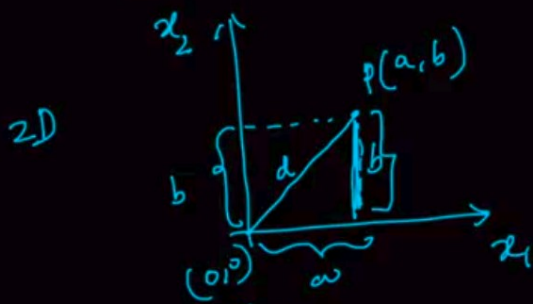


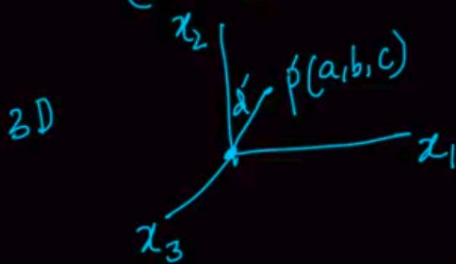
LINEAR ALGEBRA

distance of a pt. from origin:



$d = \text{dist origin \& } p$

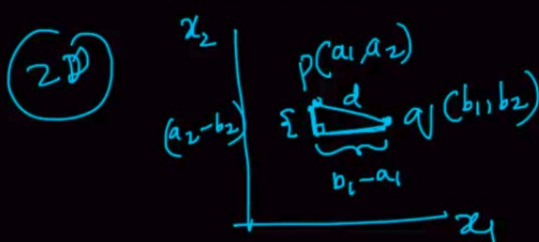
(2D) $d = \sqrt{a^2 + b^2}$ (pythagoras thm)



(3D) $d = \sqrt{a^2 + b^2 + c^2}$

(nD) $d = \sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$
 $p = [a_1, a_2, a_3, \dots, a_n]$

dist b/w 2 pts



$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$$

(3D) $p(a_1, a_2, a_3)$
 $q(b_1, b_2, b_3)$

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

(nD) $p(a_1, a_2, \dots, a_n)$
 $q(b_1, b_2, \dots, b_n)$

$$d_{pq} = \sqrt{\sum_{i=1}^n (a_i - b_i)^2}$$

✓ row vector: $A = [a_1, a_2, a_3, \dots, a_n]$ $\begin{matrix} \uparrow \\ \text{rows} \end{matrix}$ $\begin{matrix} \leftarrow \text{columns} \end{matrix}$

✓ column vector

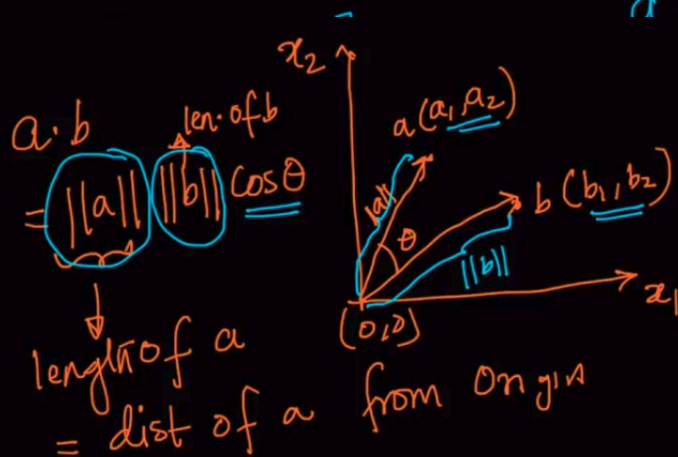
$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \quad \begin{matrix} \uparrow \\ \text{rows} \end{matrix} \quad \begin{matrix} \leftarrow \text{columns} \end{matrix}$$

$$b_{n \times 1}$$

$$A_{1 \times n}$$

$A_{m \times n}$ → double array of arrays
 $\begin{bmatrix} 1 & 2 & 3 & \dots & n \\ \vdots & & & & \end{bmatrix}$

Addition:

