Q3
Ans 3.15

According to equation 3.8, for $\gamma<1$,

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots$$

$$= \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

$$= R_{t+1} + c + \gamma(R_{t+2}+c) + \gamma^2(R_{t+3}+c) + \dots$$

$$= R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots + c(1+\gamma+\gamma^2+\dots)$$

$$= R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots + \underbrace{c\sum_{k=0}^{\infty}\gamma^k}_{V_c}$$

$$\therefore V_c = c\sum_{k=0}^{\infty}\gamma^k$$

$$= \frac{c}{1-\gamma} \qquad \because \gamma<1$$

As proved above, adding a constant to the above problem leaves the continuous task unchanged.

Ans 3.16. In case of episodic task it would be affected ~~adding~~ by adding a constant c to all the rewards.
So, let's say there is an episodic task with a terminal state ~~T~~ of length k, then :-

$$\text{expected Returns} = E[(R_{t+1}+c) + \gamma(R_{t+2}+c) + \gamma^2(R_{t+3}+c) + \dots + \gamma^{k-1}(R_{t+k}+c)]$$

$$\text{then} \quad = E[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots + \gamma^{k-1} R_{t+k} + c\underbrace{[1+\gamma+\gamma^2+\dots\gamma^k]}_{\text{II}^{\text{nd}}}] \quad \text{—} (1)$$