Assignment-15

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***Abstract*—In this document, we find the dual basis of the given basis B**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_15

1 Problem Statement

Let $\mathbf{B} = \{\alpha_1, \alpha_2, \alpha_3\}$ be the basis for $\mathbf{C}^3$ defined by

$$\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \tag{1.0.1}$$

Find the dual basis of $\mathbf{B}$

2 Solution

Let $\mathbf{B}^* = \{f_1, f_2, f_3\}$ be the dual basis of $\mathbf{B}$ such that,

$$f_i(x_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{2.0.1}$$

and

$$f_i(x_1, x_2, x_3) = \sum_{j=1}^{3} \delta_{ij} x_j \tag{2.0.2}$$

where, $(\delta_{11}, \delta_{12}, \delta_{13})$ is the solution to

$$\begin{pmatrix} 1 & 0 & -1 & 1 \\ 1 & 1 & 1 & 0 \\ 2 & 2 & 0 & 0 \end{pmatrix} \tag{2.0.3}$$

$(\delta_{21}, \delta_{22}, \delta_{23})$ is the solution to

$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 0 & 0 \end{pmatrix} \tag{2.0.4}$$

$(\delta_{31}, \delta_{32}, \delta_{33})$ is the solution to

$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{pmatrix} \tag{2.0.5}$$

Row reducing the general form of above matrices we get,

$$\begin{pmatrix} 1 & 0 & -1 & a \\ 1 & 1 & 1 & b \\ 2 & 2 & 0 & c \end{pmatrix} \xleftrightarrow[R_3 \leftarrow R_3 - 2R_1]{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 0 & -1 & a \\ 0 & 1 & 2 & -a+b \\ 0 & 0 & -2 & -2b+c \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow[R_3 \leftarrow \frac{R_3}{-2}]{R_2 \leftarrow R_2 + R_3} \begin{pmatrix} 1 & 0 & -1 & a \\ 0 & 1 & 0 & -a-b+c \\ 0 & 0 & 1 & b - \frac{c}{2} \end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + R_3} \begin{pmatrix} 1 & 0 & 0 & a+b-\frac{c}{2} \\ 0 & 1 & 0 & -a-b+c \\ 0 & 0 & 1 & b - \frac{c}{2} \end{pmatrix} \tag{2.0.8}$$

Therefore we get,

$$(\delta_{11}, \delta_{12}, \delta_{13}) = (1, -1, 0) \tag{2.0.9}$$

$$(\delta_{21}, \delta_{22}, \delta_{23}) = (1, -1, 1) \tag{2.0.10}$$

$$(\delta_{31}, \delta_{32}, \delta_{33}) = (-\frac{1}{2}, 1, -\frac{1}{2}) \tag{2.0.11}$$

Substituting the above in (2.0.2) we get,

$$f_1(x_1, x_2, x_3) = x_1 - x_2 \tag{2.0.12}$$

$$f_2(x_1, x_2, x_3) = x_1 - x_2 + x_3 \tag{2.0.13}$$

$$f_3(x_1, x_2, x_3) = -\frac{1}{2} x_1 + x_2 - \frac{1}{2} x_3 \tag{2.0.14}$$

Hence, $\{f_1, f_2, f_3\}$ is the required dual basis for $\mathbf{B}$