Image Processing and Analysis

Unit 4

Image Segmentation

Edge detection: techniques - edge linking via Hough transform; Thresholding: Intensity thresholding - Otsu's thresholding; Region-based segmentation: region growing - region splitting and merging; Morphological processing: erosion & dilation; Segmentation by morphological watersheds; basic concepts - dam construction - watershed segmentation algorithm

* Edge Detection Techniques

Steps in Edge Detection

1. Image smoothing for noise reduction
2. Detection of edge points - a local operation that extracts all potential edge-point candidates
3. Edge localization - Select from the candidate points only the points that are members of the set of points comprising an edge.

* Methods of Edge Detection

A. 'First Order Derivative / Gradient Methods

- Roberts Operator
- Sobel Operator
- Precott Operator

B. Second Order Derivative

- Laplacian
- Laplacian of Gausion

C. Optimal Edge Detection

- Canny Edge Detection

→ Masks must be derived in order to find edges corresponding to a pixel of interest.

mask for
(i) horizontal mask
(ii) vertical mask

First Order Derivative (Gradient) Methods

(1) Roberts Cross Operator

→ a gradient based edge detection operator, that uses a pair of 2x2 convolution kernel to approximate the gradient of the image in the horizontal and vertical directions.

X kernel

$$G_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Y kernel

$$G_y = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

→ Convolve these kernels with the image, and then compute the magnitude of the resulting gradient as $\sqrt{G_x^2 + G_y^2}$

→ computationally inexpensive, but more sensitive to noise

2 Sobel Operator

→ mask gives emphasis to weights. The central row of the kernal places more emphasis on the center pixel, and the outer rows assign less weight.

Xkernel

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Y kernel

$$G_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

→ The 3x3 convolution mask smoothes the image by some amount, hence it is less susceptible to noise.

→ However, it produces thicker edges

3 Prewitt Operator

→ similar to the Sobel operator, but uses slightly different masks

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad G_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

→ less affected by noise

Gaussian based Methods

1 Canny Edge Detection

Objectives:

Good Detection: minimize the number of false positives & negatives

Good Localization: thin edges, so that neighbors aren't included

Single response

Steps

1. Smooth the input image with Gaussian Filter.

2. Compute the gradient magnitude & angle using Sobel

3. Apply non-maxima suppression to the gradient magnitude image.

4. Use double or hysteresis thresholding and connectivity analysis to detect and link edges.

Disadvantages - may detect too many edges
(Adjust the Gaussian Filter's σ value)

* Edge Linking - Hough Transform

→ Hough transform is a feature extraction method for detecting simple shapes like circles, lines etc. in an image.

→ Hough transform takes the images ~~do~~ created by the edge detection operators, but most of the time - the edge map is disconnected

→ Thus, the Hough transform is used to connect the disjoined edge points.

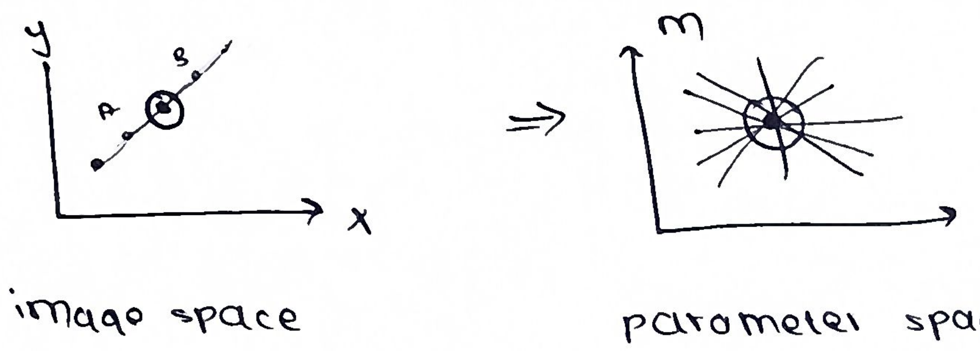
→ The equation of a line is $y = mx + c$.

→ A single point in the image space can be a part of ∞ lines.

→ Thus a point in the xy plane is transformed to one in the (c, m) plane

$$y_i = m x_i + c$$

$$\Rightarrow \quad c = -x_i m + y_i$$



→ If A and B are two points connected by a line in the image space, they would be intersecting lines in the Hough space.

