

Assignment-14

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Abstract—In this document, we find the matrix of T in the ordered basis \mathbf{B}

Download all latex-tikz codes from

[https://github.com/poojah15/
EE5609_AI20MTECH14003
/tree/master/Assignment_14](https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_14)

1 PROBLEM STATEMENT

Let T be the linear operator on \mathbf{R}^2 defined by

$$T(x_1, x_2) = (-x_2, x_1) \quad (1.0.1)$$

What is the matrix of T in the ordered basis $\mathbf{B} = \{\alpha_1, \alpha_2\}$, where $\alpha_1 = (1, 2)$ and $\alpha_2 = (1, -1)$?

2 SOLUTION

Applying the transformations on α_1 and α_2 we get,

$$T(\alpha_1) = (-2, 1) \quad (2.0.1)$$

$$T(\alpha_2) = (1, 1) \quad (2.0.2)$$

In order to write $T(\alpha_1)$ and $T(\alpha_2)$ in terms of α_1 and α_2 , we row reduce the augmented matrix

$$\begin{pmatrix} 1 & 1 & -2 & 1 \\ 2 & -1 & 1 & 1 \end{pmatrix} \xrightarrow[R_2 \leftarrow -R_2]{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & 1 & -2 & 1 \\ 0 & 1 & -\frac{5}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.3)$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{5}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.4)$$

Hence, the matrix T in ordered basis \mathbf{B} is

$$[T]_{\mathbf{B}} = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} \\ -\frac{5}{3} & \frac{1}{3} \end{pmatrix} \quad (2.0.5)$$