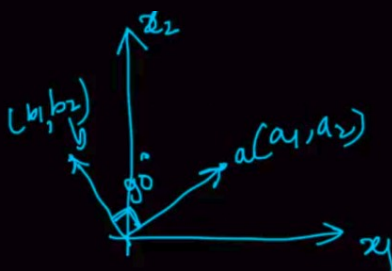


$$\|a\| = \text{len. of vcl.}$$

$$\|a\| = \sqrt{a_1^2 + a_2^2}$$

$$a \cdot b = a_1 b_1 + a_2 b_2 = \|a\| \|b\| \cos \theta$$

$$\checkmark \underline{\theta} = \cos^{-1} \left\{ \frac{a_1 b_1 + a_2 b_2}{\|a\| \|b\|} \right\}$$



$$a \cdot b = \|a\| \|b\| \cos 90^\circ$$

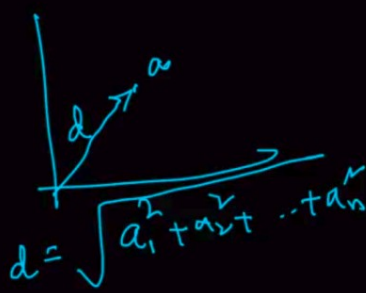
$$\underline{\underline{a \cdot b = 0}}$$

(nD) $\begin{cases} \checkmark a = [a_1, a_2, \dots, a_n] \\ \checkmark b = [b_1, b_2, \dots, b_n] \end{cases}$

$$a \cdot b = \|a\| \|b\| \cos \theta$$

$$\theta_{a,b} = \cos^{-1} \left(\frac{\sum_{i=1}^n a_i b_i}{\|a\| \|b\|} \right)$$

$$a \cdot b = \sum_{i=1}^n a_i b_i = 0 \Rightarrow \underline{\underline{a \perp b}}$$

$$\begin{aligned}
 a \cdot a &= a_1 a_1 + a_2 a_2 + \dots + a_n a_n \\
 &= a_1^2 + a_2^2 + \dots + a_n^2 \\
 &= \|a\|^2
 \end{aligned}$$


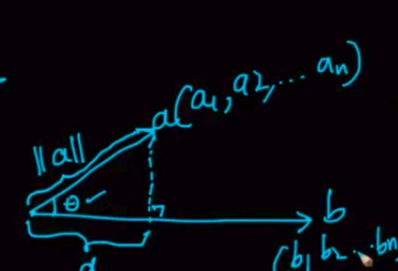
$d = \sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$

dot product \rightarrow geometrically

2D \rightarrow nD

Projection and Unit Vector

Projection



proj. of a on b
 $= d = \|a\| \cos \theta$ — (1)

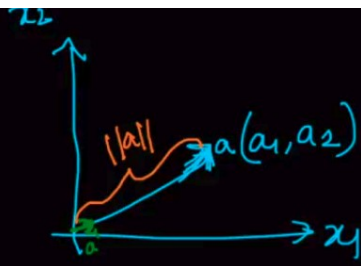
$a \cdot b = \sum_{i=1}^n a_i b_i = \|a\| \|b\| \cos \theta$