Algorithm for Hough Transform

1. Load the image
2. Determine the edges using any edge detector
3. Quantize the parameter space, P
4. Repeat for all pixels in the image:

   If the pixel is an edge pixel, then

   $c = (-x)m + y$

   $P(c, m) = P(c, m) + 1$
5. Find the local maxima in the parameter space.
6. Draw the line using the local maxima.

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Image Segmentation

* Morphological Operations

→ Morphology deals with image structure.

→ It is used to extract image components for representation and describe region shape - such as boundaries and skeletons.

→ can be used to remove imperfections

Reflection - $\hat{B} = \{w \mid w = -b, b \in B\}$

$\hat{B}$ is a set of points in B whose coordinates are replaced by $(-x, -y)$

Translation: $(B)_z = \{c \mid c = b + z, \text{ for } b \in B\}$

$(B)_z$ is a set of points in B whose coordinates are replaced by $(x + z_1, y + z_2)$

Structuring Elements

→ probe an image with a small mask called the structuring element

→ The structuring element is placed in all possible pixels of the image and compared with the corresponding neighborhood.

→ Operations are based on whether the structuring element

fits or hits

completely contains   only partly contained.

Erosion

With A and B as sets in Z^2, the erosion of A by B is

$A \ominus B$

$A \ominus B = \{ z \mid (B)_z \subseteq A \}$

shrinks
removes imperfections
split in 2 segments

A = image

B = SE

erosion is the thinning of objects in the image

→ The origin of B visits every element in the set A. For each location of the origin, B is completely contained in A, then the location is a member of the new set otherwise not.

Dilation

With A and B as sets in Z^2, the dilation of A by B, denoted by $A \oplus B$ is defined as:

$$A \oplus B = \{ z \mid (B)_z \cap A \neq \phi \}$$

→ Dilation causes a growing or thickening effect on objects

→ The extent of thickening is a function of the shape of the structuring element used.

→ Superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position

→ If at least one pixel in the SE coincides with a foreground pixel in the image underneath, then the o/p pixel is set to the foreground value.

→ If all the corresponding pixels in the image are background, the input pixel is left at the background value.

* Opening & Closing

Opening - smoothens the contour of an object and eliminates thin protrusions

$$A \circ B = (A \ominus B) \oplus B$$

erosion, then dilation

Closing - also smoothens, but it fuses narrow breaks, eliminates small holes and fill gaps in the contour

$$A \cdot B = (A \oplus B) \ominus B$$

dilation, then erosion

Example Consider the image A and the structuring element B =

0	0	0	0	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	0	1	1	0	0
0	0	0	0	0	0

1
1
1

Compute (i) A is dilated by B

(ii) A is eroded by B

Ans A is dilated by B

fully match = 1

some match = 1

no match = 0

0	0	1	1	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	1	1	0	0

(ii) A is eroded by B

fully match = 1

some match = 0

no match = 0

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	0	0	0	0
0	0	0	0	0	0

* Segmentation

Segmentation by Thresholding?

- pixels are partitioned on the basis of their intensity value

Global thresholding, using an appropriate threshold T

$$g(x,y) = \begin{cases} 1, & \text{if } f(x,y) > T \\ 0, & \text{if } f(x,y) \le T \end{cases}$$

Variable thresholding

(i) Local or regional thresholding: If T depends on the neighborhood of (x,y)

(ii) adaptive thresholding: if T is a function of (x,y)

(iii) multiple thresholding

$$g(x,y) = \begin{cases} a, & \text{if } f(x,y) > T_2 \\ b, & \text{if } T_1 < f(x,y) \le T_2 \\ c, & \text{if } f(x,y) \le T_1 \end{cases}$$

Choosing the Thresholds

→ Look for valleys in the histogram

→ Some factors affecting the suitability of the histogram for guiding the choice of the threshold are:

(i) separation between peaks

(ii) noise content in the image

(iii) relative size of object & background

(iv) uniformity of illumination

(v) uniformity of reflectance

Algorithm for Global Thresholding?

