Testing for single mean

LP:16 The mean life time of 100 electric bulbs produced by a manufacturing company is estimated to be 1570 hours with a standard deviation of 120 hours. Test the hypothesis that the mean life time of bulbs produced by the company is 1600 hours.

In [ ]:

```
import numpy as np
from scipy.stats import norm

# Given values
s_mean = 1570
p_mean = 1600
std = 120
s_size = 100

# Compute Z-statistic
z = (s_mean - p_mean) / (std / np.sqrt(s_size))

p_value = 2 * (1 - norm.cdf(abs(z)))

# Output results
print("Z-statistic:", round(z, 3))
print("p-value:", round(p_value, 4))

# Decision
alpha = 0.05
if p_value < alpha:
    print("Reject the null hypothesis")
else:
    print("Fail to reject the null hyp_sis")
```

```
Z-statistic: -2.5
p-value: 0.0124
Reject the null hypothesis
```

In [ ]:

```
from statsmodels.stats.weightstats import ztest
import numpy as np

s_mean = 1570
p_mean = 1600
std = 120
s_size = 100

# Create a sample with given summary stats (simulate data)
np.random.seed(0)
sample = np.random.normal(loc=1570, scale=120, size=100)
```

```
# Perform one-sample Z-test
z_stat, p_value = ztest(sample, value=1600, alternative='two-sided')

# Output results
print(f"Z-statistic: {z_stat:.3f}")
print(f"P-value: {p_value:.4f}")

# Decision at 5% significance level
alpha = 0.05
if p_value < alpha:
    print("Reject the null hypothesis: The mean is significantly different from
else:
    print("Fail to reject the null hypothesis: No significant difference from 16
```

```
Z-statistic: -1.878
P-value: 0.0604
Fail to reject the null hypothesis: No significant difference from 1600.
```

Testing for differnce of mean

Q1. LP:15 It is generally assumed that men are taller than women, but we would like to test at 0.01 L.O.S this, so we conduct a survey of 8000 individuals, and a summary of the heights of the males and females who participated in the survey (in inches) is given below: Male Female Sample Size 1600, 6400 Mean 172, 170 Standard Deviation 6.3, 6.4 respectively.

Let `μ_1` : mean height of males

μ 2the mean height of females.

Null Hypothesis (H_0): μ 1≤μ 2→ Men are not taller than women.

Alternative Hypothesis (H_1):

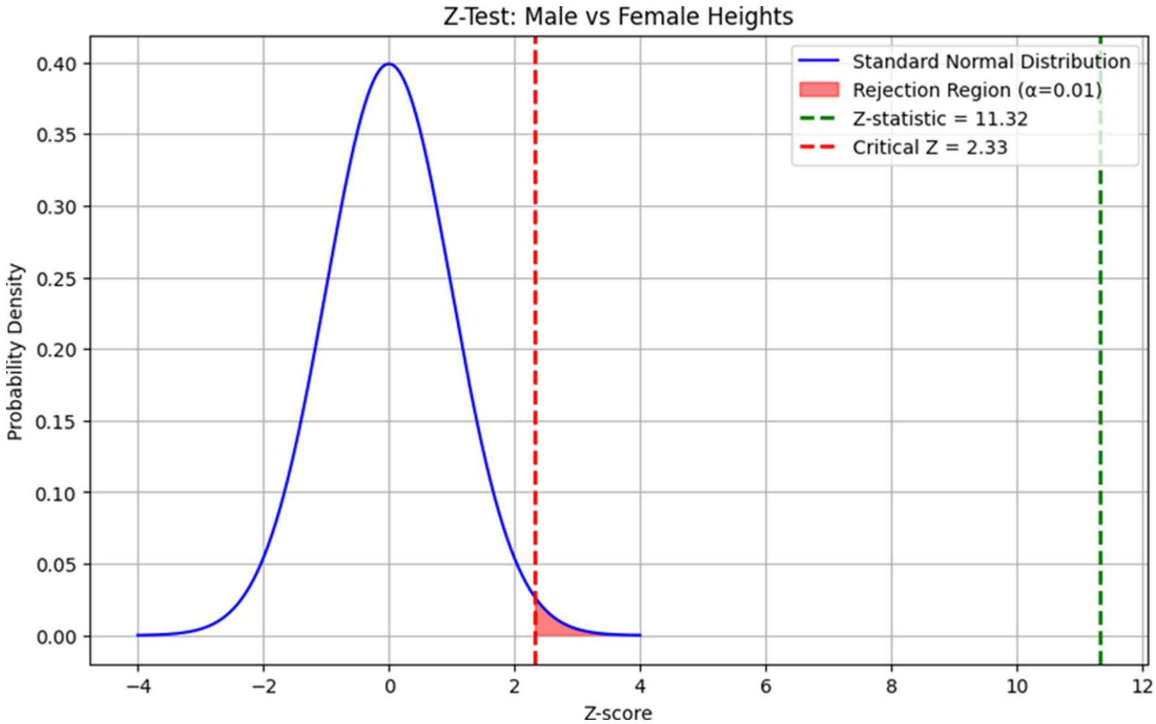
μ 1> μ 2 → Men are taller than women.

