

Assignment-3

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Abstract—In this work, we evaluate the matrix equation, to get the value of 'k'. In addition, we find the characteristic equation for the given matrix.

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_3/New_version

1 PROBLEM STATEMENT

If $\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find k so that $\mathbf{A}^2 = k\mathbf{A} - 2\mathbf{I}$.

2 SOLUTION

For a general square matrix \mathbf{A} of size $n \times n$, the characteristic equation in variable λ is defined by,

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \quad (2.0.1)$$

where, \mathbf{I} is the identity matrix of size $n \times n$. Hence, given $\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$, the characteristic equation is computed as follows:

$$\left| \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0 \quad (2.0.2)$$

$$\Rightarrow \begin{vmatrix} 3-\lambda & -2 \\ 4 & -2-\lambda \end{vmatrix} = 0 \quad (2.0.3)$$

By expanding the above determinant we get,

$$(3 - \lambda)(-2 - \lambda) + 8 = 0 \quad (2.0.4)$$

$$\Rightarrow -6 + \lambda^2 + 2\lambda - 3\lambda + 8 = 0 \quad (2.0.5)$$

$$\Rightarrow \lambda^2 - \lambda + 2 = 0 \quad (2.0.6)$$

Hence, (2.0.6) is the required characteristic equation for the given matrix \mathbf{A} . The python code containing the solution to the given problem is provided in

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_3