$$d = \frac{a \cdot b}{\|b\|} = \frac{\|a\| \|b\| \cos \theta}{\|b\|}$$

- Considering case of n-dimensional

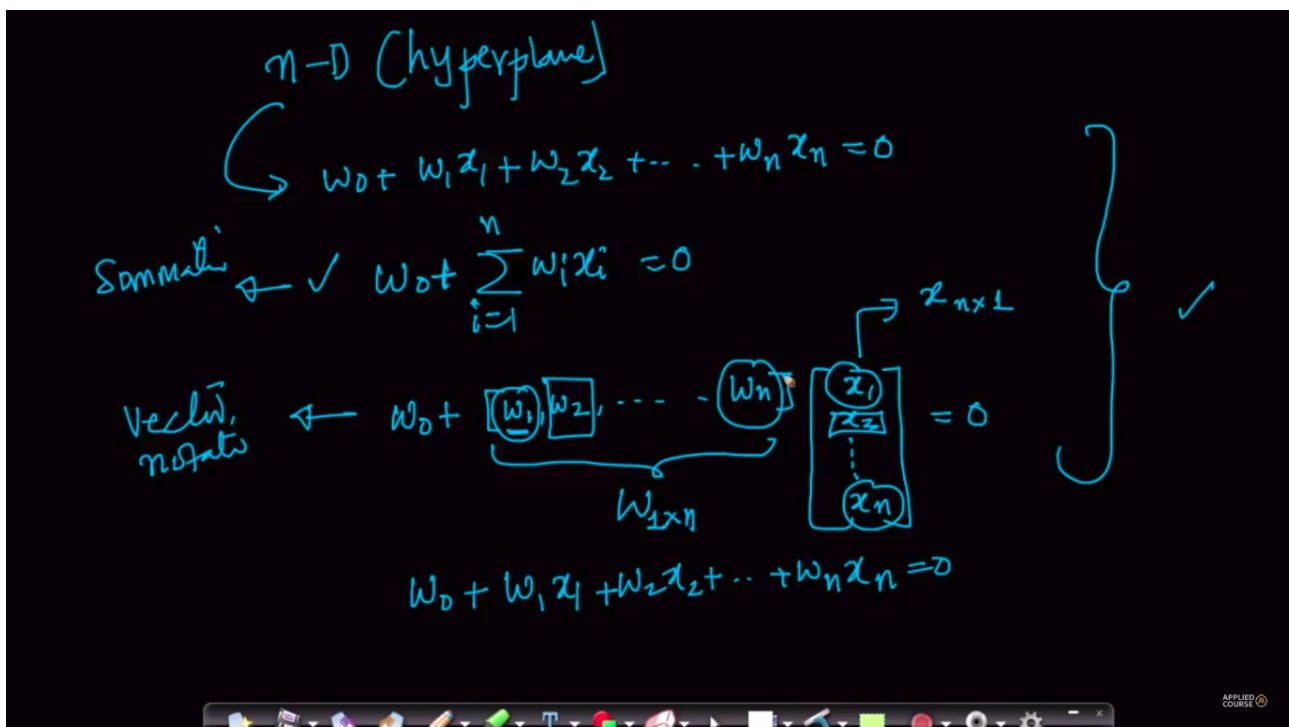
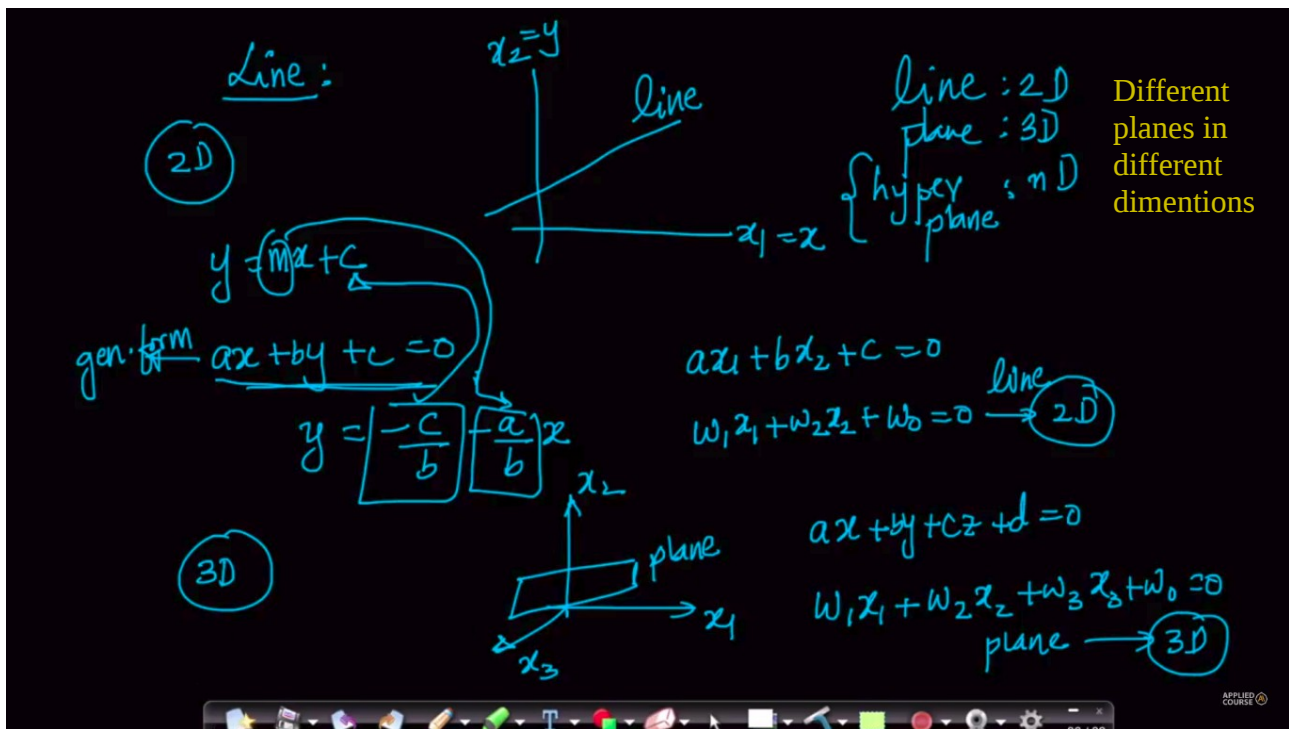
unit vector

