



No current flows through the 1Ω resistor.

Apply Kirchoff's law

$$-10 + 2I + 3I = 0$$

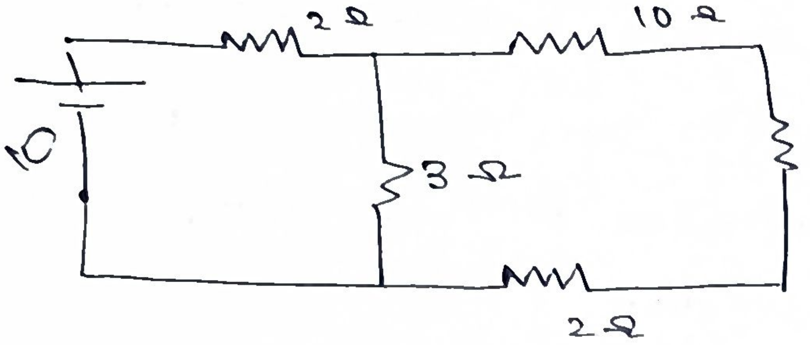
$$5I = 10 \qquad \boxed{I = 2A}$$

$$V_{AB} = I \times 3 = 6V$$

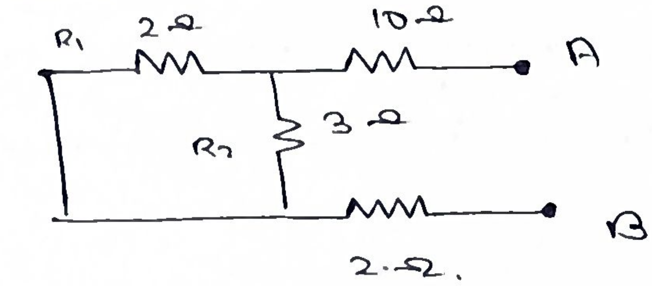
$$\boxed{V_{TH} = 6V}$$

$$\therefore \text{Current} = \frac{V_{TH}}{R_L + R_{TH}} = \frac{6}{2.2 + 5} = 0.833\ A$$

Ex2 : Find the Thevenin's equivalent circuit and the current passing through R_L if $R_L = 1\Omega$



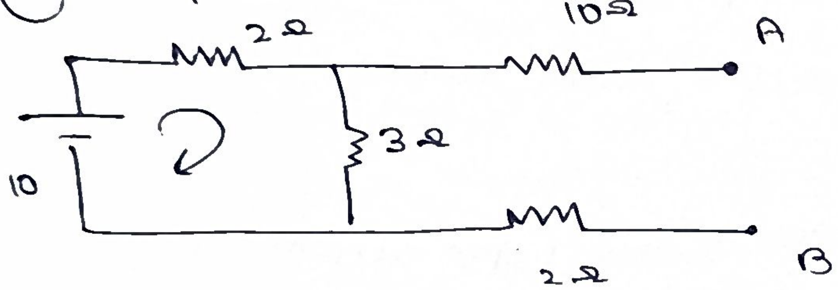
Ans ① To find R_{TH}



$$\frac{1}{R_{12}} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \qquad R_{12} = 1.2.$$

$$R_{eq} = 10 + 2 + 1.2 = \underline{13.2\,\Omega}.$$

② To find V_{th}

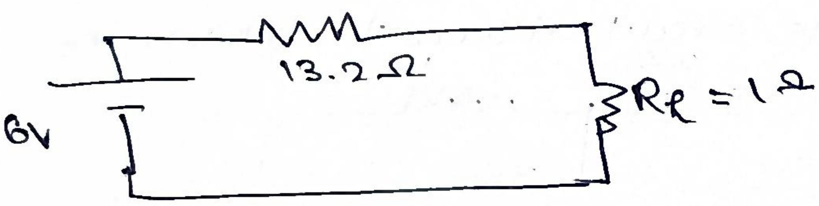


$$-10 + 5I = 0$$

$$I = 2A$$

$$V_{TH} = 2 \times 3 = 6V.$$

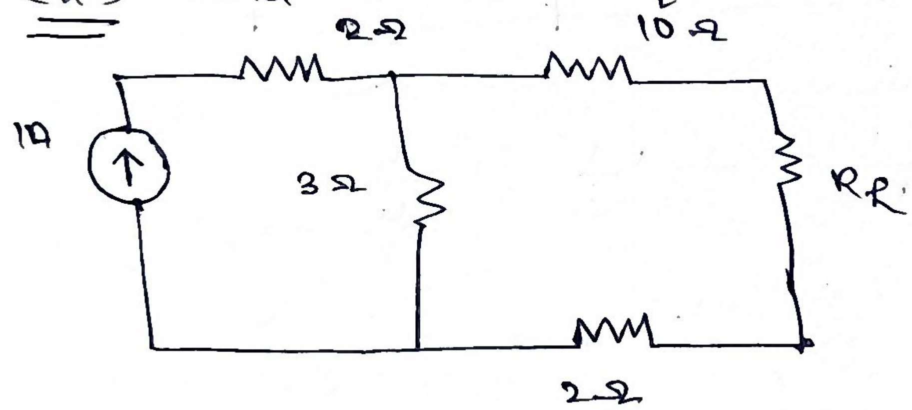
Thevenin Circuit



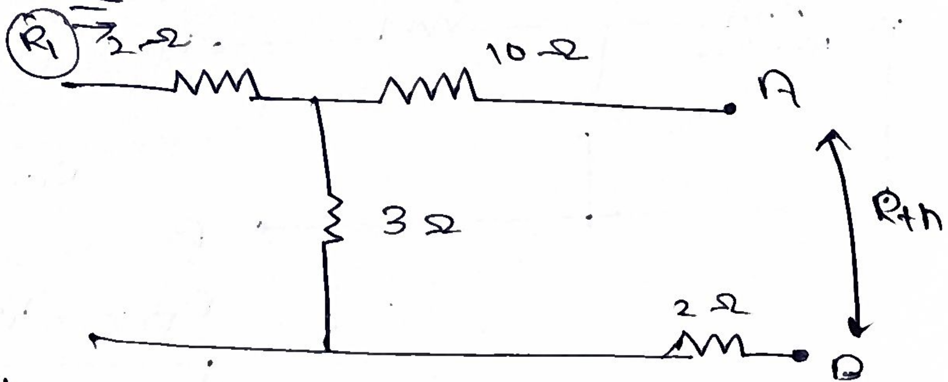
Current through $R_L = \dfrac{6}{13.2+1} = \dfrac{6}{14.2}$

$= 0.423A$

Ex3 Find Thevenin's equivalent circuit.



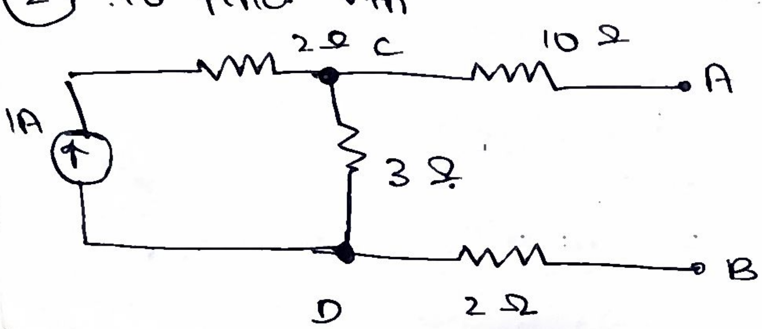
Ans ① To find R_{TH}



No need to consider R_1 as it is not connected to R_{th} in any way.

$\therefore R_{th} = 10 + 3 + 2 = 15\,\Omega$

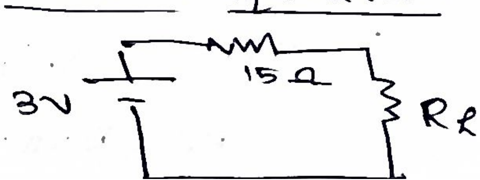
② To find V_{th}



$V_{CD} = V_{AB}$

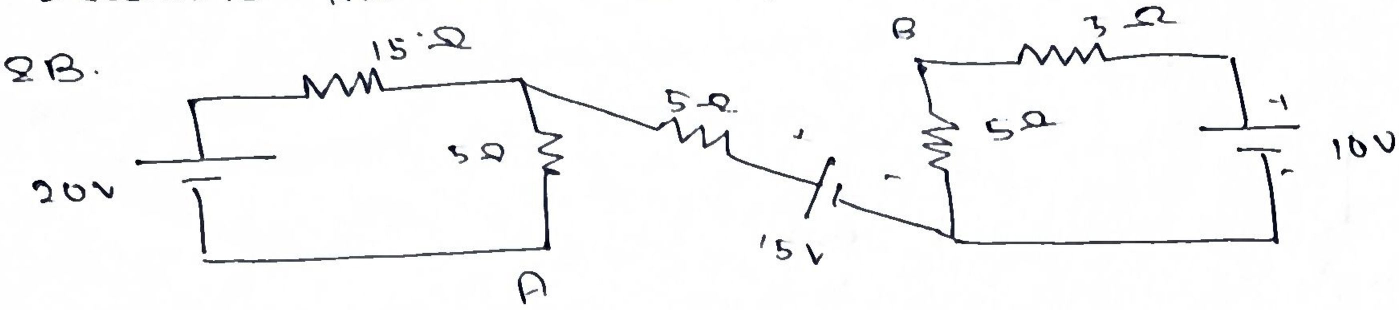
$V_{CD} = IR = 3V$ $\quad V_{AB} = V_{TH} = 3V$

Thevenin equivalent

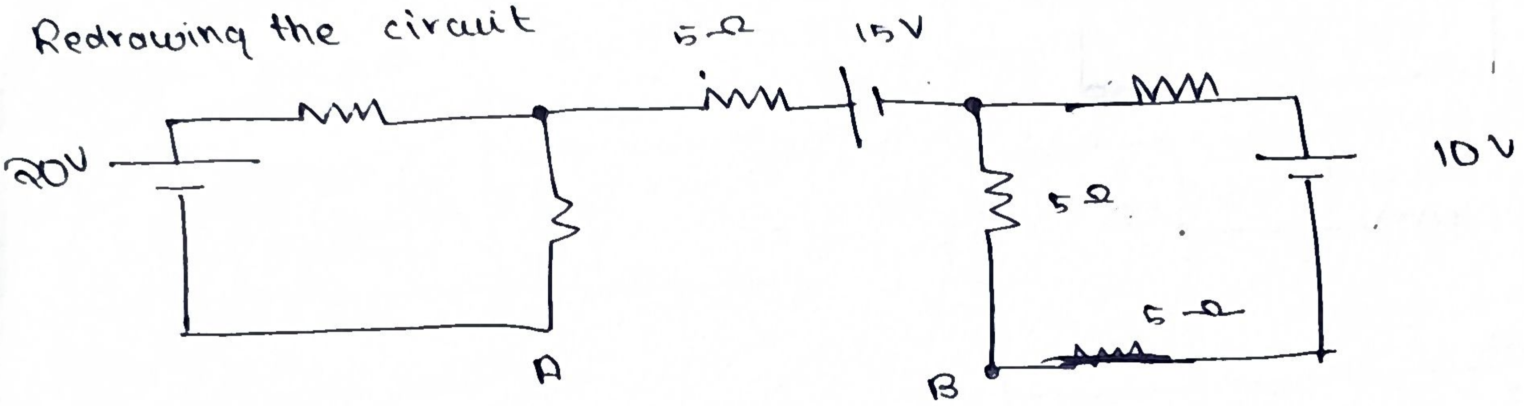


Ex4

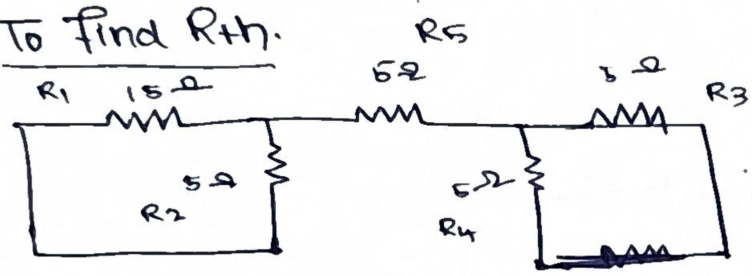
• Determine the Thevenin's equivalent circuit across the terminals A & B. (3)



Redrawing the circuit



To find Rth.



