Assignment-13

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Abstract—**In this document, we find whether the given function T from $\mathbf{R}^2$ into $\mathbf{R}^2$ is a linear transformation or not.**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_13

1 Problem Statement

Verify whether $T(x_1, x_2) = (x_1 - x_2, 0)$ is a linear transformation or not.

2 Solution

Let $\mathbf{V}$ and $\mathbf{W}$ be the vector spaces. The function $T : \mathbf{V} \to \mathbf{W}$ is called a linear transformation of $\mathbf{V}$ into $\mathbf{W}$ if the following properties are true for all $\mathbf{u}$ and $\mathbf{v}$ in $\mathbf{V}$ and for any scalar k.

$$T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v}) \tag{2.0.1}$$
$$T(k\mathbf{u}) = kT(\mathbf{u}) \tag{2.0.2}$$

Given,

$$T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{2.0.3}$$

Suppose we have $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$, then we have to show that the result of $T(\mathbf{x}+\mathbf{y})$ must be same as $T(\mathbf{x}) + T(\mathbf{y})$. Consider,

$$T(\mathbf{x}+\mathbf{y}) = T\begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix} \tag{2.0.4}$$
$$= \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix} \tag{2.0.5}$$
$$= \begin{pmatrix} x_1 + y_1 - x_2 - y_2 \\ 0 \end{pmatrix} \tag{2.0.6}$$
$$= \begin{pmatrix} x_1 - x_2 \\ 0 \end{pmatrix} + \begin{pmatrix} y_1 - y_2 \\ 0 \end{pmatrix} \tag{2.0.7}$$
$$= \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \tag{2.0.8}$$
$$T(\mathbf{x}+\mathbf{y}) = T(\mathbf{x}) + T(\mathbf{y}) \tag{2.0.9}$$

Also, we need to show that for some constant k, $T(k\mathbf{x}) = kT(\mathbf{x})$. Consider,

$$T(k\mathbf{x}) = T\begin{pmatrix} kx_1 \\ kx_2 \end{pmatrix} \tag{2.0.10}$$
$$= \begin{pmatrix} k & -k \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{2.0.11}$$
$$= k\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{2.0.12}$$
$$T(k\mathbf{x}) = kT(\mathbf{x}) \tag{2.0.13}$$

Therefore, the given function T is a linear transformation.