Group	Sample Size (n)	Mean (x̄)	Std. Dev (s)
Male	1600	172	6.3
Female	6400	170	6.4

We compare the calculated Z value with the critical Z value at α = 0.01 (one-tailed test), which is approximately 2.33.

```
import math
 import matplotlib.pyplot as plt
 import numpy as np
 from scipy.stats import norm

 # Sample data
 x1, s1, n1 = 172, 6.3, 1600    # Male
 x2, s2, n2 = 170, 6.4, 6400    # Female

 # Calculate standard error
 se = math.sqrt((s1**2)/n1 + (s2**2)/n2)

 # Calculate Z-statistic
 z = (x1 - x2) / se

 # Significance level and critical value
 alpha = 0.01
 z_critical = norm.ppf(1 - alpha)

 # Print results
 print(f"Z-statistic: {z:.3f}")
 print(f"Critical Z at 0.01 level: {z_critical:.3f}")

 if z > z_critical:
     print("Reject the null hypotsis: Men are significantly taller than women.")
 else:
     print("Fail to reject null hyposis: No significant evidence that men are tall”

 x_vals = np.linspace(-4, 4, 1000) y_vals =
 norm.pdf(x_vals)

 plt.figure(figsize=(10, 6))
 plt.plot(x_vals, y_vals, label="Standard Normal Distribution", color='blue')

 # Rejection region (right tail)
 x_fill = np.linspace(z_critical, 4, 100)
 plt.fill_between(x_fill, norm.pdf(x_fill), color='red', alpha=0.5, label='Reject

 # Z-statistic and critical value lines
 plt.axvline(z, color='green', linestyle='--', linewidth=2, label=f'Z-statistic =
 plt.axvline(z_critical, color='red', linestyle='--', linewidth=2, label=f'Critic

 # Plot details
 plt.title("Z-Test: Male vs Female Heights")
 plt.xlabel("Z-score")
 plt.ylabel("Probability Density") plt.legend()
 plt.grid(True) plt.show()
```

```
Z-statistic: 11.322
Critical Z at 0.01 level: 2.326
Reject the null hypothesis: Men are significantly taller than women.
```



Q2. LP: 17 An examination was given to two classes A and B consisting of 40 and 50 students respectively. In class A, the mean mark was 74 with a standard deviation of 8, while in class B the mean mark was 78 with a standard deviation of

7. Is there a significant difference between the performances in the two classes, at the level of significance 0.05? What about the situation at 0.01 level of significance?