1. Initial estimate of T

2. Segmentation using T

   G_1 - pixels brighter than T

   G_2 - pixels darker than or equal to T

3. compute average intensities m_1 & m_2 of G_1 & G_2

4. New threshold $T_{new} = \frac{m_1 + m_2}{2}$

5. If $|T - T_{new}| > \Delta T$, go to step 2, else stop

Otsu's Method

→ aims to find the optimal value for the global threshold

→ It is based on the interclass variance maximization

Steps

1. Let 0, 1, 2 ... L-1 denote L distinct integer intensity levels in an image with M×N pixels.

2. The normalized histogram has the components

$$p_i = \frac{n_i}{MN} \quad \text{where} \quad \sum_{i=0}^{L-1} p_i = 1 \quad p_i \geq 0$$

3. Suppose we select a threshold

$$T(k) = k, \quad 0 < k < L-1$$

and use it to threshold into 2 classes C_1 and C_2.

C_1 has intensities $[0, k]$

C_2 has intensities $[k+1, L-1]$

4. The probability that a pixel is assigned to class C_1 is

$$P_1(k) = \sum_{i=0}^{k} p_i$$

and for C_2 is:

$$P_2(k) = \sum_{i=k+1}^{L-1} p_i = 1 - p_1(k)$$

5. The mean intensity values of the pixel in C_1 is given by:

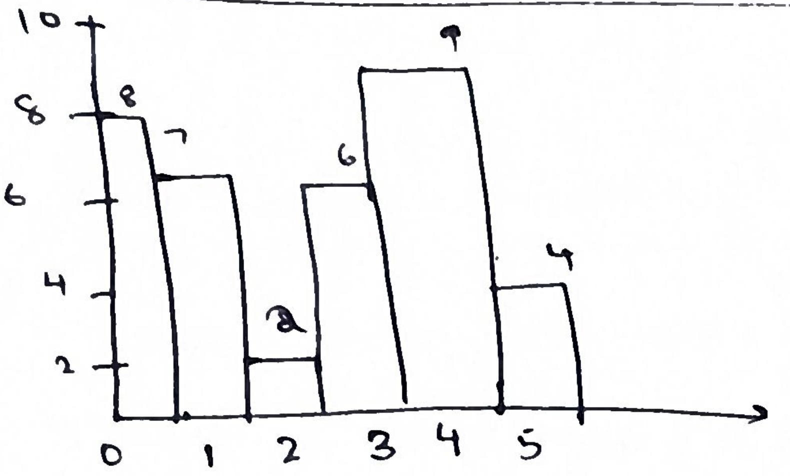
$$m_1(k) = \frac{\sum_{i=0}^{k} i p_i}{\sum_{i=0}^{k} p_i}$$

and for C_2, it is given by: $m_2(k) = \dfrac{\sum_{i=k+1}^{L-1} i p_i}{\sum_{i=k+1}^{L-1} p_i}$

Average Intensity for the entire image is given by

$$m_G = \sum_{i=0}^{L-1} i p_i, \quad \boxed{P_1 m_1 + P_2 m_2 = m_G} \ \& \ \boxed{P_1 + P_2 = 1}$$

Example 1 - Otsu's Thresholding



Threshold value = 2

Total pixels = 36

① Background

$$\text{weight} = \omega_b = \frac{8+7+2}{36} = \frac{17}{36} = 0.4722$$

Total

$$\text{Mean} = \mu_b = \frac{(0\times8)+(1\times7)+(2\times2)}{\cancel{36}\ 17} = 0.6471$$

only in class

$$\text{Variance} = \sigma^2_b = \frac{((0-0.6471)^2\times8)+((1-0.6471)^2\times7)+((2-0.6471)^2\times2)}{17}$$

only in class

$$= \frac{3.3499 + 0.8717 + 3.6606}{17}$$

$$= \underline{\underline{0.4637}}$$

② Foreground

$$\text{weight} = \omega_f = \frac{6+9+4}{36} = 0.5277$$

$\nearrow$ total

$$\text{mean} = m_f = \frac{(6\times3)+(9\times4)+(4\times5)}{19} = 3.8947$$

$\nearrow$ in class

$$\text{variance} = \sigma_f^2 = \frac{((3-3.8947)^2\times6)+((4-3.8947)^2\times9)+((5-3.8947)^2\times4)}{19}$$

