Assignment-10

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***Abstract*—In this document, we present the solution to $\mathbf{AX} = c\mathbf{X}$**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_10

1 Problem Statement

Let

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{pmatrix} \tag{1.0.1}$$

For which $\mathbf{X}$ does there exist a scalar c such that $\mathbf{AX} = c\mathbf{X}$

2 Solution

Given

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{pmatrix} \tag{2.0.1}$$

The given matrix has single eigenvalue as it is the lower triangular matrix and has equal diagonal elements. Hence $c_1 = c_2 = c_3 = 5$. To find the corresponding eigenvector, consider the following

$$(\mathbf{A} - c\mathbf{I})\mathbf{X} = 0 \tag{2.0.2}$$

$$\implies \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.3}$$

Solving the homogeneous system of linear equations by performing rref, we get

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xleftrightarrow[R_3 \leftrightarrow R_2]{R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \tag{2.0.4}$$

Hence we get,

$$x_1 = 0, x_2 = 0, x_3 = t \tag{2.0.5}$$

where, x_3 is arbitrary. Therefore,

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ t \end{pmatrix} \tag{2.0.6}$$

Hence, the given matrix has single eigenvector and is not diagonalizable.