$\frac{1}{R_{12}} = \frac{1}{5} + \frac{1}{15}$

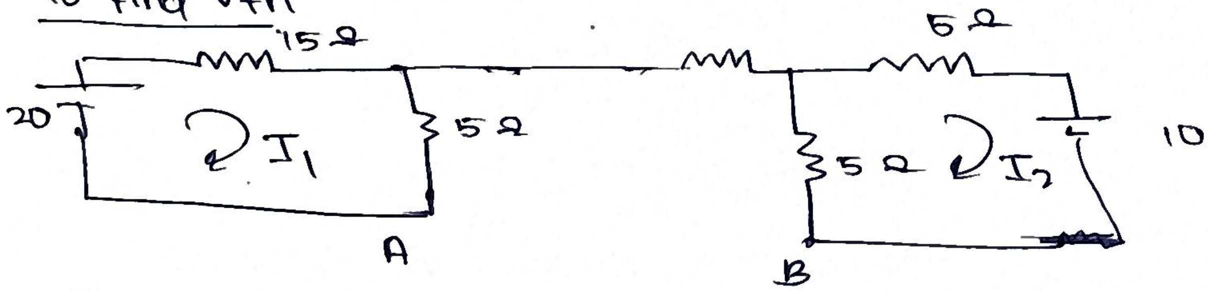
$\frac{1}{R_{12}} = \frac{\cancel{2}4}{15}$ $\quad R_{12} = \frac{15}{4}$

$\frac{1}{R_{34}} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$ $\quad R_{34} = 2.5$

$R_{TH} = \overset{3.75}{\cancel{2.142}} + 2.5 + 5 = 11.25\,\Omega$

$\boxed{R_{TH} = 11.25}$

To find Vth



$-20 + 20I_1 = 0$

$\boxed{I_1 = 1A}$

$10I_2 + 10 = 0$

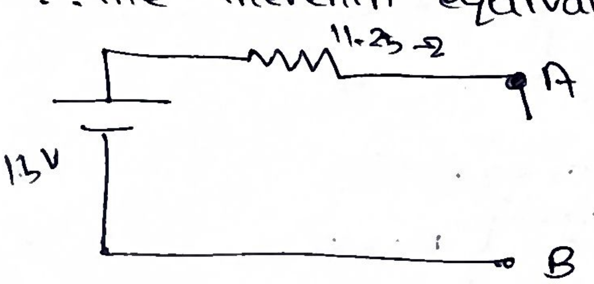
$\boxed{I_2 = -1A}$

$V_{th} = V_{AB} = 15 - V_A - V_B$

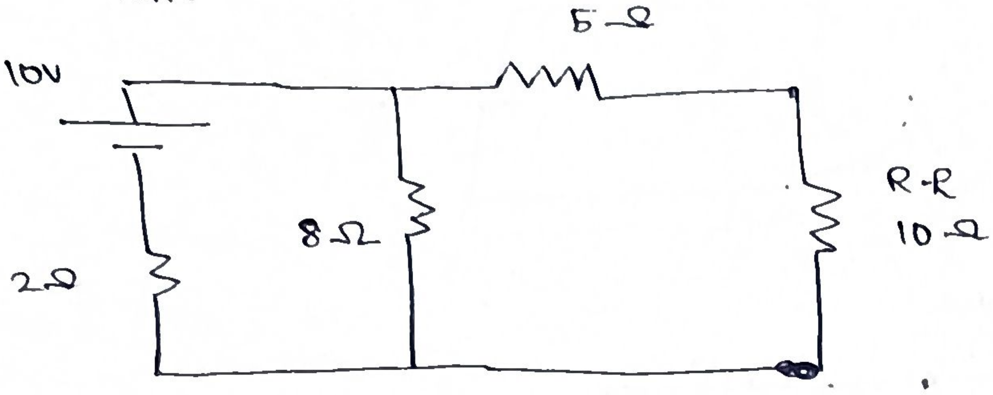
$= 15 - 1(5) - (-1)5$

$\boxed{V_{th} = 15\,V}$

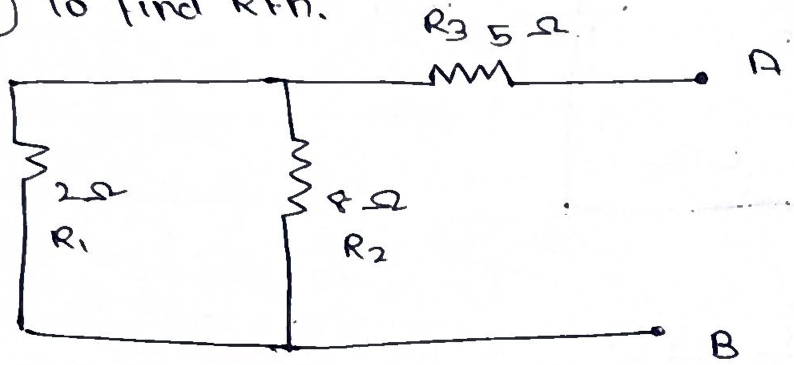
∴ The Thevenin equivalent circuit would be:



Ex5 Use Thevenin's theorem to find the current flowing in the 10Ω resistance.



① To find Rth.



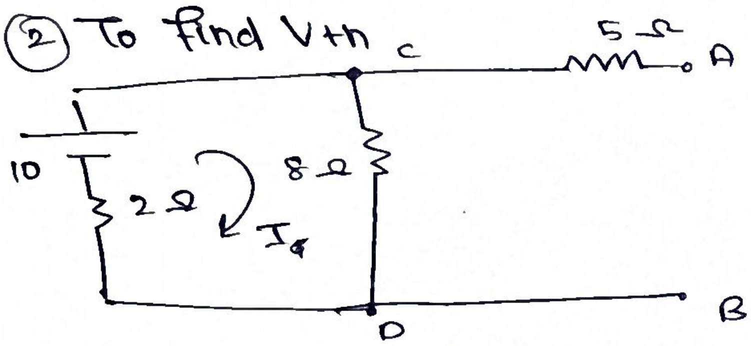
$$\frac{1}{R_{12}} = \frac{1\times4}{2\times4} + \frac{1}{8}$$

$$\frac{1}{R_{12}} = \frac{5}{8}$$

$\boxed{R_{12} = 1.6\,\Omega}$

$R_{TH} = 1.6 + 5 = 6.6\,\Omega$

② To find Vth



$$-10 + 2I + 8I = 0$$

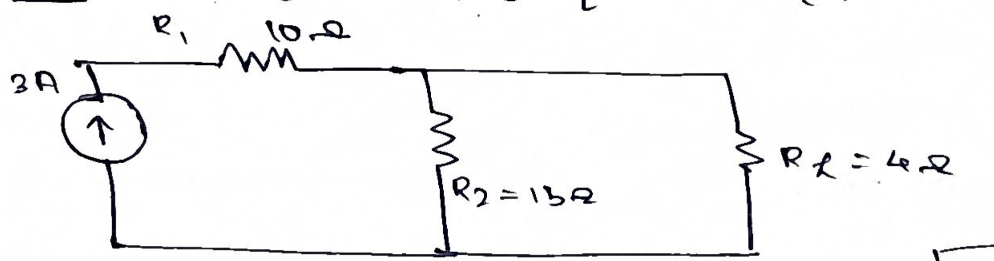
$\boxed{I = 1A}$

$V_{th} = V_{CD} = 8 \times 1 = 8V$

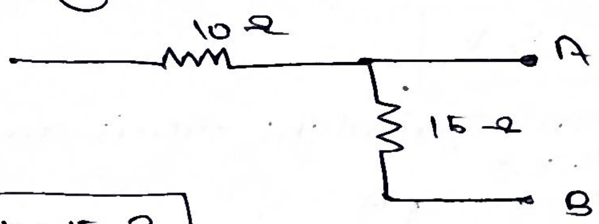
$\boxed{V_{th} = 8V}$

$$I = \frac{8}{10+6.6} = \frac{8}{16.6} = 0.4819A$$

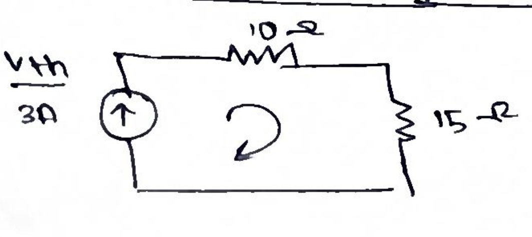
Ex6 Find the Thevenin's equivalent of the following



① To find R_{TH}



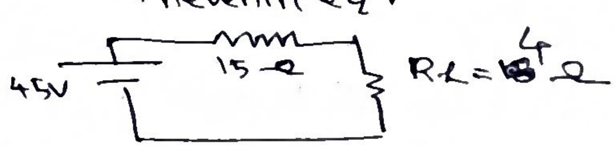
$\boxed{R_{th} = 15\,\Omega}$



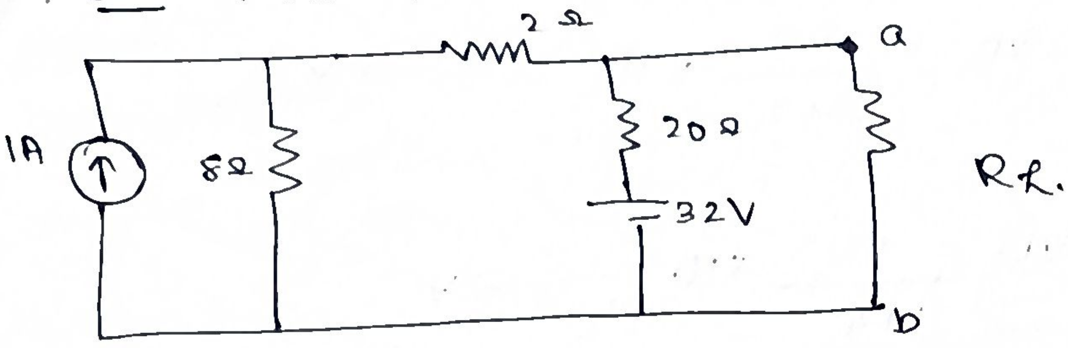
$I = 3A$

$\boxed{V_{th} = 3 \times 15 = 45V}$

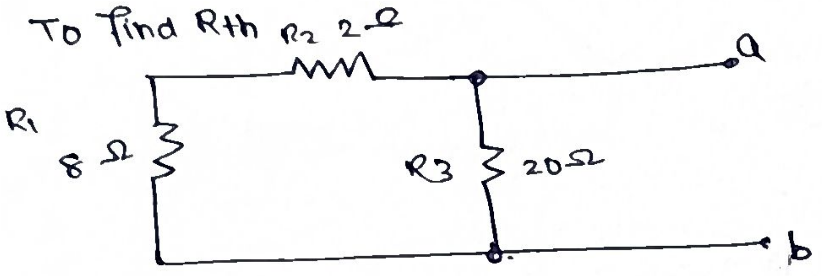
Thevenin eq.



Ex7 Find the Thevenin equivalent of the given circuit



To find Rth

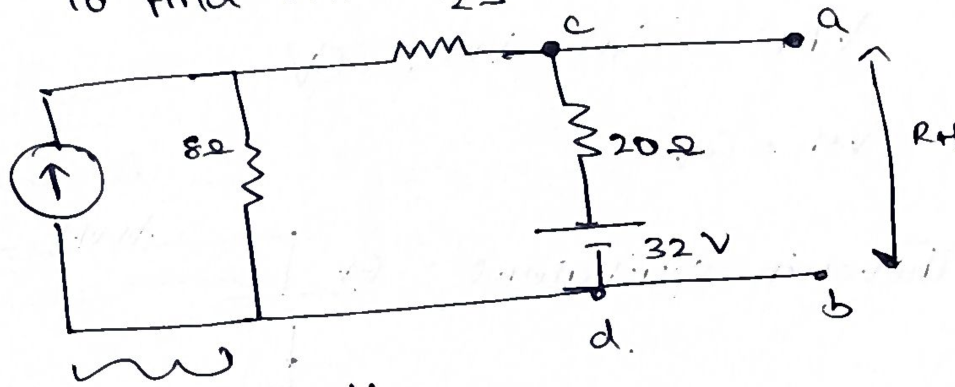


$R_{12} = 10\,\Omega$

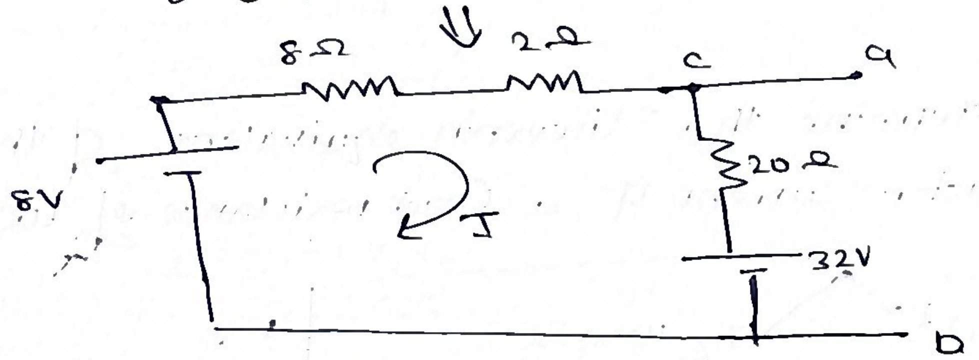
$\frac{1}{R_{123}} = \frac{1^{\times 2}}{10_{\times 2}} + \frac{1}{20} = \frac{3}{20}$