$\nearrow$ in class

$$= 0.5152$$

$$\text{within-class variance} = \sigma_w^2 = \omega_b\sigma_b^2 + \omega_f\sigma_f^2$$

$$= (0.4722 \times 0.4637) + (0.5277 \times 0.5152)$$

$$= 0.4909$$

IIIly find for other thresholds - and choose the minimum value

<u>Drawbacks of otsu's method</u>

→ may not work in the presence of noise

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* Region-based segmentation → split & merge algorithm; → region growing algorithm

1) Region-growing algorithm

→ groups pixels into larger regions

→ based on the principle of similarity

- gray level values
- color
- texture
- shape

Steps : select the seed pixel

select the seed growing criteria

termination of segmentation process

Example1

$$\begin{bmatrix} 1 & 0 & 7 & 8 & 7 \\ 0 & 1 & 8 & 9 & 8 \\ 0 & 0 & 7 & 9 & 8 \\ 0 & 1 & 8 & 8 & 9 \\ 0 & 2 & 8 & 8 & 9 \end{bmatrix}$$

seeds = 1, 9

$s_1 = 1$

$s_2 = 9$

criteria $T \leq 4$

Ans:

S1

$f(x,y) - f(x',y')$

$f(x,y) - s_1$

$f(x,y) - 9 \leq 4$

$\{7, 8, 9\} \Rightarrow B$

S2

$f(x,y) - f(x',y')$

$f(x,y) - s_2$

$T(x,y) - 1 \leq 4$

$\{0, 1, 2\}$

↓

A

$$\Rightarrow \begin{bmatrix} A & A & B & B & B \\ A & A & B & B & B \\ A & A & B & B & B \\ A & A & B & B & B \\ A & A & B & B & B \end{bmatrix}$$

(2) Region Split and Merge Algorithm

Phase I - split and continue subdividing the region, until some stopping criteria is fulfilled

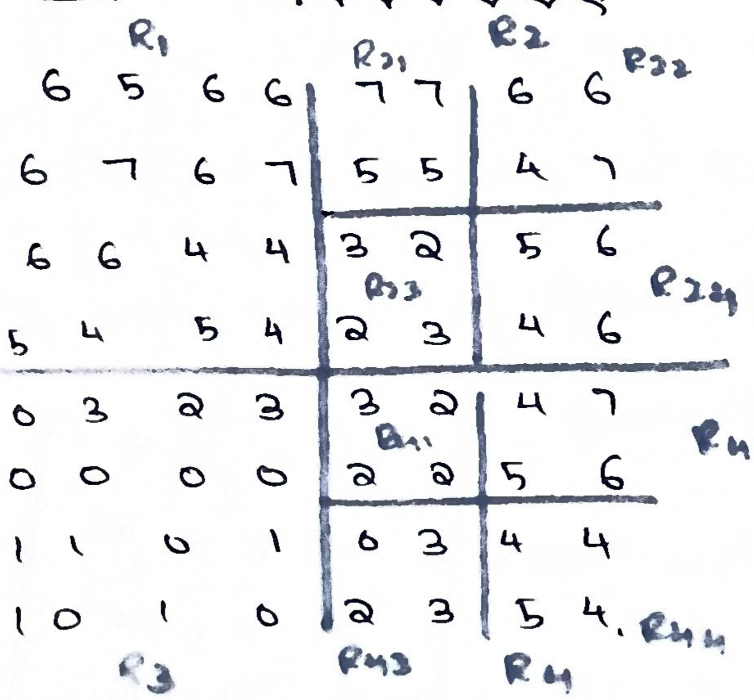
Phase II - merge adjacent regions if some criteria is satisfied

$$\left.\begin{matrix} \max(T_1) - \min(T_2) \\ \max(T_2) - \min(T_1) \end{matrix}\right\} \leq T$$

