Assignment-10

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***Abstract*—In this document, we present the solution to $\mathbf{AX} = c\mathbf{X}$**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_10

1 Problem Statement

Let

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{pmatrix} \tag{1.0.1}$$

For which $\mathbf{X}$ does there exist a scalar c such that $\mathbf{AX} = c\mathbf{X}$

2 Solution

Given

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{pmatrix} \tag{2.0.1}$$

To find $\mathbf{X}$ such that $\mathbf{AX} = c\mathbf{X}$, consider

$$(\mathbf{AX} - c\mathbf{X}) = 0 \tag{2.0.2}$$

$$\implies (\mathbf{A} - c\mathbf{I})\mathbf{X} = 0 \tag{2.0.3}$$

The characteristic polynomial for the matrix $\mathbf{A}$ is given by,

$$\det(\mathbf{A} - c\mathbf{I}) = 0 \tag{2.0.4}$$

$$\implies \begin{vmatrix} 5-c & 0 & 0 \\ 1 & 5-c & 0 \\ 0 & 1 & 5-c \end{vmatrix} = 0 \tag{2.0.5}$$

$$\implies -c^3 + 15c^2 - 75c + 125 = 0 \tag{2.0.6}$$

$$\implies c_1 = 5, c_2 = 5, c_3 = 5 \tag{2.0.7}$$

Also, since the matrix is triangular, the eigenvalues are the elements in the principal diagonal. Hence $c_1 = 5, c_2 = 5, c_3 = 5$. To find the corresponding eigenvector, consider the following

$$\mathbf{A} - 5\mathbf{I} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{2.0.8}$$

From (2.0.3), we get

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.9}$$

Solving the homogeneous system of linear equations by performing rref, we get

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xleftrightarrow[R_3 \leftrightarrow R_2]{R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \tag{2.0.10}$$

Hence we get,

$$x_1 = 0, x_2 = 0, x_3 = t \tag{2.0.11}$$

where, x_3 is arbitrary. Therefore,

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ t \end{pmatrix} \tag{2.0.12}$$

Thus the possible eigenvectors are

$$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \quad \dots \tag{2.0.13}$$

Here, we cannot find 3 independent eigenvectors corresponding to 3 eigenvalues as the eigenvalues are repeated. Also, if we try to form the modal matrix P from any three of these eigenvectors i.e., $\mathbf{P} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 1 & 2 \end{pmatrix}$, it will have determinant zero. Thus P^{-1} doesn't exist and hence the given matrix is not diagonalizable.