Assignment-13

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***Abstract*—In this document, we find whether the given function T from $\mathbf{R}^2$ into $\mathbf{R}^2$ is a linear transformation or not.**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_13

1 Problem Statement

Verify whether $T(x_1, x_2) = (x_1 - x_2, 0)$ is a linear transformation or not.

2 Solution

Let $\mathbf{V}$ and $\mathbf{W}$ be the vector spaces. The function $T : \mathbf{V} \to \mathbf{W}$ is called a linear transformation of $\mathbf{V}$ into $\mathbf{W}$ if the following properties are true for all $\mathbf{u}$ and $\mathbf{v}$ in $\mathbf{V}$ and for any scalar k.

$$T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v}) \tag{2.0.1}$$

$$T(k\mathbf{u}) = kT(\mathbf{u}) \tag{2.0.2}$$

Given,

$$T(x_1, x_2) = (x_1 - x_2, 0) \tag{2.0.3}$$

Suppose we have $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)$ and $\mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2)$, then we have to show that the result of $T(\mathbf{x}+\mathbf{y})$ must be same as $T(\mathbf{x}) + T(\mathbf{y})$. Consider,

$$T(\mathbf{x}+\mathbf{y}) = T(\mathbf{x}_1 + \mathbf{y}_1, \mathbf{x}_2 + \mathbf{y}_2) \tag{2.0.4}$$

$$= (\mathbf{x}_1 + \mathbf{y}_1 - \mathbf{x}_2 - \mathbf{y}_2, 0) \tag{2.0.5}$$

$$= (\mathbf{x}_1 - \mathbf{x}_2, 0) + (\mathbf{y}_1 - \mathbf{y}_2, 0) \tag{2.0.6}$$

$$T(\mathbf{x}+\mathbf{y}) = T(\mathbf{x}) + T(\mathbf{y}) \tag{2.0.7}$$

Also, we need to show that for some constant k, $T(k\mathbf{x}) = kT(\mathbf{x})$. Consider,

$$T(k\mathbf{x}) = T(k\mathbf{x}_1, k\mathbf{x}_2) \tag{2.0.8}$$

$$= (k\mathbf{x}_1 - k\mathbf{x}_2, k.0) \tag{2.0.9}$$

$$= k(\mathbf{x}_1 - \mathbf{x}_2, 0) \tag{2.0.10}$$

$$T(k\mathbf{x}) = kT(\mathbf{x}) \tag{2.0.11}$$

Therefore, the given function T is a linear transformation.