linear solver on which it depends. That is, the branch-and-bound procedure is guaranteed to produce an optimal solution.

SUMMARY

The ability to treat variables as integer valued, and in particular, the ability to designate variables as binary, opens up a wide variety of optimization models that can be addressed with Solver. This chapter introduced two broad classes of models that can be handled effectively with Solver. The first type of model is one that resembles a linear program but with the requirement that certain variables must be integer valued. For the purposes of using Solver, this requirement is added to the linear programming model as an additional constraint. The second type of model is one in which certain decisions exhibit an all-or-nothing structure, representing actions that are indivisible. Such decisions are modeled by binary variables, which are simply integer-valued variables no less than 0 and no greater than 1. Binary variables allow us to model the occurrence of yes/no choices and to exploit Solver, provided that the structure of the model is linear in all other respects.

In this second category, we explored three closely related model structures: set covering, set packing, and set partitioning. In the basic form of these models, the objective function coefficients and the right hand-side constants are 1's, but generalizations are also possible, as illustrated in some of the exercises at the end of this chapter.

In the next chapter, we examine a broader set of integer programming models based on the ability to represent logical constraints using binary variables. In those models, the logical conditions do not seem linear, but they can be expressed in linear forms with the help of binary variables.

EXERCISES

6.1 Callum Communications (Revisited): Revisit Example 6.1. Suppose that the objective at Callum Communications is to minimize the number of employees, rather than to minimize the total cost.

(a) What is the minimum number of employees needed at the call center?

(b) Does the solution in (a) achieve the minimum salary cost?

6.2 Make or Buy: A sudden increase in the demand for smoke detectors has left Acme Alarms with insufficient capacity to meet demand. The company has seen monthly demand from its retailers for its electronic and battery-operated detectors rise to 20,000 and 10,000, respectively, and Acme wishes to continue meeting demand. Acme's production process involves three departments: Fabrication, Assembly, and Shipping. The relevant quantitative data on production and prices are summarized below.

Department	Monthly Hours Available	Hours/Unit (Electronic)	Hours/Unit (Battery)
Fabrication	2000	0.15	0.10
Assembly	4200	0.20	0.20
Shipping	2500	0.10	0.15
Variable cost/unit		$18.80	$16.00
Retail price		$29.50	$28.00

The company also has the option to obtain additional units from a subcontractor, who has offered to supply up to 20,000 units per month in any combination of electronic and battery-operated models, at a charge of $21.50 per unit. For this price, the subcontractor will test and ship its models directly to the retailers without using Acme's production process.

(a) Acme wants an implementable schedule, so all quantities must be integers. What are the maximum profit and the corresponding make/buy levels?

(b) Compare the maximum profit in (a) to the maximum profit achievable without integer constraints. Does the integer solution correspond to the rounded-off values of the noninteger solution? By how much (in percentage terms) do the integer restrictions alter the value of the optimal objective function?

6.3 Selecting an Investment Portfolio: An investment manager wants to determine an optimal portfolio for a wealthy client. The fund has $2.5 million to invest, and its objective is to maximize total dollar return from both growth and dividends over the course of the coming year. The client has researched eight high-tech companies and wants the portfolio to consist of shares in these firms only. Three of the firms (S1–S3) are primarily software companies, three (H1–H3) are primarily hardware companies, and two (C1–C2) are Internet consulting companies. The client has stipulated that no more than 40% of the investment be allocated to any one of these three sectors. To ensure diversification, at least $100,000 must be invested in each of the eight stocks. Moreover, the number of shares invested in any stock must be a multiple of 1000.

The table below gives estimates from the investment company's database relating to these stocks. These estimates include the price per share, the projected annual growth rate in the share price, and the anticipated annual dividend payment per share.

	Stock							
	S1	S2	S3	H1	H2	H3	C1	C2
Price per share	$40	$50	$80	$60	$45	$60	$30	$25
Growth rate	0.05	0.10	0.03	0.04	0.07	0.15	0.22	0.25
Dividend	$2.00	$1.50	$3.50	$3.00	$2.00	$1.00	$1.80	$0.00