Group	n (size)	Mean ($\bar{x}$)	Std. Dev (s)
Class A	40	74	8
Class B	50	78	7

Hypotheses Null Hypothesis (H_0): $\mu\ A$

$\mu\ B$ (no difference in performance)

Alternative Hypothesis (H_1): μ A=μ B(there is a difference)

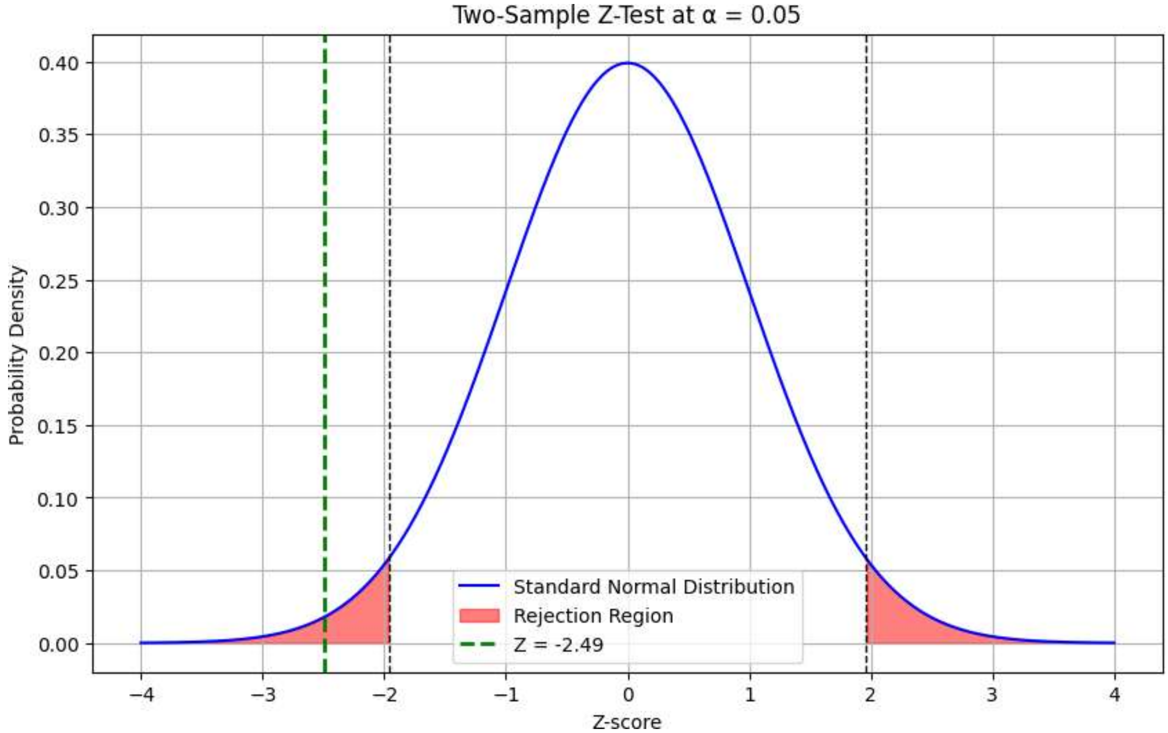
Then compare the absolute value of Z to the critical Z for:

