BUDT 758P: Decision Analytics
Assignment 4, due on 10/29 at 9:30 AM

Problem 1. The following table describes the payoffs received by the row player in a zero-sum game where the row player has four strategies A-D while the column player has five strategies 1-5:

$$V = \begin{bmatrix} 6 & -2 & 4 & 7 & 5 \\ 0 & -4 & 2 & 9 & 1 \\ 7 & -3 & 3 & 8 & 2 \\ -2 & 3 & -6 & 0 & -3 \end{bmatrix}$$

In game theory, one strategy is said to *dominate* another if one of the following happens. A row strategy i dominates another row strategy i' if $V_{i,j} \geq V_{i',j}$ for all $j = 1, 2, 3, 4, 5$ (in other words, the row player always receives more under i than under i', regardless of the column player's strategy). A column strategy j dominates another column strategy j' if $V_{i,j} \leq V_{i,j'}$ for $i = A, B, C, D$ (in other words, the row player always receives *less* if the column player picks j instead of j', regardless of the row player's own strategy).

a) Explain why, if row strategy i dominates the row strategy i', the row player will *never* play i' (with any probability).

b) Explain why, if column strategy j dominates the column strategy j', the *column* player will *never* play j' (with any probability).

c) Solve the above game in Excel and find the optimal strategies for both players. Explain why some strategies are assigned zero probability.

Problem 2. Two players simultaneously put up one or two fingers, and each player calls out a guess as to what the total sum of all the outstretched fingers will be. If you guess right, but your opponent does not, you receive a payment (from your opponent) equal to your guess; likewise, you pay the opponent in the opposite situation. In all other cases, it is a draw and no one earns anything.

Formulate and solve a linear program to identify the maximin/minimax strategies for the players. What is the value of the game? What is the optimal randomized strategy for each player? Does either player have an advantage?

Problem 3. Players A and B each pick an integer from 1 to 10 (inclusive). The game is a draw if both players pick the same number. Otherwise, the player who picks the smaller number wins, unless that number is one less than the opponent's number, in which case the opponent wins. The winner always receives \$1 from the loser, regardless of which combination of strategies produced the outcome.

Formulate and solve a linear program to identify the maximin/minimax strategies for the players. What is the value of the game? What is the optimal randomized strategy for each player? Does either player have an advantage?