Assignment-14

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***Abstract*—In this document, we find the matrix of T in the ordered basis B**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_14

1 Problem Statement

Let T be the linear operator on $\mathbf{R}^2$ defined by

$$T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix} \tag{1.0.1}$$

What is the matrix of T in the ordered basis $\mathbf{B} = \{\alpha_1, \alpha_2\}$, where $\alpha_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\alpha_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$?

2 Solution

Let $\mathbf{B}'$ be the standard ordered basis for $\mathbf{R}^2$. Then,

$$T(\alpha_1') = T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0\alpha_1 + 1\alpha_2 \tag{2.0.1}$$

$$T(\alpha_2') = T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} = -1\alpha_1 + 0\alpha_2 \tag{2.0.2}$$

Hence, the matrix of T in the standard ordered basis $\mathbf{B}'$ is

$$T_{\mathbf{B}'} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{2.0.3}$$

Given,

$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{2.0.4}$$

$$\alpha_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2.0.5}$$

Then,

$$\alpha_1' = 1\alpha_1 + 2\alpha_2 \tag{2.0.6}$$

$$\alpha_2' = 1\alpha_1 - 1\alpha_2 \tag{2.0.7}$$

So the $\mathbf{P}$ matrix is

$$\mathbf{P} = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \tag{2.0.8}$$

And

$$\mathbf{P}^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{pmatrix} \tag{2.0.9}$$

Hence

$$[T]_{\mathbf{B}} = \mathbf{P}^{-1}[T]_{\mathbf{B}'}\mathbf{P} \tag{2.0.10}$$

$$= \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \tag{2.0.11}$$

$$= \begin{pmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \tag{2.0.12}$$

$$[T]_{\mathbf{B}} = \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} \\ \frac{5}{3} & -\frac{1}{3} \end{pmatrix} \tag{2.0.13}$$

Hence, $[T]_{\mathbf{B}}$ is the required matrix for the given ordered basis $\mathbf{B}$.