95% confidence (α = 0.05) → critical Z ≈ 1.96

99% confidence (α = 0.01) → critical Z ≈ 2.576

In [ ]:
```python
import math
import matplotlib.pyplot as plt
import numpy as np
from scipy.stats import norm

# Given data
x1, s1, n1 = 74, 8, 40    # Class A
x2, s2, n2 = 78, 7, 50    # Class B

# Calculate standard error
se = math.sqrt((s1**2)/n1 + (s2**2)/n2)

# Calculate Z-statistic
z = (x1 - x2) / se

# Function to evaluate and plot for given alpha
def evaluate_test(z, alpha):
    z_crit = norm.ppf(1 - alpha/2)
    print(f"\nLevel of Significance: {alpha}")
    print(f"Z-statistic: {z:.3f}")
    print(f"Critical Z-value (two-tailed): ±{z_crit:.3f}")

    if abs(z) > z_crit:
        print("→ Reject the null hypothesis: Significant difference between the
    else:
        print("→ Fail to reject the null hypothesis: No significant difference b

    # Plotting
    x_vals = np.linspace(-4, 4, 1000)
    y_vals = norm.pdf(x_vals)

    plt.figure(figsize=(10, 6))
    plt.plot(x_vals, y_vals, label="Standard Normal Distribution", color='blue')

    # Shade rejection regions
    plt.fill_between(x_vals, y_vals, where=(x_vals <= -z_crit), color='red', alp
    plt.fill_between(x_vals, y_vals, where=(x_vals >= z_crit), color='red', alph

    # Plot Z-statistic
    plt.axvline(z, color='green', linestyle='--', linewidth=2, label=f'Z = {z:.2
    plt.axvline(-z_crit, color='black', linestyle='--', linewidth=1)
    plt.axvline(z_crit, color='black', linestyle='--', linewidth=1)

    plt.title(f"Two-Sample Z-Test at α = {alpha}")
    plt.xlabel("Z-score")
    plt.ylabel("Probability Density")
    plt.legend()
    plt.grid(True)
    plt.show()
```

```
Level of Significance: 0.05
Z-statistic: -2.490
Critical Z-value (two-tailed): ±1.960
→ Reject the null hypothesis: Significant difference between the classes.
```




```
Level of Significance: 0.01
Z-statistic: -2.490
Critical Z-value (two-tailed): ±2.576
→ Fail to reject the null hypothesis: No significant difference between the class
es.
```

```python
import math
from scipy.stats import norm

# Given data
x1, s1, n1 = 74, 8, 40    # Class A
x2, s2, n2 = 78, 7, 50    # Class B

# Calculate standard error
se = math.sqrt((s1**2)/n1 + (s2**2)/n2)

# Calculate Z-statistic
z = (x1 - x2) / se

# Function to evaluate the test
def evaluate_test(z, alpha):
    z_crit = norm.ppf(1 - alpha/2)
    print(f"\nLevel of Significance: {alpha}")
    print(f"Z-statistic: {z:.3f}")
    print(f"Critical Z-value (two-tailed): ±{z_crit:.3f}")

    if abs(z) > z_crit:
        print("→ Reject the null hypothesis: Significant difference between the
    else:
        print("→ Fail to reject the null hypothesis: No significant difference b

# Run tests for both levels of significance
evaluate_test(z, 0.05)
```

```
level of Significance: 0.05
Z-statistic: -2.490
Critical Z-value (two-tailed): ±1.960
→ Reject the null hypothesis: Significant difference between the classes.

Level of Significance: 0.01
Z-statistic: -2.490
Critical Z-value (two-tailed): ±2.576
→ Fail to reject the null hypothesis: No significant difference between the class
es.
```

One-Tailed Test

LP:18 It is generally assumed that men are taller than women, but we would like to test at 0.01 L.O.S this, so we conduct a survey of 8000 individuals, and a summary of the heights of the males and females who participated in the survey (in inches) is given below: Male Female Sample Size 1600, 6400 Mean 172, 170 Standard Deviation 6.3, 6.4 respectively.

In [ ]:

```python
import numpy as np
from scipy.stats import norm

# Sample stats
n_male = 1600
n_female = 6400

mean_male = 172
mean_female = 170

std_male = 6.3
std_female = 6.4

# Z-test statistic formula
z = (mean_male - mean_female) / np.sqrt((std_male**2 / n_male) + (std_female**2

# One-tailed p-value (right-tailed)
p_value = 1 - norm.cdf(z)

# Output
print("Z-statistic:", round(z, 3))
print("p-value:", round(p_value, 4))

# Conclusion
alpha = 0.01
if p_value < alpha:
    print("Reject the null hypothesis: Men are significantly taller than women."
else:
    print("Fail to reject the null hypothesis: No significant height difference
```

```
Z-statistic: 11.322
p-value: 0.0
Reject the null hypothesis: Men are significantly taller than women.
```