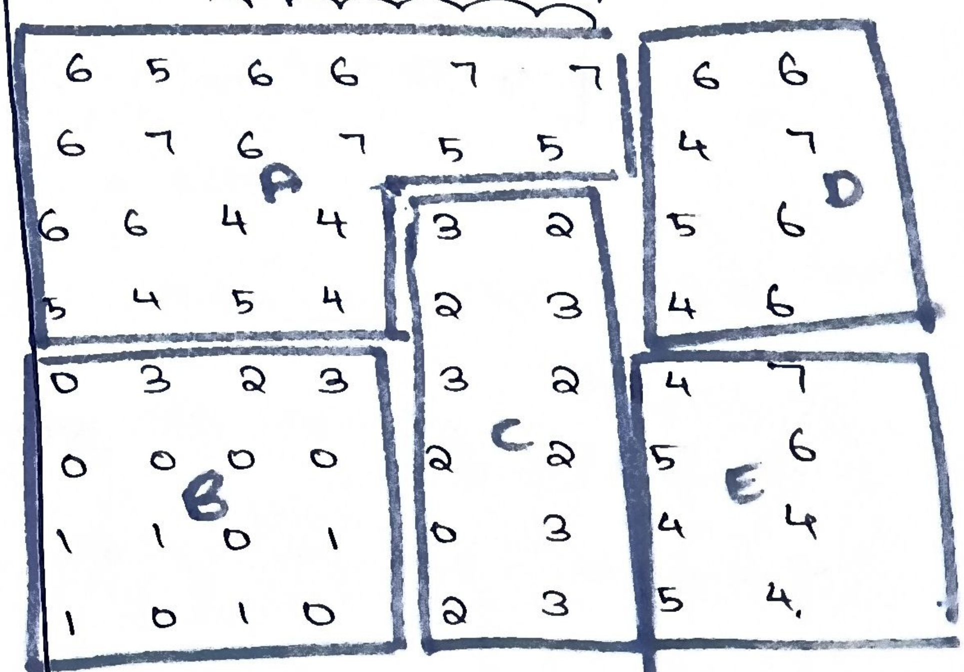
Example Apply the region split & merge algorithm w/ the $T \leq 3$

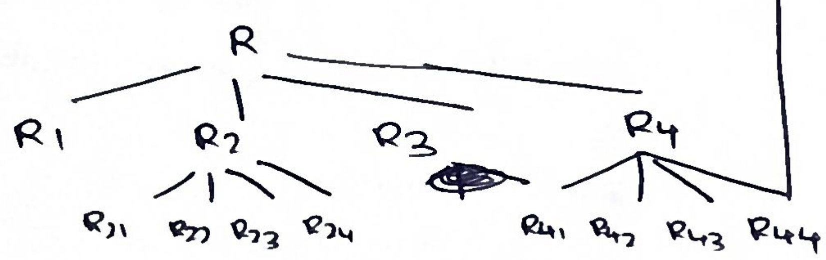
6	5	6	6	7	7	6	6
6	7	6	7	5	5	4	7
6	6	4	4	3	2	5	6
5	4	5	4	2	3	4	6
0	3	2	3	3	2	4	7
0	0	0	0	2	2	5	6
1	1	0	1	0	3	4	4
1	0	1	0	2	3	5	4.

Ans. Region Splitting



Region Merging





* Segmentation by Morphological Watersheds

→ A technique used to partition an image into distinct regions or segments based on the topography of the image intensity.

→ used to segment object, when there are intensity gradients and variations

→ Watershed segmentation involves 3 kinds of points:

(i) Points belonging to a regional minimum -

(ii) Catchment basin / watershed of a regional minimum - points at which a drop of water will certainly fall into a single minimum

(iii) Divide / watershed lines : points at which a drop of water will be equally likely to fall at more than one minimum.

* Basic Steps in Watershed Segmentation

1. Pierce holes in each regional minimum of the image.
2. The 3D topography is flooded from below gradually.
3. When the rising water in distinct catchment basins is about to merge, a dam is built to prevent the merging.
4. The dam boundaries correspond to the watershed lines to be extracted by a watershed segmentation algorithm

* Usage of markers in watershed segmentation

→ Internal markers are used to limit the number of regions by specifying the objects of interest, these can be assigned manually or automatically. They are similar to seeds in the region growing method.

→ External markers are those pixels which we are confident that they belong to the background.

