(b) Show the network diagram corresponding to the solution in (a). That is, label each of the arcs in the solution and verify that the flows are consistent with the given information.

3.8 Cash Planning: A startup investment project needs money to cover its cash flow needs. The cash income and expenditures for the period January through April are as follows.

Month	January	February	March	April	Total
Cash flow ($000)	−150	−450	500	250	150

At the beginning of May, all excess cash will be paid out to investors. There are two ways to finance the project. One is the possibility of taking out a long-term loan at the beginning of January. The interest on this loan is 1% per month, payable on the first of the month for the next 3 months. This loan can be as large as $400,000; the principal is due April 1; and no prepayment is permitted. The alternative is a short-term loan that can be taken out at the beginning of each month. This loan must be paid back at the beginning of the following month with 1.2% interest. A maximum of $300,000 may be used for this short-term loan in any month. In addition, investments may be made in a money market fund at the start of each month. This fund will pay 0.7% interest at the beginning of the following month. Assume the following about the timing of cash flows.

- For months in which there is a net cash deficit, sufficient funds must be on hand at the *start* of the month to cover the net outflow.
- For months in which there is a net cash surplus, the net inflow cannot be used until the *end* of the month (i.e., the start of the next month).

(a) What is the maximum amount that can be returned to investors? What is the optimal amount of money to borrow from each of the potential loan sources?

(b) Show the network diagram corresponding to the solution in (a). That is, label each of the arcs in the solution and verify that the flows are consistent with the given information.

(c) Explain the cost of funds for each month in the planning period. That is, if there were a $1000 change in the cash flows for any month, what would be the dollar change in the amount returned to investors?

3.9 Planning Cash: Each Fall, the treasurer of Trefny's department store does financial planning for the next 6 months, September through February. Because of the holiday season, Trefny's needs large amounts of cash during October, November, and December, whereas a large cash inflow is expected after the first of the year when customers pay off their holiday bills. The following table summarizes the predicted net cash flows (in thousands) from "business-as-usual" operations.

Month	September	October	November	December	January	February
Surplus	$20	—	—	—	30	150
Deficit	—	30	60	90	—	—

The treasurer can draw on three sources of short-term funds to meet the store's needs, although each represents a departure from "business as usual." They are as follows:

- *Accounts Receivable Loans.* A local bank will loan Trefny's funds on a month-by-month basis against a pledge on the accounts receivable balance as of the first day of a particular month. The maximum loan is 75% of the balance, and the cost of the loan is 1.5% per month, assessed on the amount borrowed. The predicted balances (in thousands) under "business-as-usual" plans are shown below.

Month	September	October	November	December	January	February
Balance	$70	50	70	110	100	50

- *Delayed Payment of Purchases.* All bills for purchases come due on the first of the month, but payments on all or part of these obligations can be delayed by 1 month. When payments are delayed this way, Trefny's loses the 2% discount it normally receives for prompt payment under "business-as-usual" operations. (Loss of this 2% discount is effectively a financing cost.) The predicted payment schedule (in thousands) without the discount is shown below.

Month	September	October	November	December	January	February
Payment	$80	90	100	60	40	50

- *Short-Term Loan.* A bank is willing to loan Trefny's any amount from $40,000 to $100,000 for 6 months, starting September 1. The principal would be paid back at the end of February, and Trefny's would not be permitted to pay off part of the loan, or add to it, during the 6-month period. The cost of the loan is 1% per month, payable at the end of each month.

In any month, excess funds can be transferred to Trefny's short-term investment portfolio, where the funds can earn 0.5% per month.

(a) Determine a plan for the treasurer that will meet the firm's cash needs at minimum cost. (Assume that all cash flows occur at the beginning of the month.) What is the cost of this plan? Equivalently, what is the maximum amount of funds on hand after February?

(b) Show the network diagram corresponding to the solution in (a). That is, label each of the arcs in the solution and verify that the flows are consistent with the given information.

3.10 Planning a National Economy: The country of Utopia has a newly appointed Minister of International Trade. She has decided that Utopia's welfare can be served best in the upcoming year by maximizing the net dollar value of Utopia's exports (i.e., the dollar value of the exports minus the cost of the