$$\hat{a} = \frac{a}{\|a\|}$$



- ① \hat{a} same direction as a
- ② $\|\hat{a}\| = 1$

- Equation of a line (2-D), Plane(3-D) and Hyperplane (n-D), Plane Passing through origin, Normal to a Plane



Hyper plane representation

Note:

By default any vector is Column Vector.

$$\pi\text{-dim} \quad \textcircled{w_0} + \underbrace{[w_1, w_2, \dots, w_n]}_{1 \times n} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = 0$$

\textcircled{x}

$$\underline{w}_{n \times 1} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

$$\underline{x}_{n \times 1} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

Planes are represented as Π in any dimension

$\Pi:$

$$\textcircled{w_0} + \underbrace{w^T}_{1 \times n} \underbrace{x}_{n \times 1} = 0$$

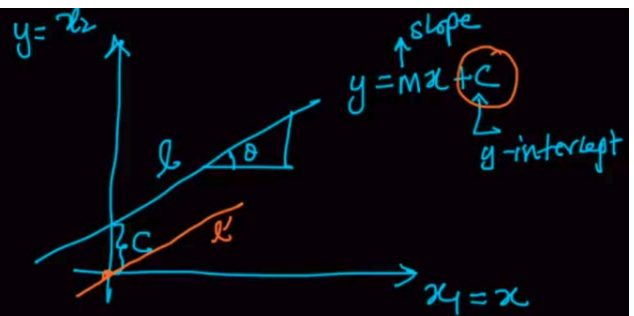
2D:

$$w_1 x_1 + w_2 x_2 + w_0 = 0$$

$$x_2 = \boxed{\frac{-w_0}{w_2}} - \boxed{\frac{w_1}{w_2}} x_1$$

$$y = C + m x$$

$$C = \frac{-w_0}{w_2}$$



l is passing through origin

$$C = 0$$

$$C = 0 \Rightarrow w_0 = 0$$

l passing through origin

line: 2D: $w_1 x_1 + w_2 x_2 = 0$

plane: 3D: $w_1 x_1 + w_2 x_2 + w_3 x_3 = 0$

hyperplane: nD: $w_1 x_1 + w_2 x_2 + \dots + w_n x_n = 0$

$$W^T x = 0$$

eqn. of a plane passing through origin

$$W^T x + w_0 = 0$$

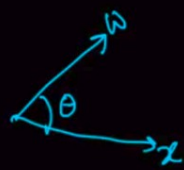
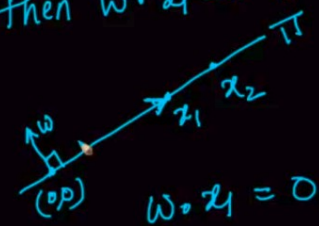
Eq. does not pass through origin