* Steps for watershed segmentation using markers

1. Use internal markers to obtain watershed lines of the gradient. The internal markers represent the desired boundaries between different regions. The flooding process creates watershed lines, which are the boundaries separating different catchment basins.
2. Once the watershed lines are identified, use them as external markers. They prevent the merging of adjacent basins during the flooding process.
3. Each region has a single internal marker & part of the background.
4. The problem is now reduced to partitioning each region into the foreground & background. Other methods like global thresholding, region growing, region splitting & merging or watershed transform can be used

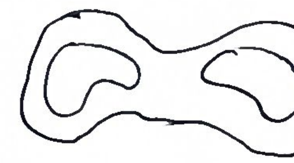
* Dam Construction Algorithm

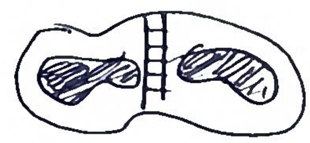
→ based on binary morphological dilation

→ Let M_1, M_2 = sets of coordinates of points in the two regional minima

$C_{n-1}(M_1)$, $C_{n-1}(M_2)$ = set of coordinates of points in the catchment basins associated with M_1, M_2 at the n-1 stage of flooding

$C[n-1]$ = union of $C_{n-1}(M_1)$, $C_{n-1}(M_2)$





→ At flooding step n-1, there are only two connected components. At ~~sta~~ flooding step n, there is only one connected component. This indicates that the water between to the 2 catchment basins has merged at the flooding step. n

→ Repeatedly dilate $C_{n-1}(M_1)$, $C_{n-1}(M_2)$ by the 3x3 structuring element

→ The dam is constructed by the points on which the dilation would cause the sets being dilated to merge. This results in a one pixel thick connected path.

* Disadvantages of watershed segmentation

→ Noise and irrelevant details make pratical application of the watershed algo difficult. — causes oversegmentation

→ use internal & external markers — criteria for defining markers incorporate the a-priori knowledge on the problem.

Numericals

A. Edge Detection Questions

1. Apply Roberts, Sobel and Prewitt operators on the pixel (1,1) in the following image :

$$\begin{bmatrix} 50 & 50 & 100 & 100 \\ 50 & 50 & 100 & 100 \\ 50 & 50 & 100 & 100 \\ 50 & 50 & 100 & 100 \end{bmatrix}$$

Ans Input image = $\begin{bmatrix} 50 & 50 & 100 \\ 50 & 50 & 100 \\ 50 & 50 & 100 \end{bmatrix}$

$\boxed{\text{Robert's operator}}$

X direction

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Y direction

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$G_x = 50 + 0 + 0 - 100$

$= -50$

$G_y = 0 + 100 - 50 + 0 = 50$

$$\text{magnitude} = \sqrt{G_x^2 + G_y^2}$$

$$= \sqrt{(50)^2 + (50)^2}$$

$$= 70.7$$

$$\approx 71$$

replace the pixel with 71

Sobel Operator

I/p image : $\begin{bmatrix} 50 & 50 & 100 \\ 50 & 50 & 100 \\ 50 & 50 & 100 \end{bmatrix}$

X kernel

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Y kernel

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$Gx =$

$-50 + \cancel{100} - \cancel{100} + 200$

$-50 + 100$

$= \underline{200}$

$Gy = \cancel{50} + \cancel{100} + \cancel{100}$

$+ \cancel{-50} - \cancel{100} - \cancel{100}$

$= \underline{0}$

magnitude $= \sqrt{(200)^2}$

$= \underline{\underline{200}}$

Prewitt Operator

I/p image $= \begin{bmatrix} 50 & 50 & 100 \\ 50 & 50 & 100 \\ 50 & 50 & 100 \end{bmatrix}$

X kernel

$$Gx = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Y kernel

$$Gy = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

$Gx = -50 + \cancel{100} +$

$-50 + 100 +$

$-50 + 100$

$= \underline{\underline{150}}$

$Gy = 50 + 50 + 100$

$-50 - 50 - 100$

$= \underline{\underline{0}}$

magnitude $= \underline{\underline{150}}$

B. Hough Transform

1. Using the Hough transform, show that the points (1,2), (2,3), (3,4) are collinear. Also find the equation of the line.

