Assignment-7

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***Abstract*—In this document, we present the procedure to obtain the equation of all lines having slope *m* that are tangents to the given curve *f*.**

Download all python codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_7

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_7

1 Problem Statement

Find the equation of all lines having slope -1 that are tangents to the curve $\frac{1}{x-1}, x \neq 1$.

2 Solution

The given curve

$$y = \frac{1}{x-1} \tag{2.0.1}$$

can be expressed as

$$xy - y - 1 = 0 \tag{2.0.2}$$

Hence, we have

$$\mathbf{V} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \mathbf{u} = \frac{1}{2}\begin{pmatrix} 0 \\ -1 \end{pmatrix}, f = -1 \tag{2.0.3}$$

Since $\left|\mathbf{V}\right| < 0$, the equation (2.0.2) represents hyperbola. To find the values of λ_1 and λ_2, consider the characteristic equation,

$$\left|\lambda \mathbf{I} - \mathbf{V}\right| = 0 \tag{2.0.4}$$

$$\implies \left|\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}\right| = 0 \tag{2.0.5}$$

$$\implies \begin{vmatrix} \lambda & \frac{-1}{2} \\ \frac{-1}{2} & \lambda \end{vmatrix} = 0 \tag{2.0.6}$$

$$\implies \lambda_1 = \frac{1}{2}, \lambda_2 = \frac{-1}{2} \tag{2.0.7}$$

In addition, given the slope -1, the direction and normal vectors are given by

$$\mathbf{m} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2.0.8}$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.9}$$

The parameters of hyperbola are as follows:

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} \tag{2.0.10}$$

$$= -\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix} \tag{2.0.11}$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.12}$$

$$axes = \begin{cases} \sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} & = \sqrt{2} \\ \sqrt{\frac{f - \mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}}{\lambda_2}} & = \sqrt{2} \end{cases} \tag{2.0.13}$$

which represents the standard hyperbola equation,

$$\frac{x^2}{2} - \frac{x^2}{2} = 1 \tag{2.0.14}$$

The points of contact are given by

$$K = \pm\sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\mathbf{n}^T\mathbf{V}^{-1}\mathbf{n}}} = \pm\frac{1}{2} \tag{2.0.15}$$

$$\mathbf{q} = \mathbf{V}^{-1}(k\mathbf{n} - \mathbf{u}) \tag{2.0.16}$$

$$\mathbf{q_1} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}\left[\frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}\right] \tag{2.0.17}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{2.0.18}$$

$$\mathbf{q_2} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}\left[\frac{-1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}\right] \tag{2.0.19}$$

$$= \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{2.0.20}$$

$\therefore$ The tangents are given by

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\left(\mathbf{x} - \begin{pmatrix} 2 \\ 1 \end{pmatrix}\right) = 0 \tag{2.0.21}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\left(\mathbf{x} - \begin{pmatrix} 0 \\ -1 \end{pmatrix}\right) = 0 \tag{2.0.22}$$

The desired equations of all lines having slope -1 that are tangents to the curve $\frac{1}{x-1}, x \neq 1$ are given by

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 3 \tag{2.0.23}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = -1 \tag{2.0.24}$$

The above results are verified in the following figure.

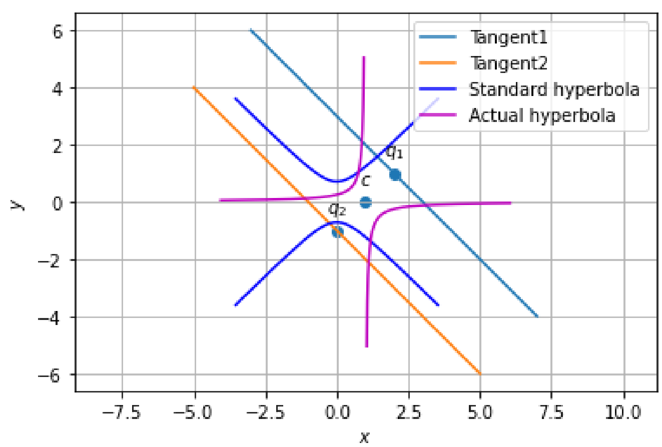

Fig. 0: The standard and actual hyperbola.