$\Pi_n: W_m^T x = 0$ ✓

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

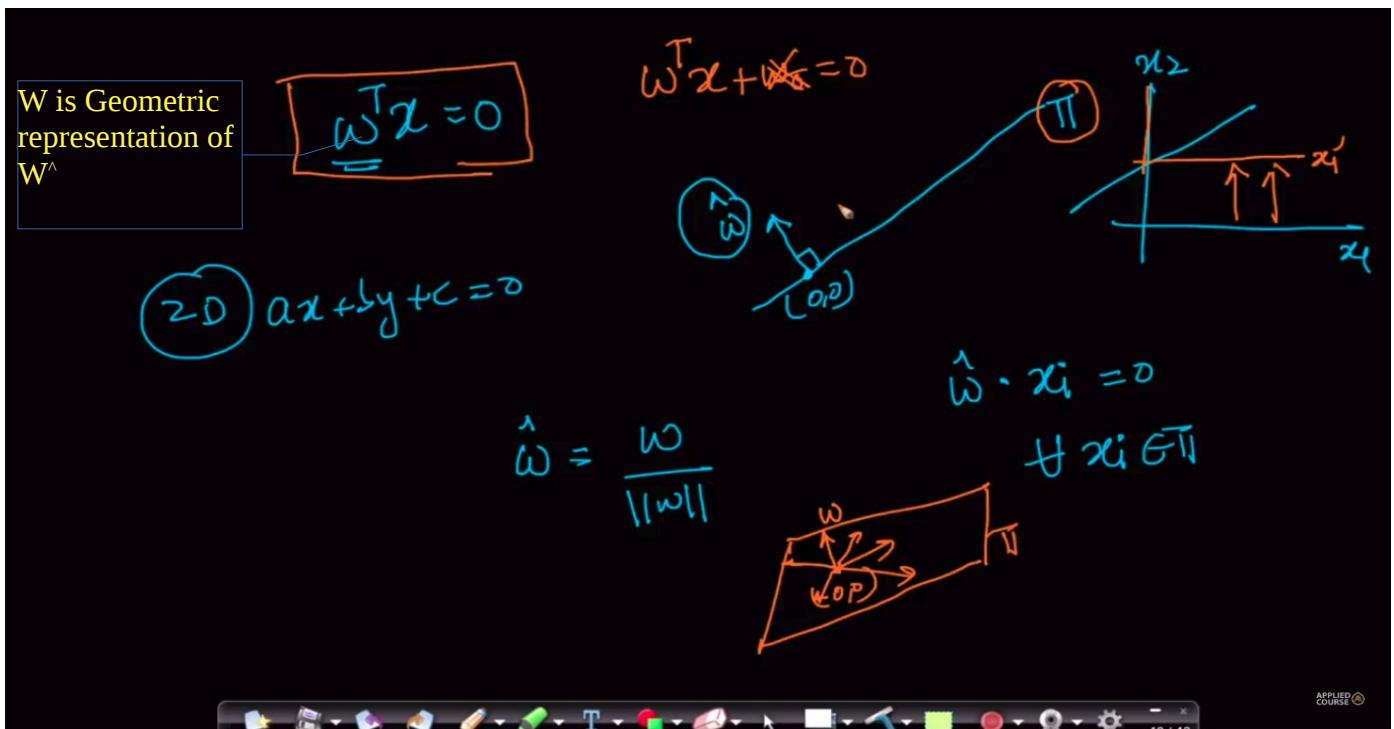
gf $w \perp \Pi$ then $w \cdot x_i = 0 \quad \forall x_i \in \Pi$