Ans For $(x,y) = (1,2)$

$$y = mx + c$$

if $m=0$, $\boxed{2 = c}$

if $c=0$, $2 = m(1)$ $\boxed{m=2}$

$$\boxed{(m,c) = (2,2)} \quad \text{—(1)}$$

For $(x,y) = (2,3)$

If $c=0$, $0 = -2m+3$

$2m = 3$

$m = 1.5$

If $m=0$ $c=3$

$$\boxed{(m,c) = (1.5,3)} \quad \text{—(2)}$$

For $(x,y) = (3,4)$

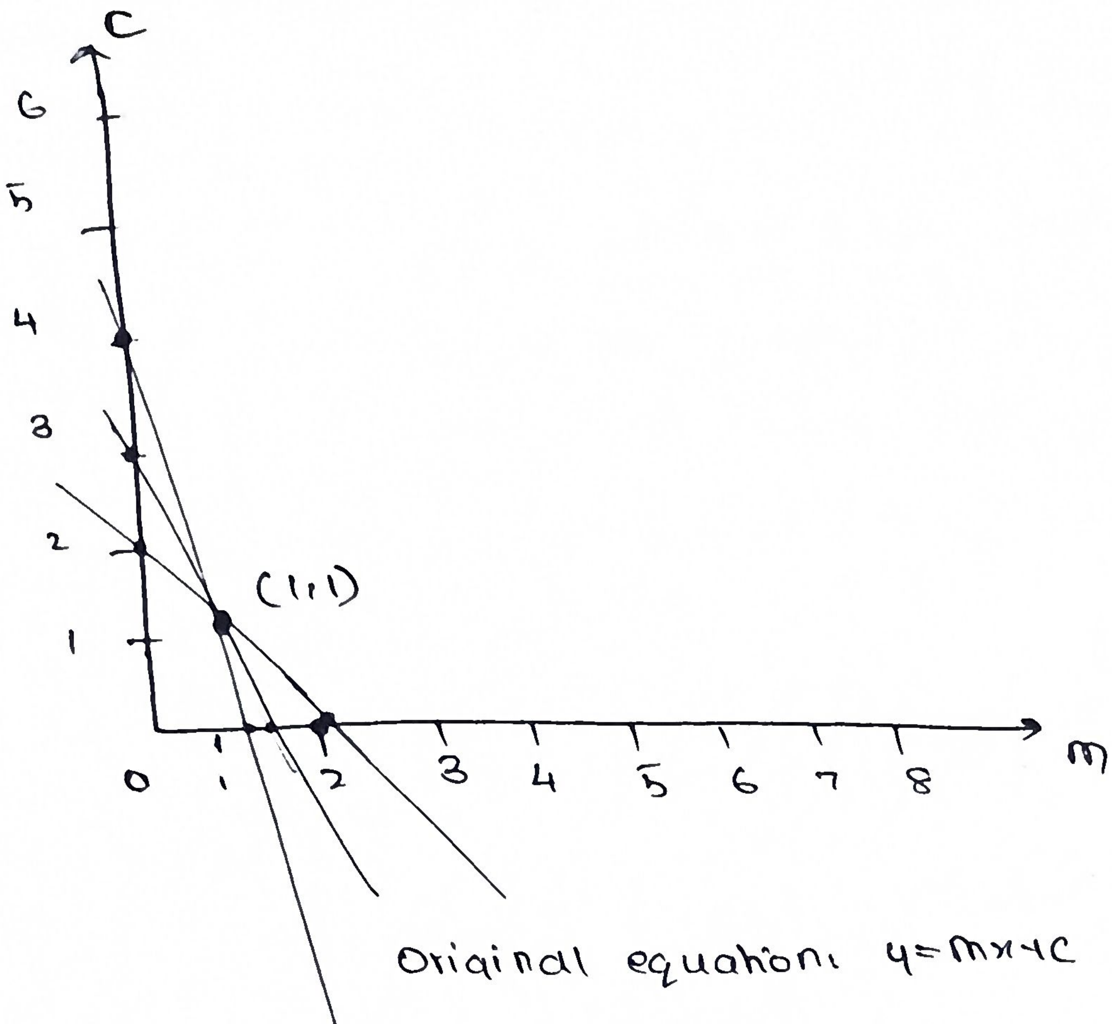
If $c=0$ $0 = -3m+4$

$m = 4/3 = 1.33$

If $m=0$, $c=4$

$$\boxed{(m,c) = (1.33,4)} \quad \text{—(3)}$$

Plot points (1), (2), (3) on the m-c plane



Original equation: $y = mx + c$

The final equation is $\boxed{y = x + 1}$

c. Thresholding

1. Apply thresholding on the following image with a threshold value of 162.

$$\begin{bmatrix} 37 & 0 & 241 & 7 & 96 \\ 13 & 105 & 199 & 243 & 6 \\ 71 & 21 & 100 & 96 & 0 \\ 44 & 29 & 7 & 0 & 241 \\ 9 & 99 & 129 & 149 & 119 \end{bmatrix}$$