In [ ]:

```python
import numpy as np
from statsmodels.stats.weightstats import ztest

# Summary statistics
mean_male = 172
std_male = 6.3
n_male = 1600

mean_female = 170
std_female = 6.4
n_female = 6400

# Simulate raw data based on the given statistics
np.random.seed(42)
male_data = np.random.normal(loc=mean_male, scale=std_male, size=n_male)
female_data = np.random.normal(loc=mean_female, scale=std_female, size=n_female)

# Perform one-tailed Z-test (alternative='larger' means male > female)
z_stat, p_val = ztest(male_data, female_data, alternative='larger')

# Print results
print("Z-statistic:", round(z_stat, 3))
print("p-value:", round(p_val, 4))

alpha = 0.01
if p_val < alpha:
    print("Reject the null hypothesis: Men are significantly taller than women."
else:
    print("Fail to reject the null hypothesis.")
```

```
Z-statistic: 13.04
p-value: 0.0
Reject the null hypothesis: Men are significantly taller than women.
```

one-sample t-test

LP:20 Ten specimens of copper wires drawn from a large lot have the following breaking strength (in Kg. weight) 578, 572, 570, 568, 572, 571, 570, 572, 596, 548. Test whether the mean breaking strength of the lot may be taken be 578kg weight?

In [ ]:

```python
import numpy as np
from scipy import stats

data = [578, 572, 570, 568, 572, 571, 570, 572, 596, 548]

mu = 578

# Perform one-sample t-test
t_stat, p_value = stats.ttest_1samp(data, mu)

# Output the results
print(f"T-statistic: {t_stat}")
print(f"P-value: {p_value}")

# Interpret the result at alpha = 0.05
alpha = 0.05
if p_value < alpha:
    print("Reject the null hypothesis: The mean is significantly diff from 578 k
else:
    print("Fail to reject null hypo_sis: The mean is not sign_cantly diff from 5
```

```
T-statistic: -1.7166927719007372
P-value: 0.12016675065728374
Fail to reject the null hypothesis: The mean is not significantly different from
578 kg.
```

LP:21 In 1950 in India the mean life expectancy was 50 years. If the life expectancies from a random sample of 11 persons are 58.2, 56.6, 54.2, 50.4, 44.2, 61.9, 57.5, 53.4, 49.7, 55.4, 57, does it confirm the expected view at 5% LOS.

Null Hypothesis (H_0):

$\mu=50$ (Life expectancy was 50 years)

Alternative Hypothesis (H_1): $\mu \neq 50$ (Life expectancy was not 50 years)

This is a two-tailed test at a 5% level of significance ($\alpha = 0.05$).

In [ ]:
```
import numpy as np
from scipy import stats

# Sample data
data = [58.2, 56.6, 54.2, 50.4, 44.2, 61.9, 57.5, 53.4, 49.7, 55.4, 57]
mu_0 = 50  # Hypothesized mean

# Perform one-sample t-test
t_statistic, p_value = stats.ttest_1samp(data, mu_0)

# Output values
print("Sample Mean:", round(np.mean(data), 2))
print("Sample Std Dev:", round(np.std(data, ddof=1), 2))
print("t-statistic:", round(t_statistic, 3))
print("p-value:", round(p_value, 4))

# Decision
alpha = 0.05
if p_value < alpha:
    print("Reject the null hypothesis: Life expectancy is significantly differen
else:
    print("Fail to reject the null hypothesis: No significant evidence of change
```

```
Sample Mean: 54.41
Sample Std Dev: 4.86
t-statistic: 3.01
p-value: 0.0131
Reject the null hypothesis: Life expectancy is significantly different from 50 ye
ars.
```

Single proportion

LP:22 A builder claims that heat pumps are installed in 70% of all homes being constructed today in the city of Richmond. Would you agree with this claim if a random survey of new homes in this city shows that 8 out of 15 had heat pumps installed? Use a 0.01 level of significance.