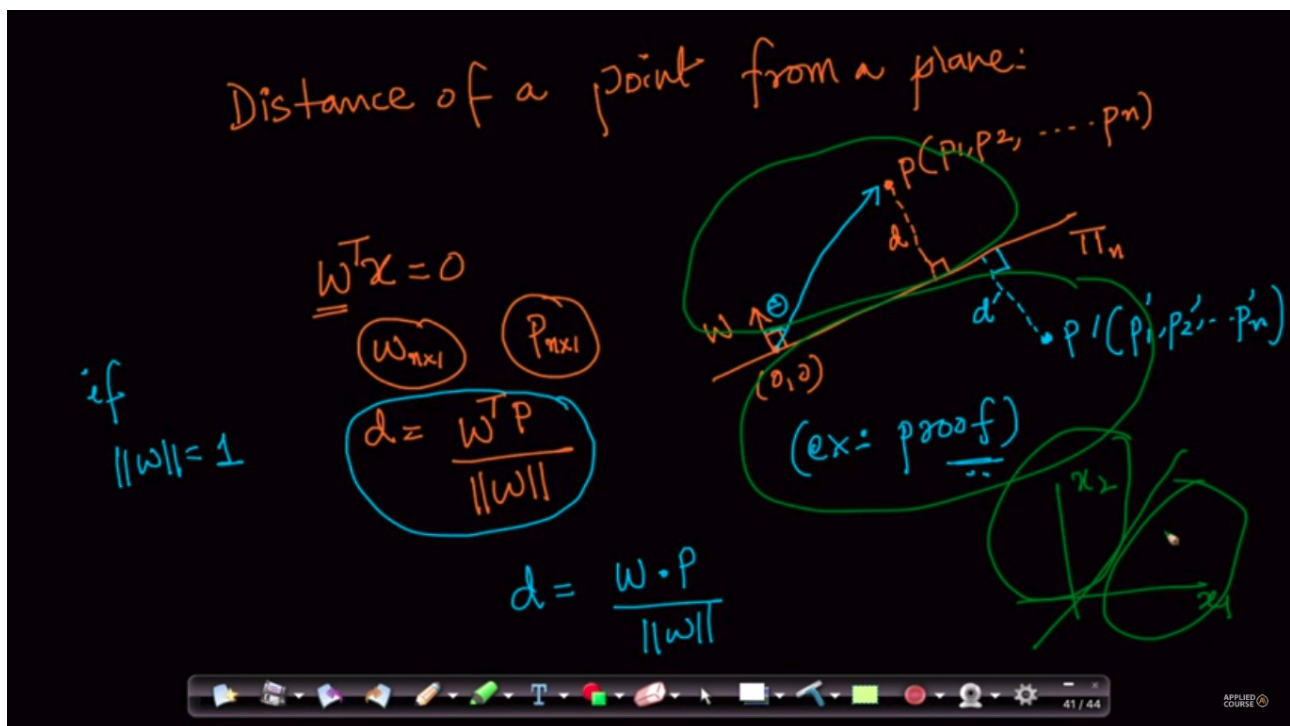
$$w \cdot x = W^T x = \|w\| \|x\| \cos \theta_{w,x} = 0$$

$$w \perp x \Rightarrow \theta_{w,x} = 90^\circ$$

Note: Normally we assume our planes passing through origin



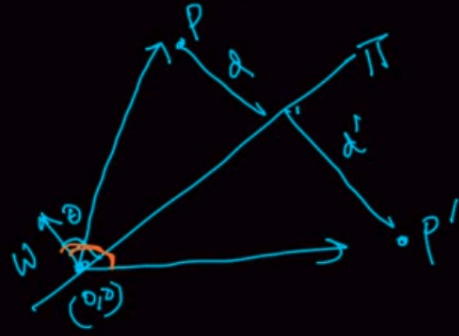
Distance of a point from a Plane/Hyperplane, Half-Spaces



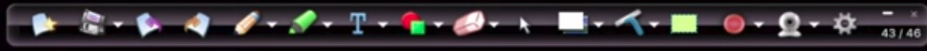
- Line separates 2D into half spaces n 3D divides plane into half spaces.
- Both distances will be in opposite direction

$$d = \frac{w \cdot p}{\|w\|} = \underline{\underline{+ve}}$$

$$d' = \frac{w \cdot p'}{\|w\|} = -ve$$

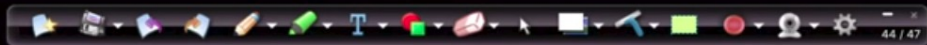
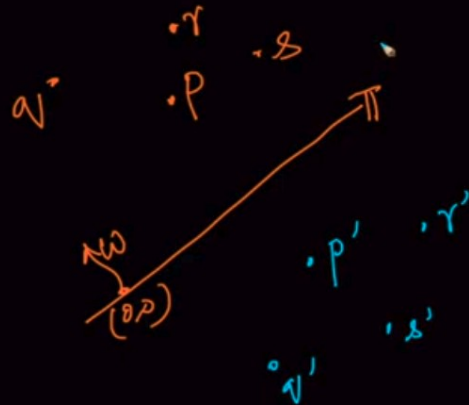


$$w \cdot p = \|w\| \|p\| \cos \theta_{w,p}$$




$$\frac{w \cdot p}{\|w\|} = +ve$$

$$\frac{w \cdot p'}{\|w\|} = -ve$$



Equation of a Circle (2-D), Sphere (3-D) and Hypersphere (n-D)

