Assignment-4

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***Abstract*—In this work, we evaluate the determinant of a matrix.**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_4

Download the python code from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_4

1 Problem Statement

Evaluate $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

2 Theory

The determinant of a matrix of order three can be determined by expressing it in terms of second order determinants and is called expansion of a determinant along either a row or column. Consider a determinant of square matrix $\mathbf{A} = [a_{ij}]_{3x3}$

$$i.e., \left|\mathbf{A}\right| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \tag{2.0.1}$$

Now, the expansion of determinant of $\mathbf{A}$ that is, $\left|\mathbf{A}\right|$ can be written as

$$\left|\mathbf{A}\right| = (-1)^{1+1}a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2}a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3}a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \tag{2.0.2}$$

or

$$\left|\mathbf{A}\right| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22}) \tag{2.0.3}$$

Generally, the properties of determinants are used while evaluating the determinant. We have used row/column reduction method and then compute the determinant of a matrix.

3 Solution

$$Given, \left|\mathbf{A}\right| = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} \tag{3.0.1}$$

$$\xleftrightarrow{R_1 \leftarrow R_1+R_2+R_3} \begin{vmatrix} 2(x+y) & 2(x+y) & 2(x+y) \\ y & x+y & x \\ x+y & x & y \end{vmatrix} \tag{3.0.2}$$

$$= 2(x+y)\begin{vmatrix} 1 & 1 & 1 \\ y & x+y & x \\ x+y & x & y \end{vmatrix} \tag{3.0.3}$$

$$\xleftrightarrow[C_3 \leftarrow C_3 - C_1]{C_2 \leftarrow C_2 - C_1} 2(x+y)\begin{vmatrix} 1 & 0 & 0 \\ y & x & x-y \\ x+y & -y & -x \end{vmatrix} \tag{3.0.4}$$

Expanding the determinant from (3.0.4), we get

$$= 2(x+y)(-x^2 - ((-y)(x-y))) \tag{3.0.5}$$

$$= 2(x+y)(-x^2 + xy - y^2) \tag{3.0.6}$$

$$= -2x^3 + 2x^2y - 2xy^2 - 2x^2y + 2xy^2 - 2y^3 \tag{3.0.7}$$

$$= -2(x^3 + y^3) \tag{3.0.8}$$

$$\therefore \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = -2(x^3 + y^3) \tag{3.0.9}$$