$\boxed{R_{TH} = \frac{20}{3}\,\Omega}$

To find Vth



⇓



$-8 + 10I + 20I + 32 = 0$

$30I + 24 = 0$

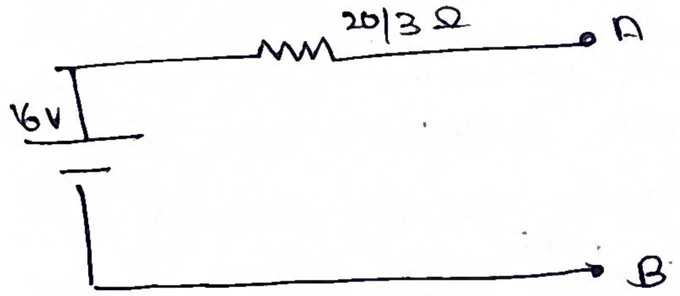
$I = -0.8A$

(note opp direction of assumed current)

$V_{CD} = V_{TH} = 32 - 20 \times (0.8)$

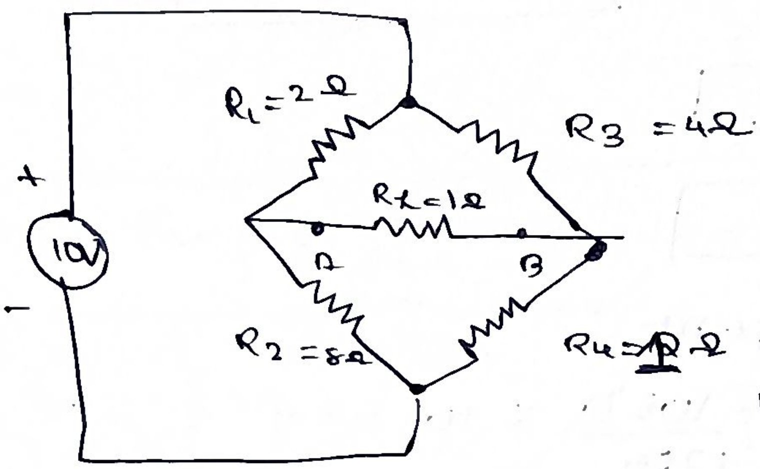
$= 32 - 16 = \underline{\underline{16V}}$

Thevenin's circuit

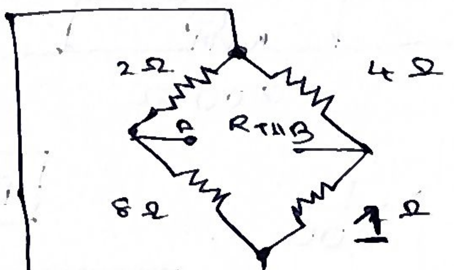


Thevenin's theorem for bridge circuits

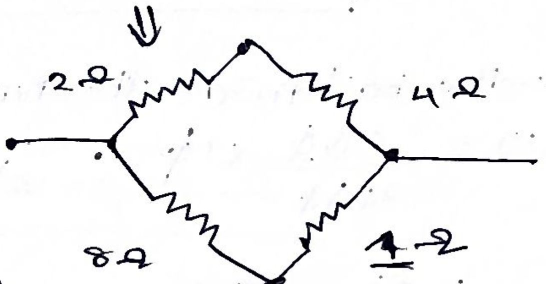
Find the Thevenin's equivalent circuit



(i) Rth



⇓

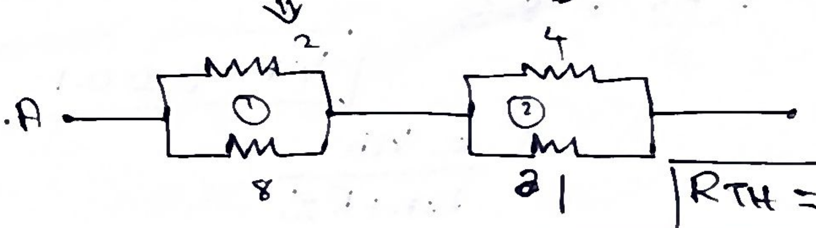


$\frac{1}{R_{TH}}$

$\frac{1}{R_1} = \frac{1^{\times 4}}{2_{\times 4}} + \frac{1}{8}$

$\frac{1}{R_1} = \frac{5}{8}$ $R_1 = 1.6\,\Omega$

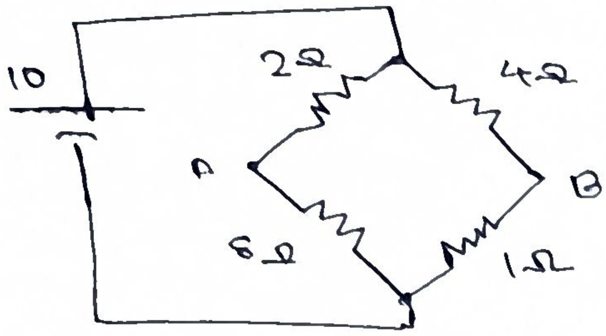
⇓



$\frac{1}{R_2} = \frac{1^{\times 2}}{1} + \frac{1}{4} = \frac{5}{4}$ $R_2 = 0.8$

$\boxed{R_{TH} = 2.4\,\Omega}$

V_{th}



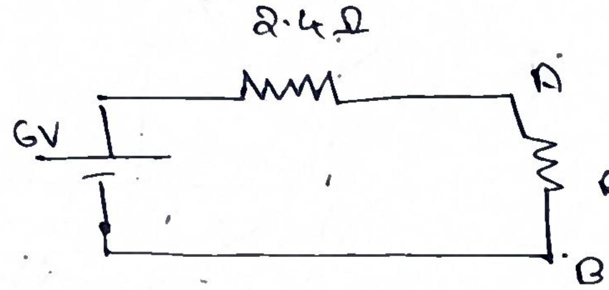
$$V_A = \frac{8}{8+2} \times 10 = 8V$$

$$V_B = \frac{1}{1+4} \times 10 = 2V$$

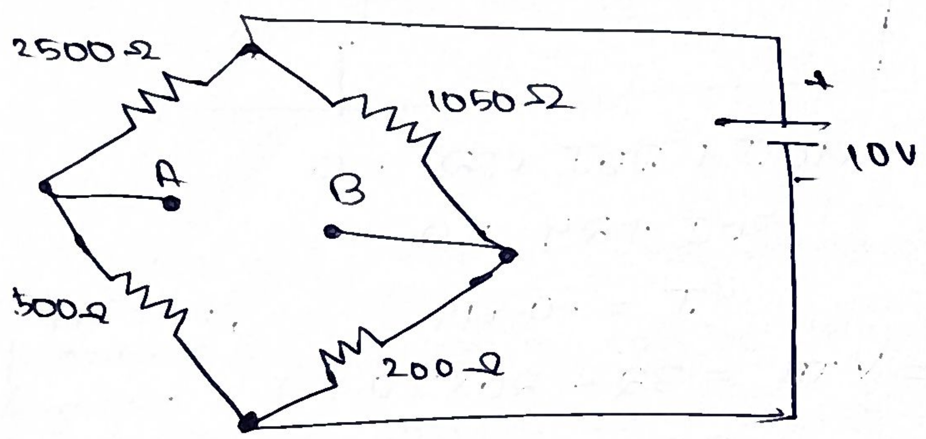
$$V_{AB} = V_A - V_B = 6V$$

$V_{th} = 6V$

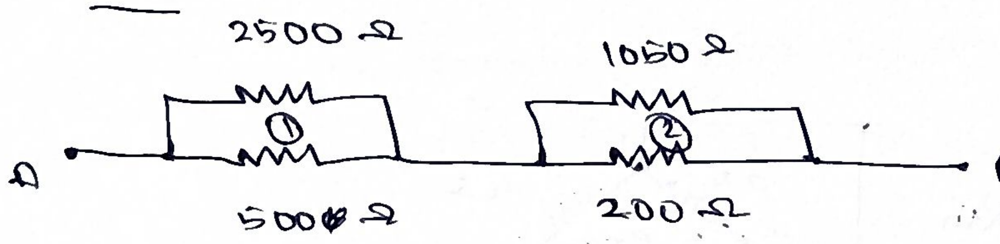
Thevenin equivalent :



2. Determine the Thevenin equivalent of the given circuit. Also find the ~~load~~ current if a load resistance of 100Ω is connected across AB



Ans. To find R_{th}



$$\frac{1}{R_1} = \frac{1}{2500} + \frac{1 \times 5}{500 \times 5}$$

$$\frac{1}{R_1} = \frac{6}{2500}$$

$$\boxed{R_1 = 416.67Ω}$$

$$\frac{1}{R_2} = \frac{1}{1050} + \frac{1}{200}$$

$$\boxed{R_2 = 168Ω}$$

$$\boxed{R_{th} = 584.67Ω}$$

To find V_{th} : ~~Take the bottom as the null point~~

~~$V_A = \frac{500}{500+2500} \times 10 = \frac{500 \times 10}{3000}$~~

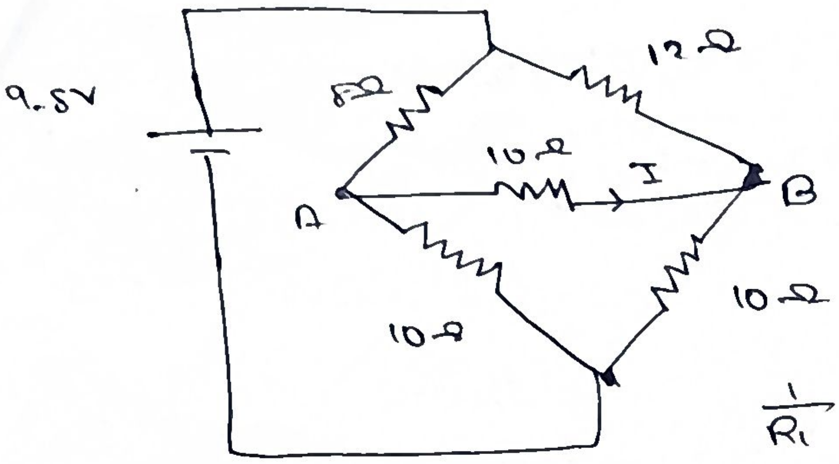
$$V_B = \frac{1050}{1250} \times 10 = 8.4$$

$$V_A = \frac{2500}{3000} \times 10 = \frac{25}{3} = 8.3$$

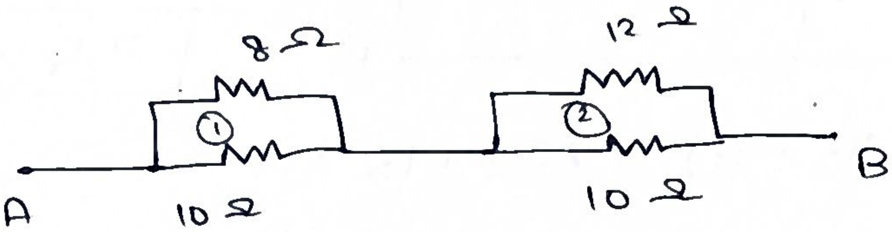
$$\boxed{V_{th} = 0.1}$$

$$I = \frac{V_{th}}{R_{th} + R_L}$$

3. Find the current through the load resistance with Thevenin's theorem



(i) To find R_{th}



$\frac{1}{R_1} = \frac{1}{8} + \frac{1}{10}$ $\frac{1}{R_2} = \frac{1}{10} + \frac{1}{12}$

$R_1 = 4.44\,\Omega$ $R_2 = 5.45$

$\boxed{R_{th} = 9.89}$

(ii) To find V_{th}

$V_A = \frac{10}{18} \times 9.8 = 5.444\,V$

$V_B = \frac{10}{22} \times 9.8 = 4.45\,V$

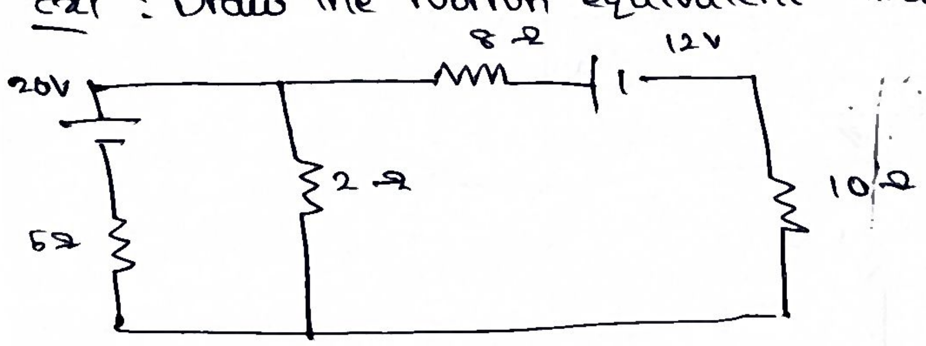
$V_A - V_B = 0.98\,V = V_{th}$

$I_{th} = \frac{0.98}{9.89 + 10}$

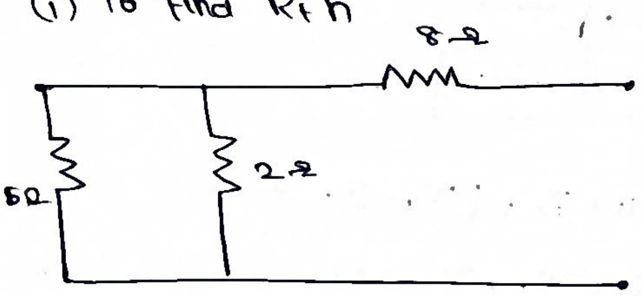
$= 0.049\,A$

Norton's Theorem : Any linear bilateral circuit consisting of energy sources and resistances can be replaced with an equivalent circuit consisting of a Norton's current source I_N in parallel to a Norton resistance connected to a load resistance.