Circle:



$$x^2 + y^2 = r^2$$


$P(x_1, x_2)$

$$C: (h, k); \underline{(x-h)^2 + (y-k)^2 = r^2}$$

$\begin{cases} x_1^2 + x_2^2 \leq r^2 \Rightarrow P \text{ lies inside the circle} \\ x_1^2 + x_2^2 > r^2 \Rightarrow P \text{ lies outside the circle} \\ x_1^2 + x_2^2 = r^2 \Rightarrow P \text{ lies on the circle} \end{cases}$

3D x_1, x_2, x_3

Sphere



$$x_1^2 + x_2^2 + x_3^2 = r^2$$

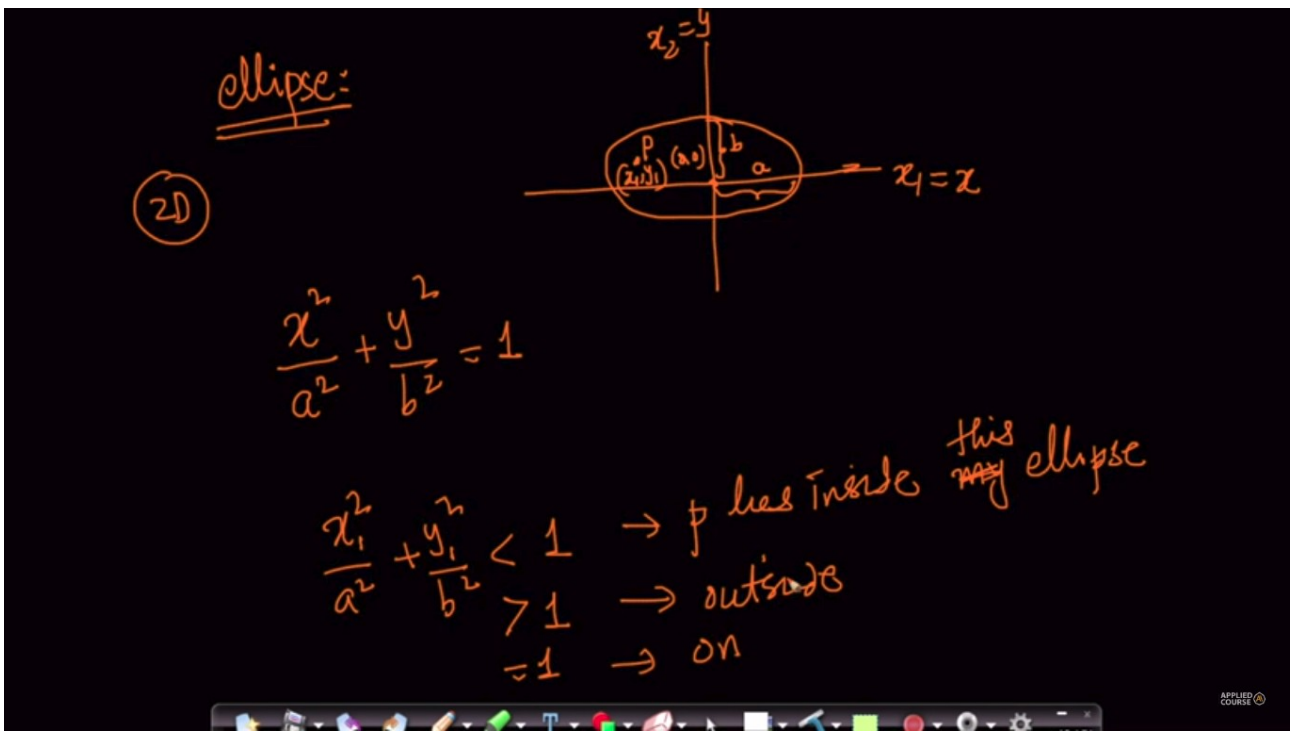
$\sum_{i=1}^n x_i^2 = r^2$

nD x_1, x_2, \dots, x_n