In [ ]:

```
import numpy as np
from scipy.stats import norm

# Given data
n = 15
x = 8
p0 = 0.70  # hypothesized proportion
alpha = 0.01

# Sample proportion
phat = x / n

# Standard error
se = np.sqrt(p0 * (1 - p0) / n)

# Z-statistic
z = (phat - p0) / se

# Two-tailed p-value
p_value = 2 * (1 - norm.cdf(abs(z)))

# Output
print("Sample Proportion:", round(phat, 4))
print("Z-statistic:", round(z, 3))
print("p-value:", round(p_value, 4))

# Conclusion
if p_value < alpha:
    print("Reject the null hypothesis: Evidence that true proportion differs fro
else:
    print("Fail to reject the null hypothesis: No significant evidence that the
```

```
Sample Proportion: 0.5333
Z-statistic: -1.409
p-value: 0.159
Fail to reject the null hypothesis: No significant evidence that the true proport
ion differs from 70%.
```

In [ ]:

```
import numpy as np
from statsmodels.stats.proportion import proportions_ztest

# Given data
n = 15
x = 8
p0 = 0.70  # hypothesized proportion
alpha = 0.01

# Perform the z-test
stat, p_value = proportions_ztest(count=x, nobs=n, value=p0, alternative='two-si

# Output
print("Z-statistic:", round(stat, 3))
print("p-value:", round(p_value, 4))

# Conclusion
if p_value < alpha:
    print("Reject the null hypothesis: Evidence that true proportion differs fro
else:
    print("Fail to reject the null hypothesis: No significant evidence that the
```

```
Z-statistic: -1.294
p-value: 0.1957
Fail to reject the null hypothesis: No significant evidence that the true proport
ion differs from 70%.
```

Difference of proportion

LP:24 Many consumers think that automobiles built on Mondays are more likely to have serious defects than those built on any other day of the week. To support this theory a random sample of 100 cars built on Monday is selected and inspected. Of these 8 are found to have serious defects. A random sample of 200 cars produced on other days reveals 12 with serious defects. Do these data support the stated connection?

In [ ]:

```python
import numpy as np
from scipy.stats import norm

# Sample data
n1, x1 = 100, 8     # Monday cars
n2, x2 = 200, 12    # Other day cars

# Sample proportions
p1 = x1 / n1
p2 = x2 / n2

# Pooled proportion
p_pooled = (x1 + x2) / (n1 + n2)

# Standard error
se = np.sqrt(p_pooled * (1 - p_pooled) * (1/n1 + 1/n2))

# Z-statistic
z = (p1 - p2) / se

# One-tailed p-value (right tail)
p_value = 1 - norm.cdf(z)

# Output
print("Proportion (Monday):", round(p1, 4))
print("Proportion (Other Days):", round(p2, 4))
print("Pooled Proportion:", round(p_pooled, 4))
print("Z-statistic:", round(z, 3))
print("p-value:", round(p_value, 4))
```

```
# Conclusion
alpha = 0.05
if p_value < alpha:
    print("Reject H₀: Monday cars have significantly more defects.")
else:
    print("Fail to reject H₀: No significant evidence that Monday cars have more
```

```
Proportion (Monday): 0.08
Proportion (Other Days): 0.06
Pooled Proportion: 0.0667
Z-statistic: 0.655
p-value: 0.2563
Fail to reject H₀: No significant evidence that Monday cars have more defects.
```

In [ ]:

```
from statsmodels.stats.proportion import proportions_ztest

# Data
count = [8, 12]        # Number of cars with defects: Monday, Other days
nobs = [100, 200]      # Total cars inspected: Monday, Other days

# Perform one-tailed z-test (H1: p1 > p2)
stat, p_value = proportions_ztest(count=count, nobs=nobs, alternative='larger')

# Output
print("Z-statistic:", round(stat, 3))
print("p-value:", round(p_value, 4))

# Conclusion
alpha = 0.05
if p_value < alpha:
    print("Reject H₀: Monday cars have significantly more defects.")
else:
    print("Fail to reject H₀: No significant evidence that Monday cars have more
```

```
Z-statistic: 0.655
p-value: 0.2563
Fail to reject H₀: No significant evidence that Monday cars have more defec
```