Ex: Draw the Norton equivalent circuit.



(i) To find R_{th}

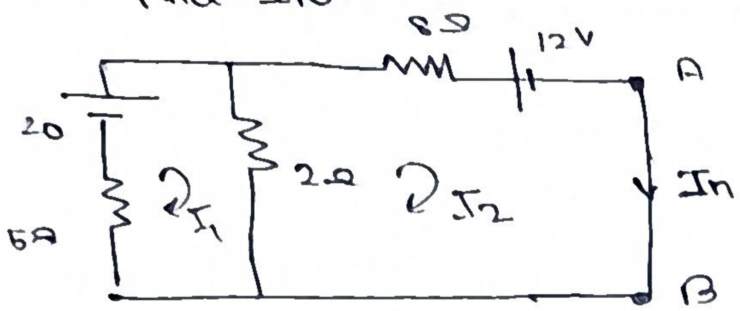


$5 \| 2' + 8$

$\frac{10}{7} + 8 = 9.42\,\Omega$

$\boxed{R_N = 9.42\,\Omega}$

To Find I_N



$$5I_1 - 20 + 2(I_1 - I_2) = 0$$

$$5I_1 - 20 + 2I_1 - 2I_2 = 0$$

$$\boxed{7I_1 - 2I_2 = 20}$$

$$2(I_2 - I_1) + 8I_2 + 12 = 0$$

$$2I_2 - 2I_1 + 8I_2 + 12 = 0$$

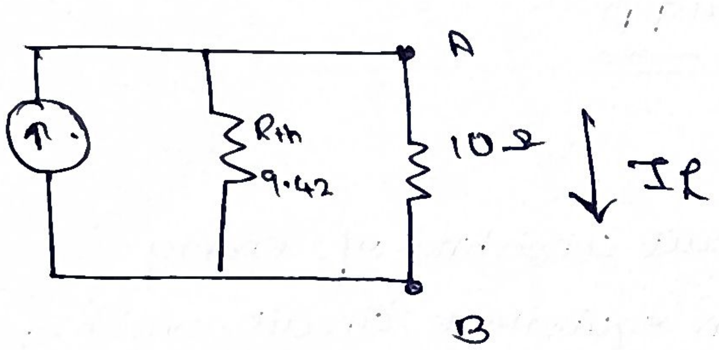
$$10I_2 - 2I_1 + 12 = 0$$

$$\boxed{2I_1 - 10I_2 = 12}$$

$$I_1 = 2.6A$$

$$\boxed{I_2 = I_N = -0.66A}$$

∴ The Norton equivalent circuit would be:

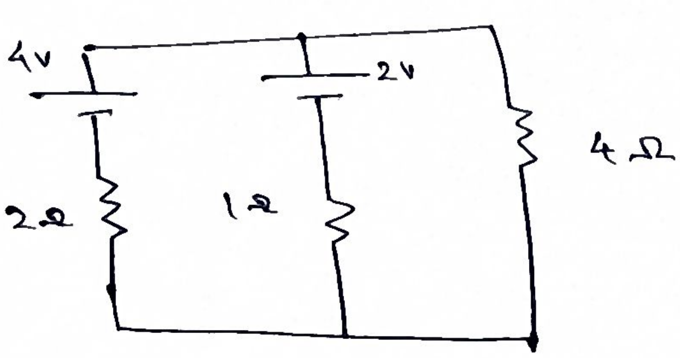


$$I_\ell = I_N \times \frac{R_{TH}}{R_{TH} + 10}$$

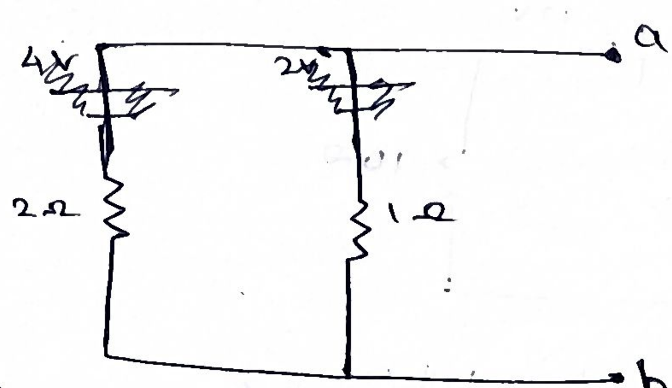
$$= -0.666 \times \frac{9.4}{9.4 + 10}$$

$$\boxed{I_\ell = -0.323A}$$

2. Find the current flowing through the 4Ω resistance using Norton's theorem

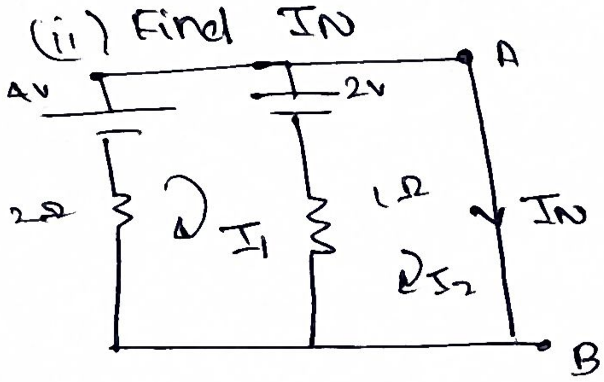


(i) Find R_N



$$\frac{1}{R_N} = \frac{1}{2} + \frac{1}{1} \Rightarrow R_N = \frac{2 \times 1}{3} \quad \boxed{R_N = \frac{2}{3}\Omega}$$

(ii) Find I_N



$$\cancel{2I - 4 + 2 + I = 0}$$

$$\cancel{2I - 2 = 0}$$

$$\cancel{3I = 2}$$

$$\cancel{I_N = \frac{2}{3}}$$

$$2I_1 - 4 + 2 + (I_1 - I_2) = 0$$

$$2I_1 - 2 + I_1 - I_2 = 0$$

$$\boxed{3I_1 - I_2 = 2}$$

$I_2 - I_1 - 2 = 0$

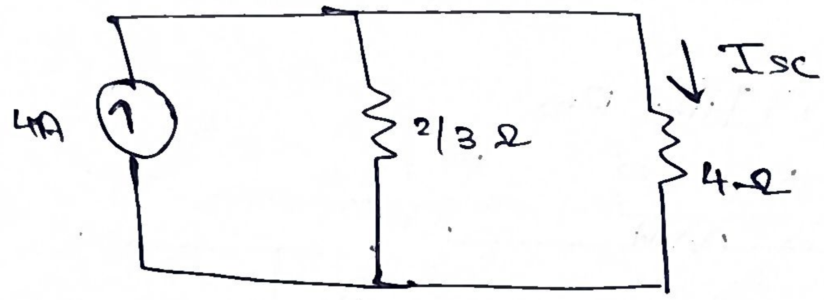
$I_2 - I_1 = 2$

$\boxed{I_1 - I_2 = -2}$

$I_1 = 2A$

$I_2 = I_N = 4A$

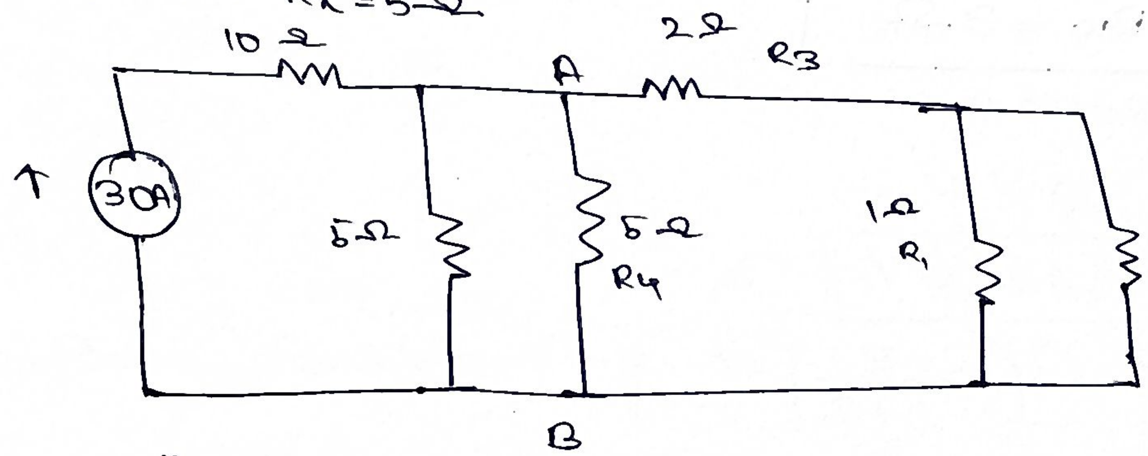
The Norton circuit is



$$I_{sc} = \frac{R_{TH}}{R_{TH} + 4} \times I_N$$

$$= \frac{2/3}{2/3 + 4} \times 4 = 0.571A$$

3. Determine the current through AB in the network shown using Norton's theorem. $R_L = 5\Omega$



$$\frac{1}{R_{12}} = \frac{1}{1} + \frac{1}{1}$$

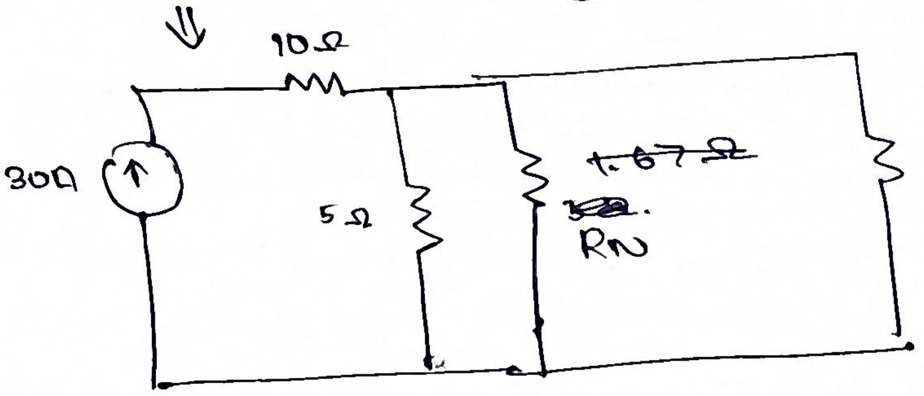
$R_{12} = 0.5$

$R_{123} = 2.5\Omega$

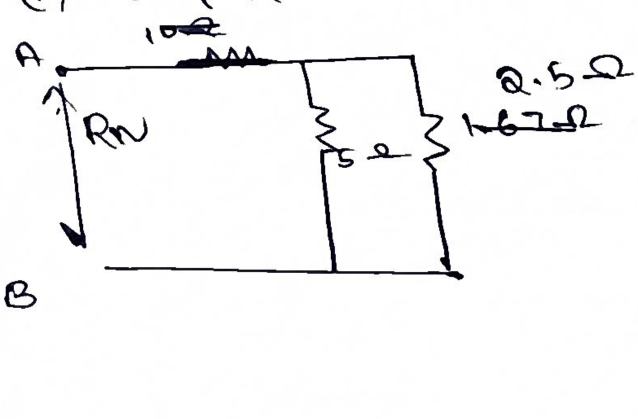
$$\frac{1}{R_{1234}} = \frac{1}{2.5} + \frac{1}{5}$$

~~$R_{1234} = 1.67\Omega$~~

⇓



(i) To find R_N

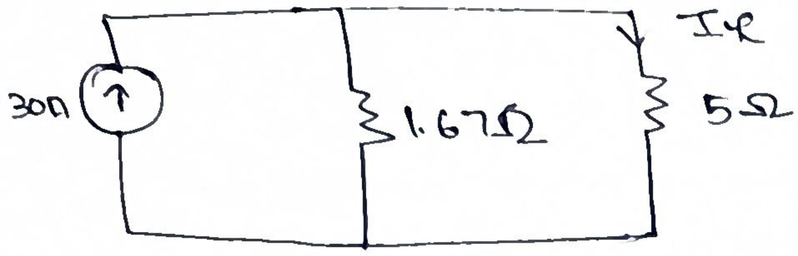


$$\frac{1}{R_N} = \frac{1}{5} + \frac{1}{2.5}$$

$\boxed{R_N = 1.67\Omega}$

(don't include 10 Ω as once the current source is short-circuited, no current flows through it.

