Assignment-4

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Abstract—**In this work, we evaluate the determinant of a matrix.**

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1 PROBLEM STATEMENT

Evaluate $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

2 THEORY

The determinant of a matrix of order three can be determined by expressing it in terms of second order determinants and is called expansion of a determinant along either a row or column. Consider a determinant of square matrix $\mathbf{A} = [a_{ij}]_{3x3}$

$$i.e., |\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad (2.0.1)$$

Now, the expansion of determinant of $\mathbf{A}$ that is, $|\mathbf{A}|$ can be written as

$$|\mathbf{A}| = (-1)^{1+1}a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2}a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$+ (-1)^{1+3}a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \quad (2.0.2)$$

or

$$|\mathbf{A}| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23})$$

$$+ a_{13}(a_{21}a_{32} - a_{31}a_{22}) \quad (2.0.3)$$

Generally, the properties of determinants are used while evaluating the determinant. We have used row/column reduction method and then compute the determinant of a matrix.

3 SOLUTION

$$Given, |\mathbf{A}| = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} \quad (3.0.1)$$

$$\xleftarrow{R_1 \leftarrow R_1 + R_2 + R_3} \begin{vmatrix} 2(x+y) & 2(x+y) & 2(x+y) \\ y & x+y & x \\ x+y & x & y \end{vmatrix} \quad (3.0.2)$$

$$= 2(x+y) \begin{vmatrix} 1 & 1 & 1 \\ y & x+y & x \\ x+y & x & y \end{vmatrix} \quad (3.0.3)$$

$$\xleftarrow[C_3 \leftarrow C_3 - C_1]{C_2 \leftarrow C_2 - C_1} 2(x+y) \begin{vmatrix} 1 & 0 & 0 \\ y & x & x-y \\ x+y & -y & -x \end{vmatrix} \quad (3.0.4)$$

Expanding the determinant from (3.0.4), we get

$$= 2(x+y)\left[-x^2 - \{(-y)(x-y)\}\right] \quad (3.0.5)$$

$$= 2(x+y)\left(-x^2 + xy - y^2\right) \quad (3.0.6)$$

$$= -2x^3 + 2x^2y - 2xy^2 - 2x^2y + 2xy^2 - 2y^3 \quad (3.0.7)$$

$$= -2\left(x^3 + y^3\right) \quad (3.0.8)$$

$$\therefore \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = -2\left(x^3 + y^3\right) \quad (3.0.9)$$