Assignment-10

Pooja H
AI20MTECH14003

***Abstract*—In this document, we present the solution to $\mathbf{AX} = c\mathbf{X}$**

Download all latex-tikz codes from

```
https://github.com/poojah15/
    EE5609_AI20MTECH14003/tree/master/
    Assignment_10
```

1 Problem Statement

Let

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{pmatrix} \tag{1.0.1}$$

For which $\mathbf{X}$ does there exist a scalar c such that $\mathbf{AX} = c\mathbf{X}$

2 Solution

Given

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{pmatrix} \tag{2.0.1}$$

To find $\mathbf{X}$ such that $\mathbf{AX} = c\mathbf{X}$, consider

$$(\mathbf{AX} - c\mathbf{X}) = 0 \tag{2.0.2}$$

$$\implies (\mathbf{A} - c\mathbf{I})\mathbf{X} = 0 \tag{2.0.3}$$

Finding the corresponding eigenvalues and eigenvectors:

$$\det(\mathbf{A} - c\mathbf{I}) = 0 \tag{2.0.4}$$

$$\implies \begin{vmatrix} 5-c & 0 & 0 \\ 1 & 5-c & 0 \\ 0 & 1 & 5-c \end{vmatrix} = 0 \tag{2.0.5}$$

$$\implies c = 5 \tag{2.0.6}$$

$$\therefore \mathbf{A} - c\mathbf{I} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{2.0.7}$$

From (2.0.3), we get

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.8}$$

Solving the homogeneous system of linear equations by performing rref, we get

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xleftrightarrow[R_3 \leftrightarrow R_2]{R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \tag{2.0.9}$$

Hence we get,

$$\mathbf{x}_1 = 0, \mathbf{x}_2 = 0, \mathbf{x}_3 = t \tag{2.0.10}$$

where, t is some constant. Therefore,

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} t \tag{2.0.11}$$

Hence for $\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, there exists a scalar $c = 5$ such that $\mathbf{AX} = c\mathbf{X}$.