

Assignment-10

Pooja H
AI20MTECH14003

Abstract—In this document, we present the solution to $\mathbf{AX} = c\mathbf{X}$

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_10

1 PROBLEM STATEMENT

Let

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{pmatrix} \quad (1.0.1)$$

For which \mathbf{X} does there exist a scalar c such that $\mathbf{AX} = c\mathbf{X}$

2 SOLUTION

Given

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{pmatrix} \quad (2.0.1)$$

To find \mathbf{X} such that $\mathbf{AX} = c\mathbf{X}$, consider

$$(\mathbf{AX} - c\mathbf{X}) = 0 \quad (2.0.2)$$

$$\Rightarrow (\mathbf{A} - c\mathbf{I})\mathbf{X} = 0 \quad (2.0.3)$$

Finding the corresponding eigenvalues and eigenvectors:

$$\det(\mathbf{A} - c\mathbf{I}) = 0 \quad (2.0.4)$$

$$\Rightarrow \begin{vmatrix} 5-c & 0 & 0 \\ 1 & 5-c & 0 \\ 0 & 1 & 5-c \end{vmatrix} = 0 \quad (2.0.5)$$

$$\Rightarrow c = 5 \quad (2.0.6)$$

$$\therefore \mathbf{A} - c\mathbf{I} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad (2.0.7)$$

From (2.0.3), we get

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.8)$$

Solving the homogeneous system of linear equations by performing rref, we get

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \xleftrightarrow[R_3 \leftrightarrow R_2]{R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad (2.0.9)$$

Hence we get,

$$x_1 = 0, x_2 = 0, x_3 = t \quad (2.0.10)$$

where, t is some constant. Therefore,

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} t \quad (2.0.11)$$

Hence for $\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, there exists a scalar $c = 5$ such that $\mathbf{AX} = c\mathbf{X}$.