Assignment-16

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***Abstract*—In this document, we solve for rank, basis and dimension of the column space of a matrix.**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_16

1 Problem Statement

Let $\mathbf{A}$ be a 4×4 matrix. Suppose that the null space $N(\mathbf{A})$ of $\mathbf{A}$ is

$$\left\{(x, y, z, w) \in \mathbf{R}^4 : x + y + z = 0, x + y + w = 0\right\} \tag{1.0.1}$$

Then which one of the following is correct

1) dim(column space($\mathbf{A}$)) = 1
2) dim(column space($\mathbf{A}$)) = 2
3) rank($\mathbf{A}$) = 1
4) $\mathbf{S} = \{(1, 1, 1, 0), (1, 1, 0, 1)\}$ is a basis of $N(\mathbf{A})$

2 Solution

The nullspace is given by

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.1}$$

Row reducing the above matrix we get,

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xleftrightarrow[R_2 \leftarrow R_2 \times -1]{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.2}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.3}$$

3 Answers for different cases

dim(C(**A**)) = 1	**False**. Because the number of pivot variables are 2 as obtained in (2.0.3)
dim(C(**A**)) = 2	**True**. Since the number of pivot variables are 2, the rank of **A** is 2. $\therefore dim(C(\mathbf{A})) = 2 \quad [\because dim(C(\mathbf{A})) = rank(\mathbf{A})]$
rank(**A**) = 1	**False**. Because the rank(**A**) = 2, as the number of pivot variables are 2
$\mathbf{S} = \{(1, 1, 1, 0), (1, 1, 0, 1)\}$ is a basis of $N(\mathbf{A})$	**False**. Let, $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ Consider, $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 0 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ Similarly, $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 0 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ Hence, the given vectors do not form the basis.