

# Assignment-4

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**Abstract**—In this work, we evaluate the determinant of a matrix.

Download all latex-tikz codes from

[https://github.com/poojah15/EE5609\\_AI20MTECH14003/tree/master/Assignment\\_4](https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_4)

Download the python code from

[https://github.com/poojah15/EE5609\\_AI20MTECH14003/tree/master/Assignment\\_4](https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_4)

## 1 PROBLEM STATEMENT

Evaluate  $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

## 2 THEORY

The determinant of a matrix of order three can be determined by expressing it in terms of second order determinants and is called expansion of a determinant along either a row or column. Consider a determinant of square matrix  $A = [a_{ij}]_{3 \times 3}$

$$i.e., |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad (2.0.1)$$

Now, the expansion of determinant of A that is,  $|A|$  can be written as

$$|A| = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \quad (2.0.2)$$

or

$$|A| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22}) \quad (2.0.3)$$

Generally, the properties of determinants are used while evaluating the determinant. We have used

row/column reduction method and then compute the determinant of a matrix.

## 3 SOLUTION

$$\text{Given, } |A| = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} \quad (3.0.1)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + R_2 + R_3} \begin{vmatrix} 2(x+y) & 2(x+y) & 2(x+y) \\ y & x+y & x \\ x+y & x & y \end{vmatrix} \quad (3.0.2)$$

$$= 2(x+y) \begin{vmatrix} 1 & 1 & 1 \\ y & x+y & x \\ x+y & x & y \end{vmatrix} \quad (3.0.3)$$

$$\xleftrightarrow{\substack{C_2 \leftarrow C_2 - C_1 \\ C_3 \leftarrow C_3 - C_1}} 2(x+y) \begin{vmatrix} 1 & 0 & 0 \\ y & x & x-y \\ x+y & -y & -x \end{vmatrix} \quad (3.0.4)$$

Expanding the determinant from (3.0.4), we get

$$= 2(x+y)(-x^2 - ((-y)(x-y))) \quad (3.0.5)$$

$$= 2(x+y)(-x^2 + xy - y^2) \quad (3.0.6)$$

$$= -2x^3 + 2x^2y - 2xy^2 - 2x^2y + 2xy^2 - 2y^3 \quad (3.0.7)$$

$$= -2(x^3 + y^3) \quad (3.0.8)$$

$$\therefore \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = -2(x^3 + y^3) \quad (3.0.9)$$