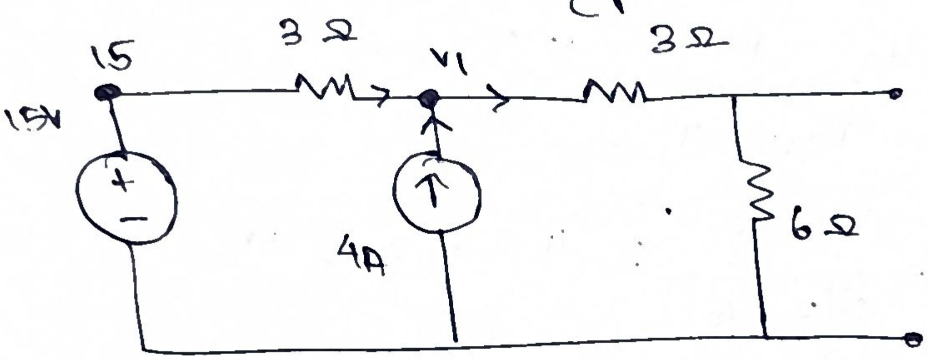
Norton's current = 30A (all the current flows through A B)



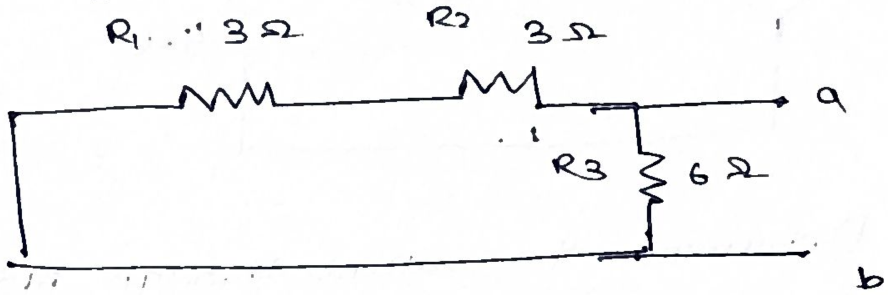
$$I_L = \frac{R_{TH}}{R_{TH} + R_L} \times I_N$$

$$= \frac{1.67}{1.67+5} \times 30 = 7.5A$$

4. Find the value of I_N in the circuit



(i) Find R_N



$R_{12} = 6\Omega$

$$\frac{1}{R_{123}} = \frac{1}{6} + \frac{1}{6}$$

$$\boxed{R_{eq} = R_N = 3\Omega}$$

(ii) Find I_N

by nodal analysis

$$\cancel{15-3} \quad \frac{15-V_1}{3} + 4 = \frac{V_1}{3}$$

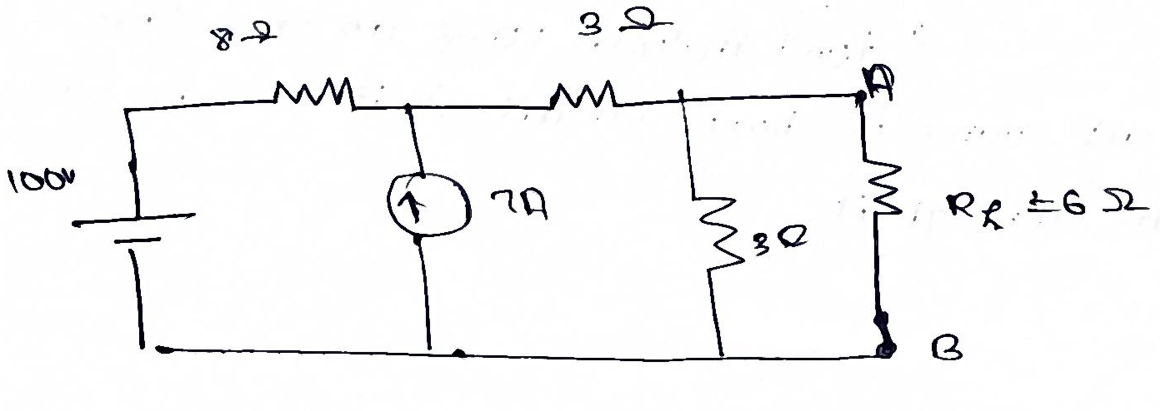
$$15 - V_1 + 12 = V_1$$

$$27 = 2V_1$$

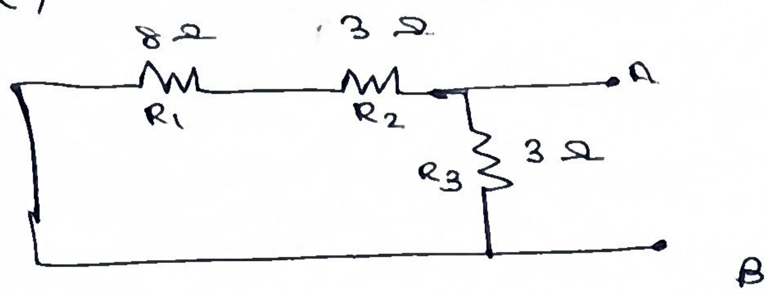
$$\boxed{V_1 = 13.5}$$

$$I_N = \frac{V_1}{R_N} = \frac{13.5}{3} = 4.5A$$

5. Find the value of I_N and I_{sc}



(i) Find R_N

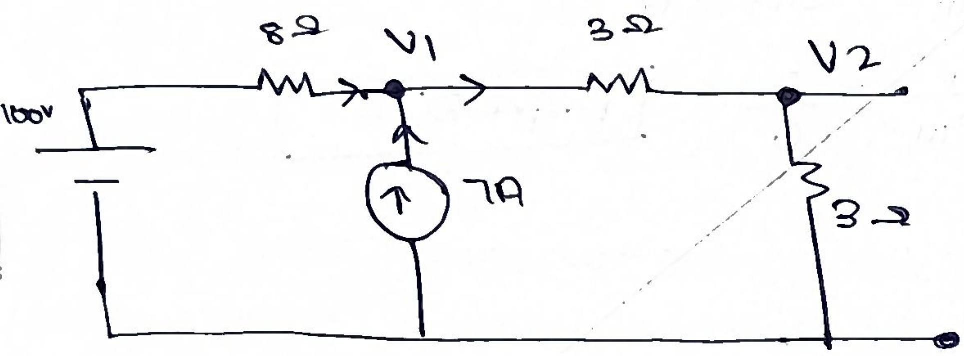


$R_{12} = 11\,\Omega$

$$\frac{1}{R_{123}} = \frac{1}{11} + \frac{1}{3}$$

$$\boxed{R_{123} = 2.357\,\Omega = R_N}$$

(ii) find I_N



by nodal analysis

$$\frac{100 - V_1}{8} + 7 = \frac{V_1 - 0}{6}$$

$$3(100 - V_1) + 168 = 4V_1$$

$$300 - 3V_1 + 168 = 4V_1$$

$\cancel{4V_1 = 468}$ $7V_1 = 468$

$$V_1 = 66.857$$

$$\boxed{V_1 = 66.857\,V}$$

~~$I_N = \frac{66.857}{}$~~ $V_2 = V_1 \left(\frac{3}{3+3} \right)$

$$V_2 = \frac{V_1}{2}$$

$$\boxed{V_{th} = 33.428\,V}$$

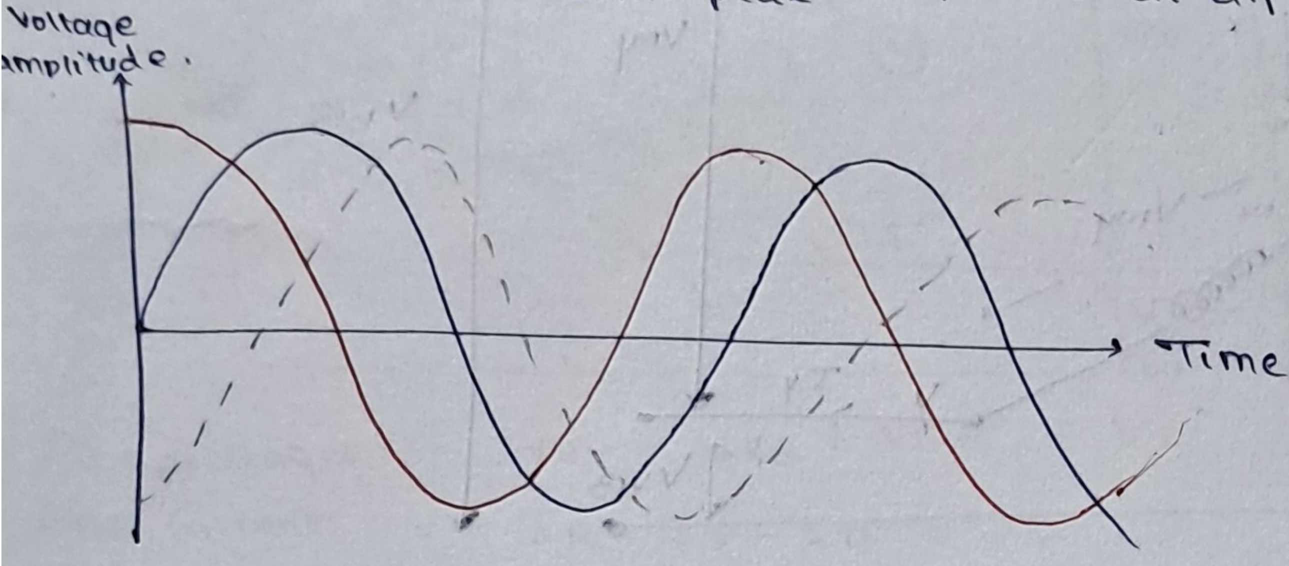
$$I_N = \frac{33.428}{2.357} = 14.18\,A$$

$$I_{sc} = I_N \times \frac{R_{TH}}{R_{TH} + R_L}$$

$$= 14.18 \times \frac{2.357}{8.357} \approx \underline{\underline{4A}}$$

3-Phase AC

In a 3phase AC, 3 conductors can carry 3 alternating electrical quantities at the same frequency. The electrical quantity in these 3 conductors reach the same peak amplitude at different time instances.



The 3 phase power system can be viewed as a combination of 3 separate single phase systems with a 120° phase difference.