Ans: $\begin{bmatrix} 0 & 0 & 255 & 0 & 0 \\ 0 & 0 & 255 & 255 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 255 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

★★★

2. Otsu's Thresholding - see page ____

D. Region Growing

1. See example on page. ______

2. Consider the given 3 bit image. Assume a seed value of 6, segment the following image using 8- connectivity and a threshold = 3

2	2	7	2	1
1	7	6	6	2
7	6	6	5	7
2	4	5	4	2
1	2	5	1	1

Ans: seed = 6

threshold = 3

The condition for thresholding would be:

abs(pixel - seed) < 3

Applying the thresholding condition.

0	0	1	0	0
0	1	1	1	0
1	1	1	1	1
0	1	1	1	0
0	0	1	0	0

$\Rightarrow$

a	a	b	c	c
a	b	b	b	c
b	b	b	b	b
d	b	b	b	e
d	d	b	e	e

(5 segments)

Considering the 8 connectivity of each pixel - assign labels to each

E. Region Split and Merge Algorithm.

Example 1. See pg. ________

2. Apply the region splitting algorithm on the following image:

with $T \leq 4$

5	6	6	6	7	7	6	6
6	7	6	7	5	5	4	7
6	6	4	4	3	2	5	6
5	4	5	4	2	3	4	6
0	3	2	3	3	2	4	7
0	0	0	0	2	2	5	6
1	1	0	1	0	3	4	4
1	0	1	0	2	3	5	4

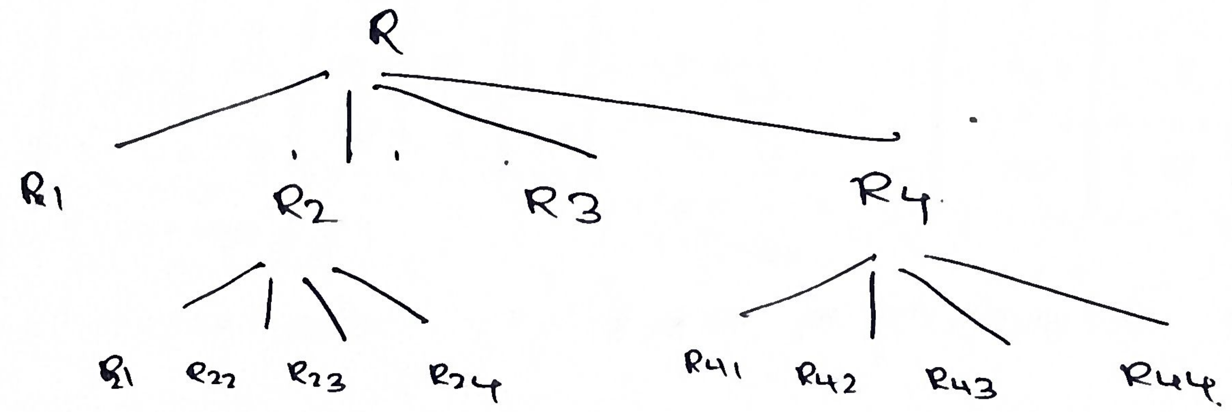
Ans:

R1 R21 R2 R22

5	6	6	6	7	7	6	6
6	7	6	7	5	5	4	7
6	6	4	4	3	2	5	6
5	4	5	4	2	3	4	6
0	3	2	3	3	2	4	7
0	0	0	0	2	2	5	6
1	1	0	1	0	3	4	4
1	0	1	0	2	3	5	4

R23 R24 R41 R42 R43 R44

R3 R4



F: Erosion and Dilation

1. See pg. ____

2. Apply erosion and dilation on the image A, using the structuring element B.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Ans For erosion - only if the image segment & overlapping structuring element completely match - make the pixel value 1.

$$\text{new image} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

dilation

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

G. Opening and closing

1. For the given image, open using the structuring element

A =

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
1	1	1	1	1
0	0	0	0	1

B =

1	1	1

Ans : First erode, then dilate.

erosion

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
1	1	1	1	1
0	0	0	0	0

dilation - same thing