Problem Statement:
To verify whether the lines passing through the given set of points are parallel or not

Solution-1 *Using the vector representation*
Given the points, $A = \begin{pmatrix} 4 \\ 7 \\ 8 \end{pmatrix}, B = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, and \quad C = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}, D = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$
•Compute the direction vector for the given set of points

$$B - A = \begin{pmatrix} -2 \\ -4 \\ -4 \end{pmatrix} \tag{1}$$

$$D - C = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} \tag{2}$$

•Check whether one of the direction vector is the scalar multiple of the other direction vector
Here, from (1) and (2), $B - A = k(D - C)$. In this example, k = -1.
Hence, the lines are parallel.

Solution-2 *Using the matrix representation and rank of a matrix*
Represent the direction vectors in the matrix form and perform row reduction:

$$i.e., M = \begin{pmatrix} B - A & D - C \end{pmatrix}^T$$

$$M = \begin{pmatrix} -2 & -4 & -4 \\ 2 & 4 & 4 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} -2 & -4 & -4 \\ 0 & 0 & 0 \end{pmatrix}$$

Here, the rank of the matrix is 1. This implies that the lines are parallel.

Solution-3 *Using the cross product of the vectors*
•Compute the cross product of the direction vectors
The cross product of the direction vectors given in (1) and (2) is:

$$\begin{pmatrix} -2 \\ -4 \\ -4 \end{pmatrix} X \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -16 + 16 \\ -8 + 8 \\ -8 + 8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The zero vector infers that the lines are parallel.

Solution-4 Let the lines be parallel and the first two points pass through $n^T\mathbf{x} = \mathbf{c}$ i.e.

$$n^T x1 = c1 => x1^T n = c1, \quad n^T x2 = c2 => x2^T n = c2 \tag{3}$$

and the second two points pass through $n^T\mathbf{x} = \mathbf{c}$ Then

$$n^T x3 = c3 => x3^T n = c3, \quad n^T x4 = c4 => x4^T n = c4 \tag{4}$$

Putting equations (3) and (4) together, we obtain

$$\begin{pmatrix} x1^T \\ x2^T \\ x3^T \\ x4^T \end{pmatrix} \vec{n} = \begin{pmatrix} c1 \\ c2 \\ c3 \\ c4 \end{pmatrix} \tag{5}$$

Now if this equation has a solution, then $\vec{n}$ exists and the lines will be parallel.