Advantages of 3 phase AC

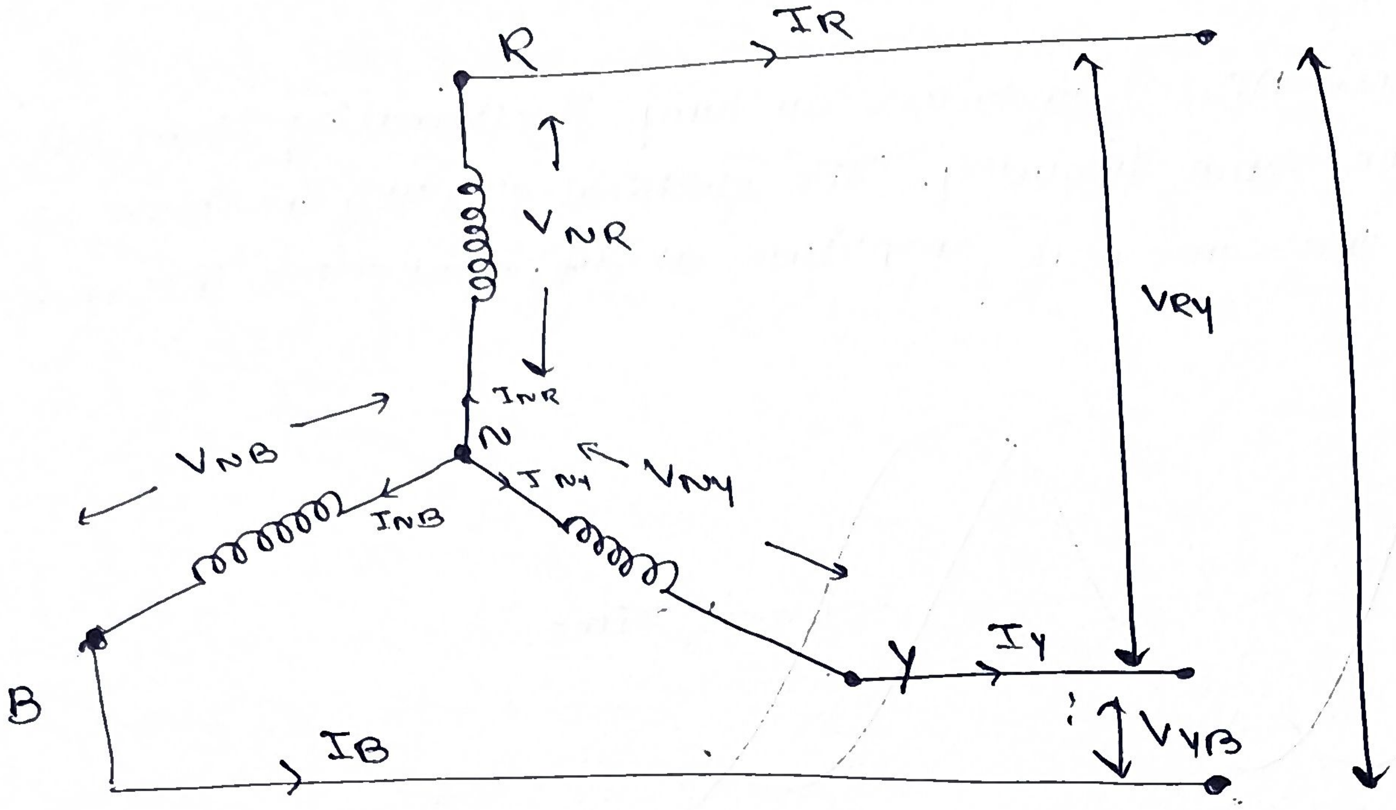
- Most electric power is generated and distributed in three-phase.
- The instantaneous power in a 3-phase system is constant.
- more economic
- higher power factor and efficiency

Phase Sequence: It is the sequence or order in which the alternating quantities attain their peak positive values.

Phase are labelled R Y B
↓ ↓ ↓
red yellow blue

RYB = positive phase sequence
BYR = negative phase sequence

Star Connection: It is a type of connection where one terminal from each winding is connected to a common point and the remaining three terminals are connected to the circuit.



Line Quantities : (a quantity that is measured from line to line)

Line voltage: V_{RY}, V_{YB}, V_{BR} $\quad V_\ell$

Line current: I_R, I_Y, I_B $\quad I_\ell$

Phase Quantities : (quantity measured between a line and a neutral pt)

Phase voltage: V_{NR}, V_{NY}, V_{NB} $\quad V_{PH}$

Phase current: I_{NR}, I_{NY}, I_{NB} $\quad I_{PH}$.

In a balanced system,

line voltages: $V_{RY} = V_{YB} = V_{BR}$

line currents: $I_R = I_Y = I_B$

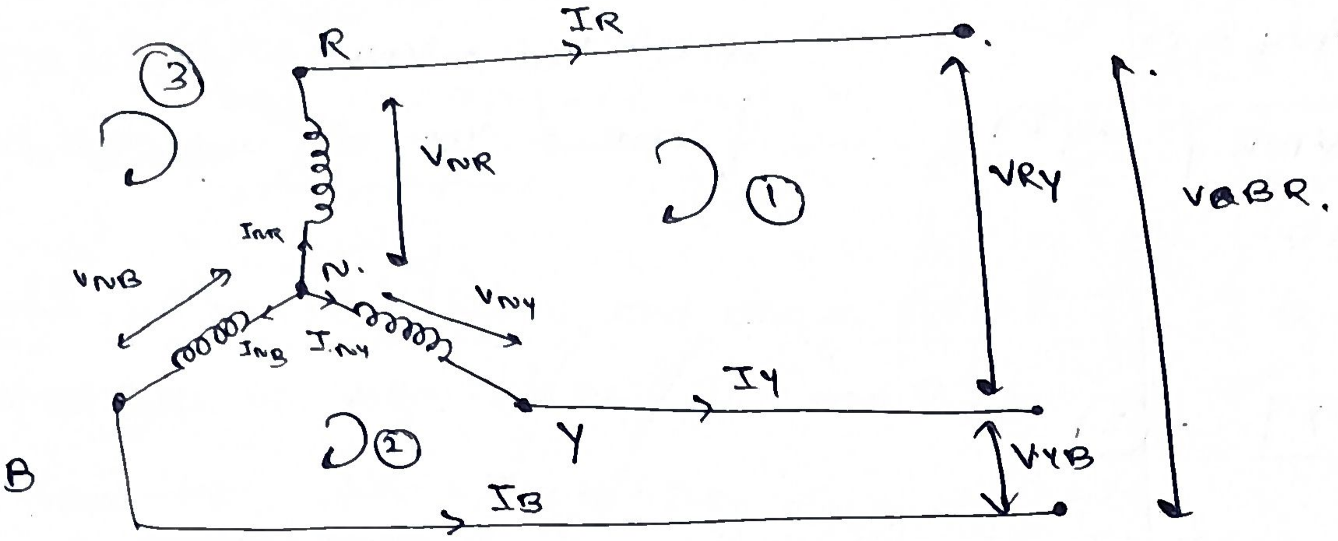
phase currents: $I_{NR} = I_{NY} = I_{NB}$

phase voltages: $V_{NR} = V_{NY} = V_{NB}$

In a star connection, using Kirchoff's laws:

$$\boxed{V_{NR} + V_{NY} + V_{NB} = 0}$$

Relation between line and phase quantities for a balanced 3 phase star connected system:



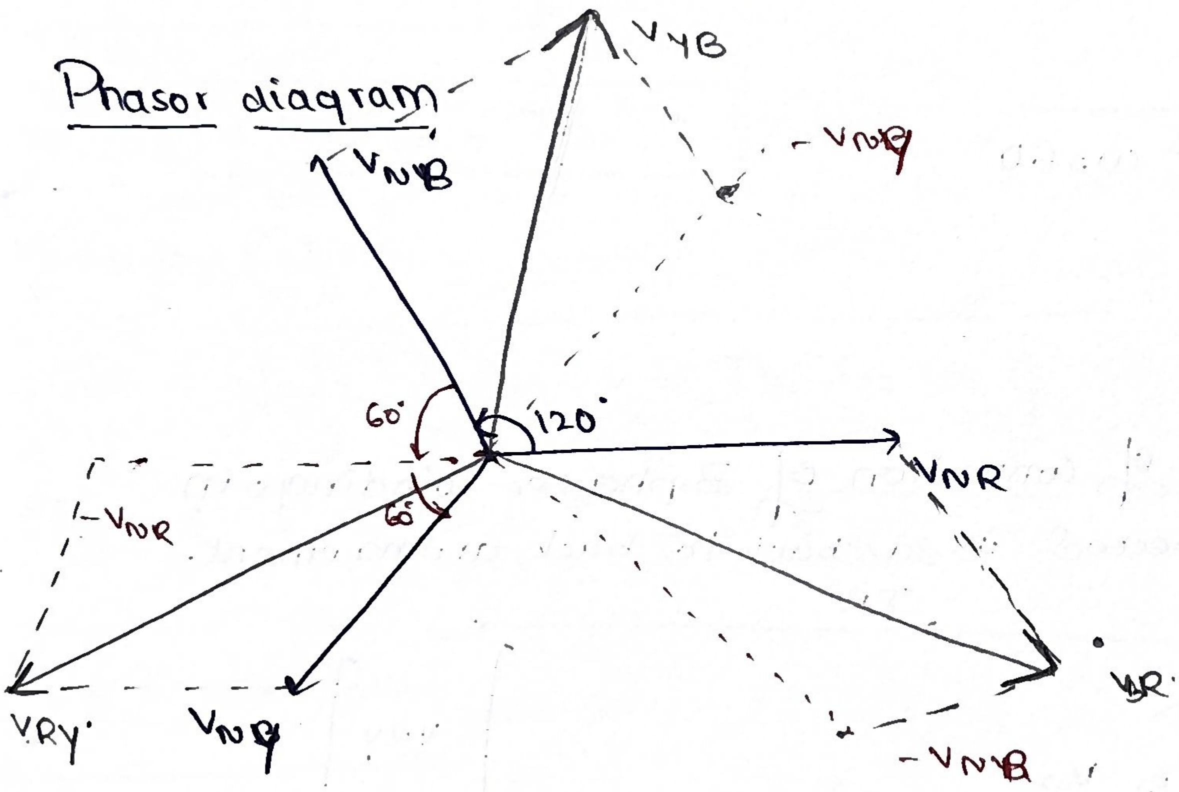
line voltages = $V_{RY} = V_{YB} = V_{BR} = V_L$

line currents = $I_R = I_Y = I_B = I_L$

phase voltages = $V_{NR} = V_{NY} = V_{NB} = V_{PH}$

phase currents = $I_{NR} = I_{NY} = I_{NB} = I_{PH}$.

Phasor diagram



Relation between line and phase currents

$I_{NR} = I_R$

$I_{NY} = I_Y$

$I_{NB} = I_B$

$I_{NR} = I_{NY} = I_{NB} = I_{PH}$

$I_R = I_Y = I_B = I_L$

$\Rightarrow \boxed{I_{PH} = I_L}$

Relation between line and phase voltages

Apply KVL to Loop 1

$V_{NR} + V_{RY} - V_{NY} = 0$

$$\boxed{V_{RY} = V_{NY} - V_{NR}} \quad -(1)$$

Apply KVL to Loop 2

$V_{NY} + V_{YB} - V_{NB} = 0$

$$\boxed{V_{YB} = V_{NB} - V_{NY}} \quad -(2)$$

Apply KVL to Loop 3

$V_{NB} + V_{BR} - V_{NR} = 0$

$$\boxed{V_{BR} = V_{NR} - V_{NB}}$$

From eqn (1)

$$V_{RY} = \sqrt{V_{NY}^2 + V_{NR}^2 + 2V_{NY}V_{NR}\cos\phi}$$

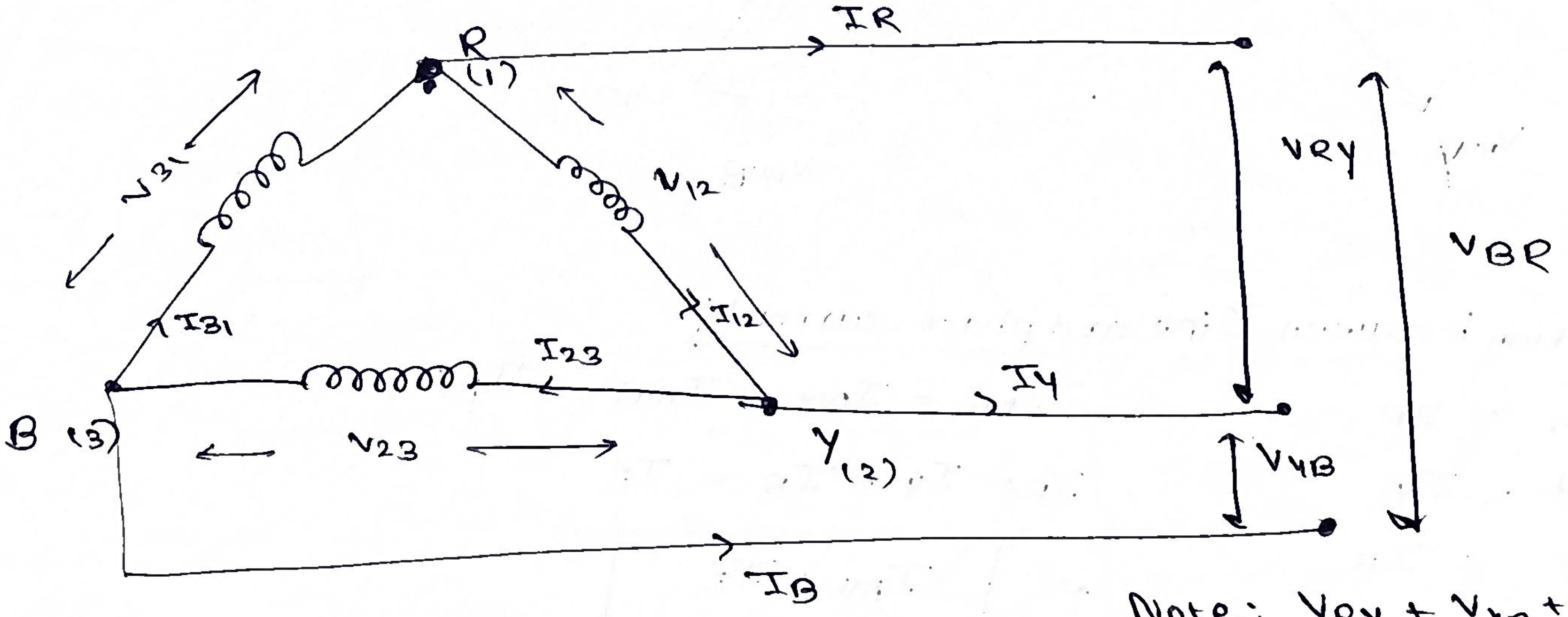
$V_{NY} = V_{NR} = V_{PH}$

$V_{RY} = V_L$

$$V_L = \sqrt{V_{PH}^2 + V_{PH}^2 + 2V_{PH}^2\cos 60^\circ}$$

$$\boxed{V_L = \sqrt{3}\, V_{PH}}$$

Delta Connection : A type of connection of 3 phases windings in which all the coils are connected in a back to back arrangement.