hyper sphere

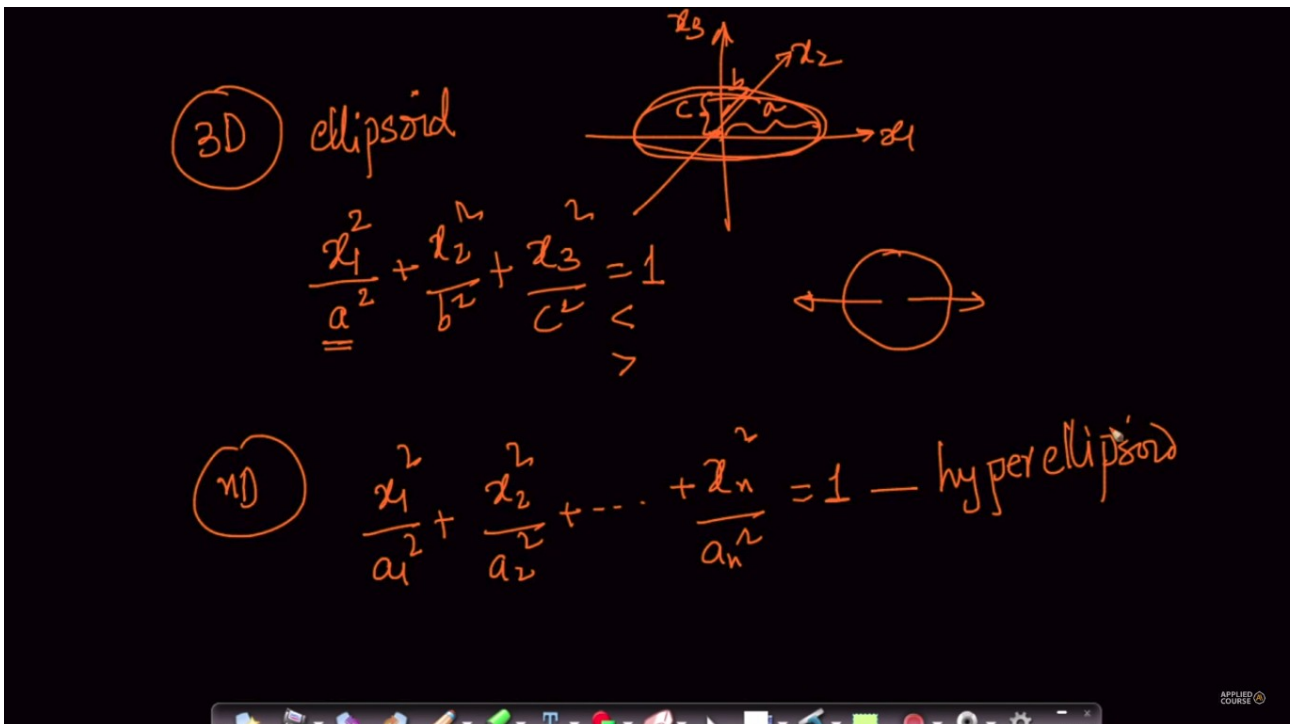
$$x_1^2 + x_2^2 + \dots + x_n^2 = r^2$$

Equation of ellipse , ellipsoid and Hyperellipsoid



2D - ellipse

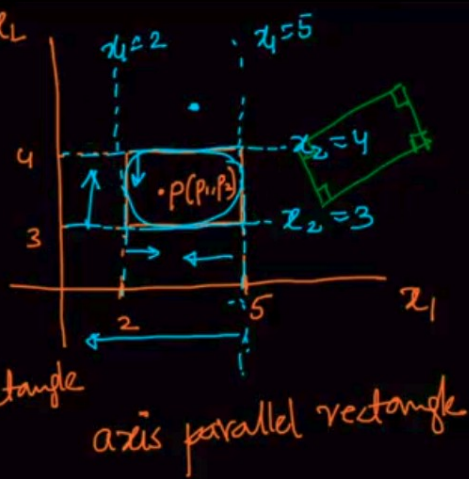
3D - ellipsoid



Squares, rectangle:

(2D)

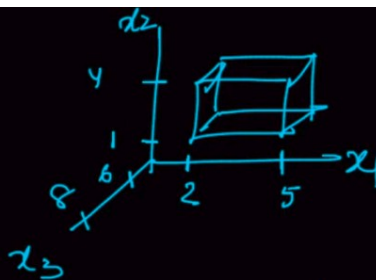
if $p_1 \leq 5$ & $p_1 > 2$
 if $p_2 > 3$ & $p_2 < 4$
 then p lies
 inside
 this rectangle



$p_1 < 5$ $p_1 > 5$

Hyper Cube, Hyper Cuboid

(3D) Cuboid



if-else
 →

(nD) hyper cuboid