# Assignment-6

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***Abstract*—In this document, we find the value of k such that the equation represents a pair of straight lines.**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_6

## 1 Problem Statement

Find the value of k such that $6x^2+11xy-10y^2+x+31y+k=0$ represent pairs of straight lines.

## 2 Theory

The general equation of second degree is given by

$$ax^2+2bxy+cy^2+2dx+2ey+f=0 \tag{2.0.1}$$

and can be expressed as

$$\mathbf{x}^T\mathbf{V}\mathbf{x}+2\mathbf{u}^T\mathbf{x}+f=0 \tag{2.0.2}$$

where

$$\mathbf{V}=\mathbf{V}^T=\begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{u}^T=\begin{pmatrix} d & e \end{pmatrix} \tag{2.0.4}$$

Let the pair of straight lines be given by

$$\mathbf{n}_1^T\mathbf{x}=c_1 \tag{2.0.5}$$

$$\mathbf{n}_2^T\mathbf{x}=c_2 \tag{2.0.6}$$

Equating their product with (2.0.2), we get

$$(\mathbf{n}_1^T\mathbf{x}-c_1)(\mathbf{n}_2^T\mathbf{x}-c_2)$$
$$\implies \mathbf{x}^T\mathbf{V}\mathbf{x}+2\mathbf{u}^T\mathbf{x}+f=0 \tag{2.0.7}$$

(2.0.7) represents a pair of straight lines if

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix}=0 \tag{2.0.8}$$

## 3 Solution

Given,

$$6x^2+11xy-10y^2+x+31y+k=0 \tag{3.0.1}$$

Equating (3.0.1) to (2.0.2), we get

$$\mathbf{V}=\begin{pmatrix} 6 & \frac{11}{2} \\ \frac{11}{2} & -10 \end{pmatrix} \tag{3.0.2}$$

$$\mathbf{u}^T=\begin{pmatrix} \frac{1}{2} & \frac{31}{2} \end{pmatrix} \tag{3.0.3}$$

Substituting $\mathbf{V}$ and $\mathbf{u}^T$ in (2.0.8), we obtain

$$\begin{vmatrix} 6 & \frac{11}{2} & \frac{1}{2} \\ \frac{11}{2} & -10 & \frac{31}{2} \\ \frac{1}{2} & \frac{31}{2} & k \end{vmatrix}=0 \tag{3.0.4}$$

$$\implies 6\left(-10k-\left(\frac{31}{2}\right)^2\right)-\frac{11}{2}\left(\frac{11}{2}k-\frac{31}{4}\right)$$
$$+\frac{1}{2}\left(\frac{11}{2}\times\frac{31}{2}+5\right)=0 \tag{3.0.5}$$

$$\implies \frac{-361}{4}k-\frac{5415}{4}=0 \tag{3.0.6}$$

$$\implies \boxed{k=-15} \tag{3.0.7}$$

Substituting (3.0.7) in (3.0.1), we get

$$6x^2+11xy-10y^2+x+31y-15=0 \tag{3.0.8}$$

Hence the solution.

## 4 Graphical Illustration

Firstly, obtain linear equation of the form $(ax+by+c)$ of (3.0.8) by considering

$$6x^2+11xy-10y^2 \tag{4.0.1}$$

$$\implies 6x^2-4xy+15xy-10y^2 \tag{4.0.2}$$

$$\implies 2x(3x-2y)+5y(3x-2y) \tag{4.0.3}$$

$$\implies (3x-2y)(2x+5y) \tag{4.0.4}$$

Equation (4.0.4) can be written as,

$$(3x - 2y + m)(2x + 5y + n) = 0 \quad (4.0.5)$$

$$\implies 6x^2 + 11xy - 10y^2 + (3n + 2m)x + (-2n + 5m)y = 0 \quad (4.0.6)$$

Equating (3.0.8) and (4.0.6),

$$3n + 2m = 1 \quad (4.0.7)$$

$$-2n + 5m = 31 \quad (4.0.8)$$

Solving for m and n we obtain,

$$m = 5 \quad (4.0.9)$$

$$n = -3 \quad (4.0.10)$$

Substituting (4.0.9) and (4.0.10) in (4.0.5), we obtain

$$(3x - 2y + 5)(2x + 5y - 3) = 0 \quad (4.0.11)$$

Hence (4.0.11) represent equation of two straight lines. Graphically,

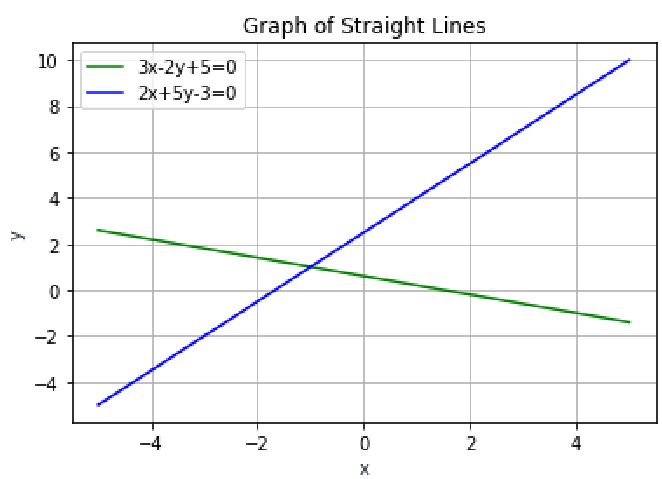


Fig. 1: Plot of two straight lines.