Note: $V_{RY} + V_{YB} + V_{BR} = 0$

Line currents : $I_R = I_Y = I_B = I_L$

Line voltages : $V_{RY} = V_{YB} = V_{BR} = V_L$

phase currents : $I_{12} = I_{23} = I_{31} = I_{PH}$

phase voltages : $V_{12} = V_{23} = V_{31} = V_{PH}$

Relation between line and phase voltage

$V_{12} = V_{RY}$

$V_{23} = V_{YB}$ $\Rightarrow \boxed{V_L = V_{PH}}$

$V_{31} = V_{BR}$

Relation between line and phase currents.

using KCL on the vertices R, Y and B.

at R ~~$I_R = I_{31}$~~ $I_R + I_{12} = I_{31}$

$\boxed{I_R = I_{31} - I_{12}}$ — (1)

at Y $I_{12} = I_Y + I_{23}$

$\boxed{I_Y = I_{12} - I_{23}}$

at B $\boxed{I_B = I_{23} - I_{31}}$

Consider (1)

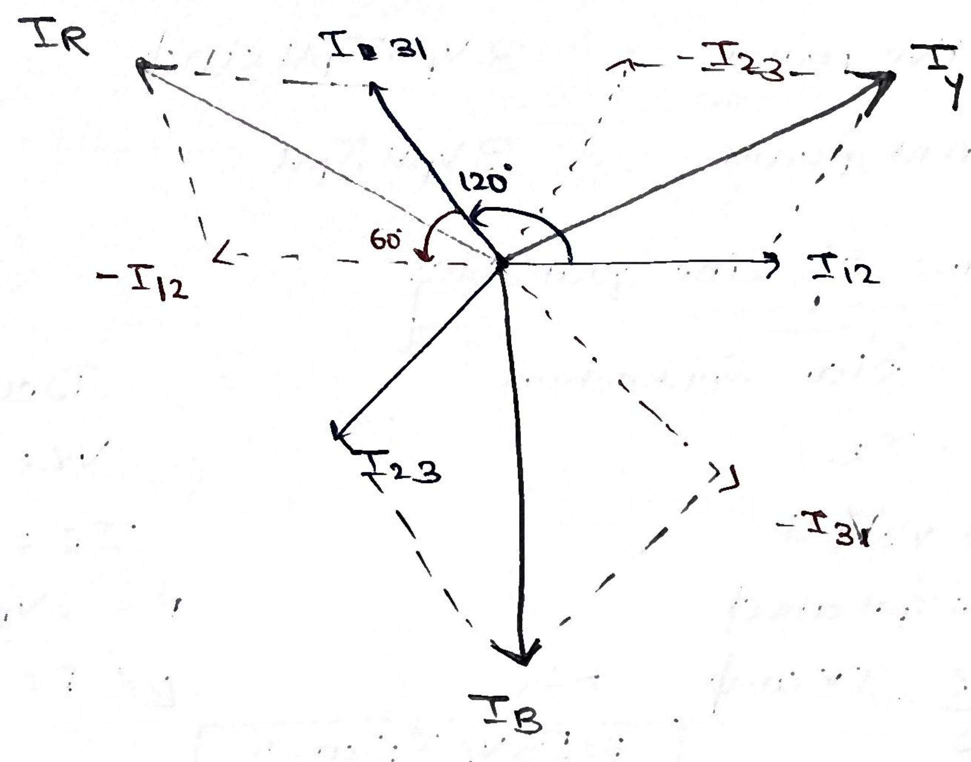
$I_R = \sqrt{I_{31}^2 + I_{12}^2 + 2 I_{31} I_{12} \cos\phi}$

$I_R = \sqrt{I_{PH}^2 + I_{PH}^2 + 2 I_{PH}^2 \cos 60^\circ}$

$I_L = \sqrt{3}\, I_{PH}$

$\boxed{I_L = \sqrt{3}\, I_{PH}}$

phasor diagram



Power in 3 phase AC

In a single phase AC

power factor $= \cos\phi$

$= \frac{R}{Z}$

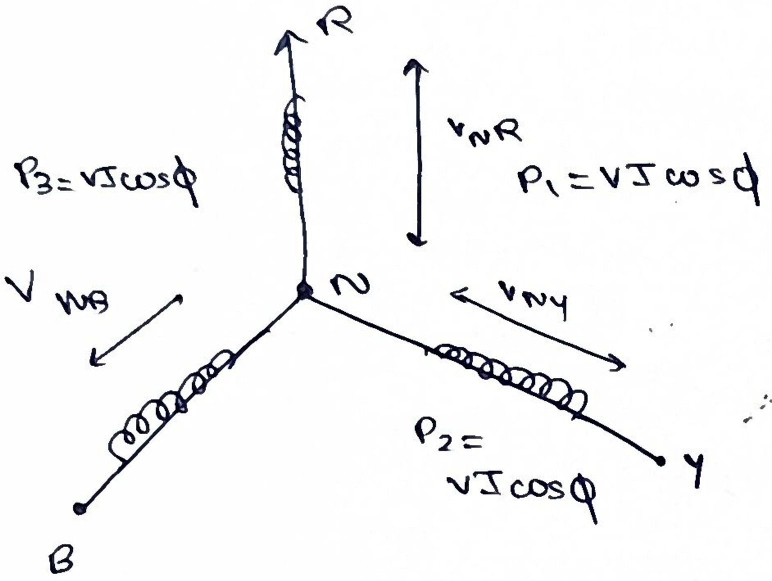
Active power $= V_{rms} I_{rms} \cos\phi$ (P)

Reactive power $= V_{rms} I_{rms} \sin\phi$ (Q)

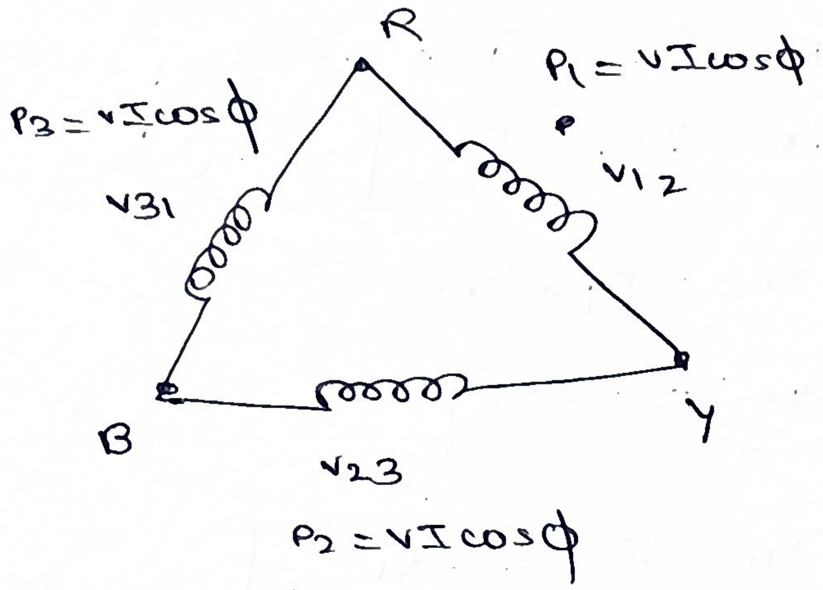
Apparent power $= V_{rms} I_{rms}$ (S)

In terms of phase quantities

Star Connection



Delta Connection



In both star and delta connections, the ~~line~~ phase voltages are equal, as are the phase currents

$\Rightarrow P = P_1 + P_2 + P_3$

$= 3VI\cos\phi$

$$\boxed{P = 3V_{pH} I_{pH} \cos\phi}$$

			unit
$\therefore$ Active power	$=$	$3V_{pH} I_{pH} \cos\phi$	watts
Reactive power	$=$	$3V_{pH} I_{pH} \sin\phi$	Volt Ampere Reactive
Apparent power	$=$	$3V_{pH} I_{pH}$	Volt Ampere

In terms of line quantities

Star Connection

$I_{pH} = I_L$

$V_L = \sqrt{3} V_{pH}$

$P = 3V_{pH} I_{pH} \cos\phi$

$P = 3 \cdot \frac{V_L}{\sqrt{3}} \cdot I_L \cos\phi$

Delta Connection

$V_L = V_{pH}$

$I_L = \sqrt{3} I_{pH}$

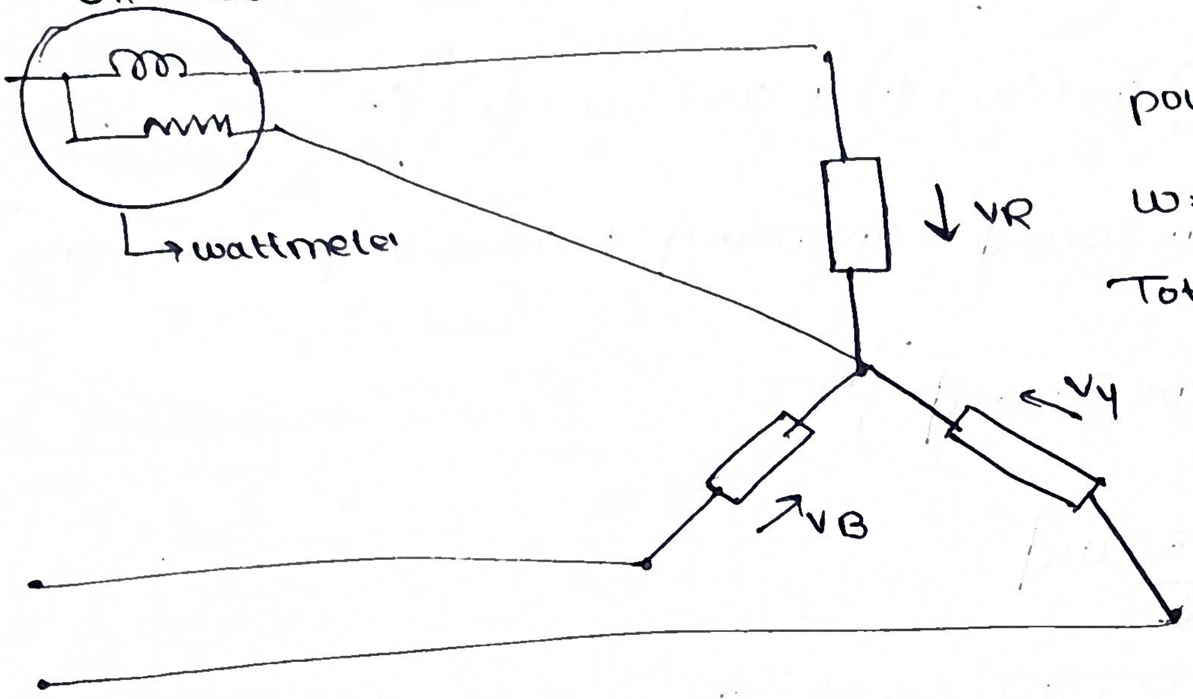
$P = 3V_{pH} I_{pH} \cos\phi$

$P = 3V_L \frac{I_L}{\sqrt{3}} \cos\phi$

$\Rightarrow$ $$\boxed{P = \sqrt{3} V_L I_L \cos\phi}$$

Measurement of power and power factor in 3 phase circuits

One watt meter method



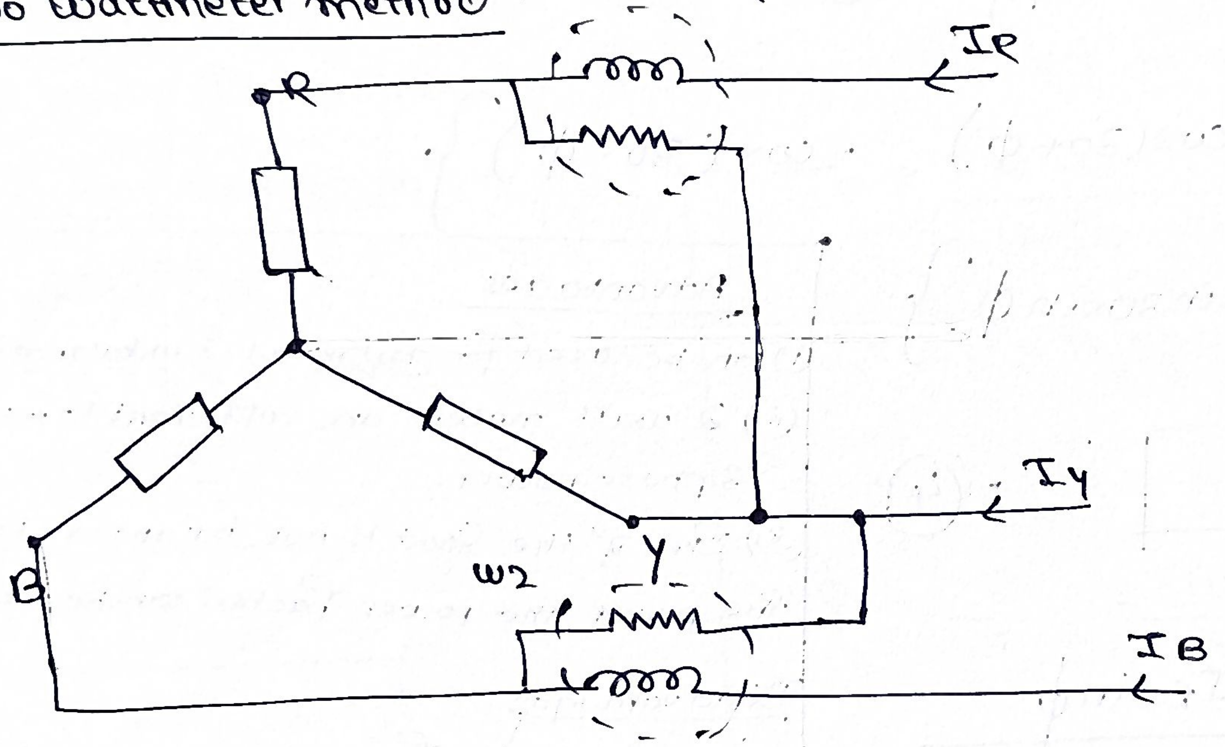
power reading of wattmeter

$W = V_{ph} I_{ph} \cos\phi$

Total power $= 3 V_{ph} I_{ph} \cos\phi$

$\boxed{P = 3W}$

Two wattmeter method.



$W_1 = I_R V_{RY} \cos\phi$ (VRY since one end is connected to R, the other at Y)

$W_2 = I_B V_{BY} \cos\phi$

$V_{RY} = V_R - V_Y = V_R + (-V_Y)$

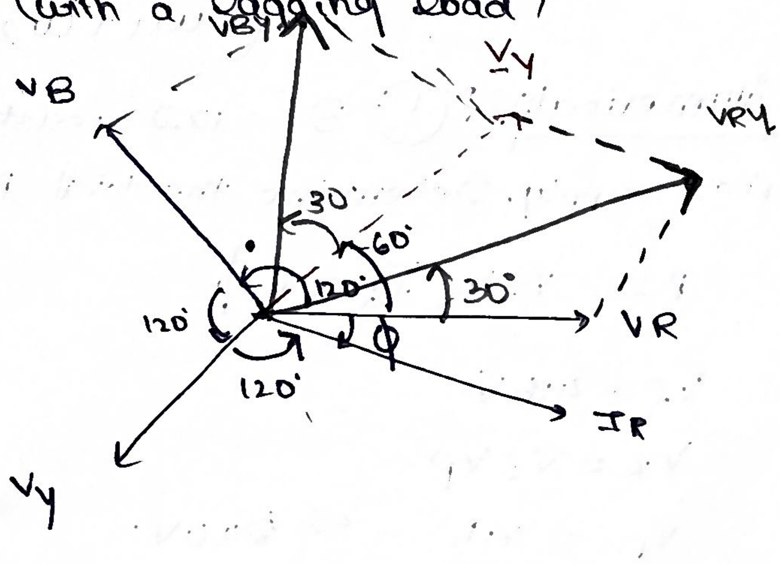
$\therefore \boxed{W_1 = I_R V_{RY} \cos(30+\phi)}$ — (1)

$\Rightarrow \boxed{W_1 = I_R V_L \cos(30+\phi)}$

|||ly $V_{BY} = V_B - V_Y = V_B + (-V_Y)$

$\boxed{W_2 = I_R V_L \cos(30-\phi)}$ → (2)

phasor diagram (with a lagging load)



$$W_1 = I_L V_L \cos(30+\phi)$$

$$W_2 = I_L V_L \cos(30-\phi)$$

$$W_1 + W_2 = I_L V_L \{\cos(30+\phi) + \cos(30-\phi)\}$$

$$= I_L V_L \{\cos 30 \cos\phi - \sin 30 \sin\phi + \cos 30 \cos\phi + \sin 30 \sin\phi\}$$

$$= I_L V_L \{2 \cos 30 \cos\phi\}$$

$$= 2 V_L I_L \cdot \frac{\sqrt{3}}{2} \cos\phi$$

$$\boxed{W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi} \quad -(3)$$

$$W_1 - W_2 = I_L V_L \{\cos(30+\phi) - \cos(30-\phi)\}$$

$$= I_L V_L \{2 \sin 30 \sin\phi\}$$

$$\boxed{W_2 - W_1 = I_L V_L \sin\phi} \quad -(4)$$

(4) ÷ (3)

$$\frac{W_2 - W_1}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi}$$

$$\therefore \tan\phi = \sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)$$

Advantages

(i) can be used for balanced & unbalanced loads

(ii) 2 watt meters are sufficient to measure 3 phase power.

(iii) Even if the load is not balanced, both the power and power factor can be measured.

Disadvantages

(i) not applicable for a 3 phase - 4 wire system

(ii) The sign of W_1 & W_2 must be identified and noted down correctly, otherwise it may lead to the wrong result.

Numericals: (1) 3 12Ω resistors are connected in star to a 415 V, three-phase supply. Determine the total power dissipated by the resistors.

$$P = \sqrt{3} V_L I_L \cos\phi$$

$$V_L = 415\,V$$

$$V_L = \sqrt{3} V_P$$

$$V_P = \frac{415}{\sqrt{3}} \approx 240\,V.$$

$$I_P = \frac{V_P}{Z_P}$$

$$I_P = \frac{240}{12} = 20\,A$$

In a star connection, $I_L = I_P = 20\,A$

$$\therefore P = \sqrt{3} \cdot 415 \times 20 = 14376\,W$$

2: The input power to a 3 phase AC motor is measured as 5kW. If the voltage and current to the motor are 400V and 8.6A, determine the power factor of the system.

$P = 5000W$

$V_L = 400V$

$I_L = 8.6A$

$P = \sqrt{3} V_L I_L \cos\phi$

power factor $= \cos\phi$

$\cos\phi = \frac{P}{V_L I_L} = \frac{5000}{\sqrt{3}\, 400 \times 8.6} = 0.839$

3. 3 identical coils, each of resistance 10Ω and inductance 42mH are connected in (a) star and (b) in delta to a 415V, 50Hz, three-phase supply. Determine the total power dissipated in each case

(a) In star connection

$X_L = 2\pi f L$

$= 2\pi \times 50 \times 42 \times 10^{-3}$

$= 13.19\,\Omega$

$Z_p = \sqrt{R^2 + X_L^2}$

$= \sqrt{100 + (13.19)^2}$

$= 16.55\,\Omega.$

$V_L = 415V$

Power $= \sqrt{3} V_L I_L \cos\phi$

~~$I_L = V_L$~~

$I_p = \frac{V_p}{Z_p}$

$V_L = \sqrt{3} V_p$

$V_p = \frac{415}{\sqrt{3}} \approx 240V.$

$I_p = \frac{240}{16.55} = 14.50A$

$I_L = I_p = 14.50A$

$P = \sqrt{3} \times 415 \times 14.50 \times \cos\phi$

$\cos\phi = \frac{R}{Z} = \frac{10}{16.55} = 0.6042$

$P = \sqrt{3} \times 415 \times 14.50 \times 0.6042$

$P = 6297.3W$

(ii) Delta connection

$V_L = 415\,V$ $Z_P = 16.55$

$V_L = V_P = 415\,V$

$I_L = \sqrt{3} I_P$.

$I_P = \frac{V_P}{Z_P}$

Note: Loads connected in delta dissipate three times the power when connected in star, and take a line curren three times greater.

$I_P = \frac{415}{16.55} = 25.075\,A$

$I_L = \sqrt{3} I_P = \sqrt{3} \times 25.075$

$= 43.43\,A$

$P = \sqrt{3} \times 43.43 \times 415 \times 6.6042$

$= 1886.16\,W$

4. The input power to a 3phase AC motor is measured as 5kw. If the voltage and the current to the motor are 400V and 8.6A respectively, determine the power factor of the system.

$V_L = 400V$

$I_L = 8.6A$

(repeated)

$P = 5kW = 5000\,W$

$P = \sqrt{3} V_L I_L \cos\phi$

$\cos\phi = \frac{5000}{\sqrt{3} \times 400 \times 8.6} = 0.839$

5. 2 wattmeters are connected to measure the input power to a balanced 3-phase load by the 2-wattmeter method. If the instrument readings are 8kW and 4kW, determine the total power input and the load power factor

Total power = 12kW

$\tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$

$\tan\phi = \sqrt{3}\left(\frac{4}{12}\right) \Rightarrow \tan\phi = \frac{1}{\sqrt{3}} = 30°$

power factor $= \cos 30 = \sqrt{3}/2$

6. Two wattmeters connected to a 3-phase motor indicate the total power input to be 12kW. The power factor is 0.6 Determine the readings of each wattmeter

$\cos\phi = 0.6$

$\phi = \cos^{-1}(0.6)$

$= 53.13°$

$$\tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$\tan(53.13) = \sqrt{3}\left(\frac{W_1 - W_2}{12}\right)$$

$P_1 - P_2 = 9.237$

$P_1 + P_2 = 12$

$$\boxed{\begin{array}{l} P_1 = 10.62\,kW \\ P_2 = 1.38\,kW \end{array}}$$

7. Three loads, each of resistance 30 Ω are connected in a star to a 415V, 3phase supply. Determine

(i) the system phase voltage

$V_L = 415V$

$V_L = \sqrt{3}V_p$

$V_p = \frac{415}{\sqrt{3}} \approx 240V$

(ii) the phase current

$R = 30\,\Omega$

$I_p = \frac{V_p}{R} = \frac{240}{30} = 8A$

(iii) the line current

$I_p = I_L = 8A$

8. A 415V, 3phase AC motor has a power output of 12.75kW and operates at a power factor of 0.77 lagging with an efficiency of 85%. If the motor is delta connected, determine:

(a) the power input

$$\text{Efficiency} = \frac{\text{power o/p}}{\text{power input}} \times 100$$

$$\text{power input} = \frac{100}{85} \times 12750$$

$$= 15{,}000 \, W$$

$$= \underline{\underline{15 kW}}$$

(b) the line current

$P = \sqrt{3} V_L I_L \cos\phi$

$\cos\phi = 0.77$

$V_L = 415 V$

$P = 15{,}000$

$$I_L = \frac{P}{\sqrt{3} V_L \cos\phi} = 27.10 A$$

(c) the phase current

$I_L = \sqrt{3} I_p$

$$I_p = \frac{27.10}{\sqrt{3}} = \underline{\underline{15.647 A}}$$

9. A 400V, 3 phase star connected alternator supplies a delta connected load, each phase of which has a resistance of 30 Ω and inductive reactance 40 Ω. Calculate the current supplied by the alternator and the output power, kVA of the alternator.

(a) current supplied by the alternator.

$V_L = 400V$

$R = 30 \Omega$

$X_L = 40 \Omega$

$Z_p = \sqrt{X_L^2 + R^2} = 50 \Omega$

(ignore alternator, only consider the delta connected load)

~~$V_L = \sqrt{3} V_p$~~

~~$V_p = \frac{400}{\sqrt{3}}$~~

$V_p = V_L = 400 V$

$$I_p = \frac{400}{50} = 8A$$

$I_l = \sqrt{3} I_p$

$I_l = 8\sqrt{3} = 13.856 \Omega$

output power.

$P = \sqrt{3} V_L I_l \cos\phi$

$\cos\phi = \frac{R}{Z} = \frac{30}{50} = 0.6$

$P = \sqrt{3} \times 400 \times 13.856 \times 0.6$

$= 5759.83\ W$