Assignment-6

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***Abstract*—In this document, we find the value of k such that the equation represents a pair of straight lines.**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_6

1 Problem Statement

Find the value of k such that $6x^2 + 11xy - 10y^2 + x + 31y + k = 0$ represent pairs of straight lines.

2 Solution

Given,

$$6x^2 + 11xy - 10y^2 + x + 31y + k = 0 \tag{2.0.1}$$

Equating to the general equation of second degree

$$i.e., ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.2}$$

we get,

$$a = 6, b = \frac{11}{2}, c = -10, d = \frac{1}{2}, e = \frac{31}{2}, f = k \tag{2.0.3}$$

Compute the slopes of lines given by the roots of the polynomial $-10m^2 + 11m + 6$

$$i.e., m_i = \frac{-b \pm \sqrt{-\left|\mathbf{V}\right|}}{c}, where \mathbf{V} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{2.0.4}$$

$$\implies m = \frac{\frac{-11}{2} \pm \frac{19}{2}}{-10} \tag{2.0.5}$$

$$\implies m_1 = \frac{-2}{5}, m_2 = \frac{3}{2} \tag{2.0.6}$$

Let the pair of straight lines be given by

$$\mathbf{n}_1^T \mathbf{x} = c_1 \tag{2.0.7}$$

$$\mathbf{n}_2^T \mathbf{x} = c_2 \tag{2.0.8}$$

Here,

$$\mathbf{n}_1 = k_1 \begin{pmatrix} -m_1 \\ 1 \end{pmatrix} = k_1 \begin{pmatrix} \frac{2}{5} \\ 1 \end{pmatrix} \tag{2.0.9}$$

$$\mathbf{n}_2 = k_2 \begin{pmatrix} -m_2 \\ 1 \end{pmatrix} = k_2 \begin{pmatrix} \frac{-3}{2} \\ 1 \end{pmatrix} \tag{2.0.10}$$

We know that,

$$\mathbf{n}_1 * \mathbf{n}_2 = \begin{pmatrix} a \\ 2b \\ c \end{pmatrix} \tag{2.0.11}$$

Substituting (2.0.9) and (2.0.10) in the above equation, we get

$$k_1 \begin{pmatrix} \frac{2}{5} \\ 1 \end{pmatrix} * k_2 \begin{pmatrix} \frac{-3}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \\ -10 \end{pmatrix} \tag{2.0.12}$$

$$\implies k_1 k_2 = -10 \tag{2.0.13}$$

By inspection, we get the values, $k_1 = 5, k_2 = -2$. Substituting the values of k_1 and k_2 in (2.0.9) and (2.0.10) respectively, we get

$$\mathbf{n}_1 = \begin{pmatrix} 2 \\ 5 \end{pmatrix} \tag{2.0.14}$$

$$\mathbf{n}_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{2.0.15}$$

Using Teoplitz matrix representation, the convolution of $\mathbf{n}_1$ with $\mathbf{n}_2$, is as follows:

$$\begin{pmatrix} 2 & 0 & 5 \\ 5 & 2 & 0 \\ 0 & 5 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \\ -10 \end{pmatrix} = \begin{pmatrix} a \\ 2b \\ c \end{pmatrix} \tag{2.0.16}$$

Hence, $\mathbf{n}_1$ and $\mathbf{n}_2$ satisfies (2.0.11). We have,

$$c_2 \mathbf{n}_1 + c_1 \mathbf{n}_2 = -2\mathbf{u} \tag{2.0.17}$$

where

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} \tag{2.0.18}$$

Substituting (2.0.14), (2.0.15) in (2.0.17), we get

$$\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix} \begin{pmatrix} c_2 \\ c_1 \end{pmatrix} = -2 \begin{pmatrix} \frac{1}{2} \\ \frac{31}{2} \end{pmatrix} \tag{2.0.19}$$

Solving for c_1 and c_2, the augmented matrix is,

$$\begin{pmatrix} 2 & 3 & -1 \\ 5 & -2 & -31 \end{pmatrix} \xleftrightarrow[R_2 \leftarrow R_2 - 5R_1]{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & \frac{3}{2} & \frac{-1}{2} \\ 0 & \frac{-19}{2} & \frac{-57}{2} \end{pmatrix} \tag{2.0.20}$$

$$\xleftrightarrow[R_1 \leftarrow R_1 - \frac{3}{2}R_2]{R_2 \leftarrow \frac{R_2}{-19/2}} \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & 3 \end{pmatrix} \tag{2.0.21}$$

Hence we obtain,

$$c_1 = 3, c_2 = -5 \tag{2.0.22}$$

We know that,

$$f = k = c_1 c_2 \tag{2.0.23}$$

$$\implies \boxed{k = -15} \tag{2.0.24}$$

Hence the solution. Using (2.0.7) and (2.0.8), the equation of pair of straight lines is given by,

$$\begin{pmatrix} 2 & 5 \end{pmatrix} \mathbf{x} = 3 \tag{2.0.25}$$

$$\begin{pmatrix} 3 & -2 \end{pmatrix} \mathbf{x} = -5 \tag{2.0.26}$$

Graphically,

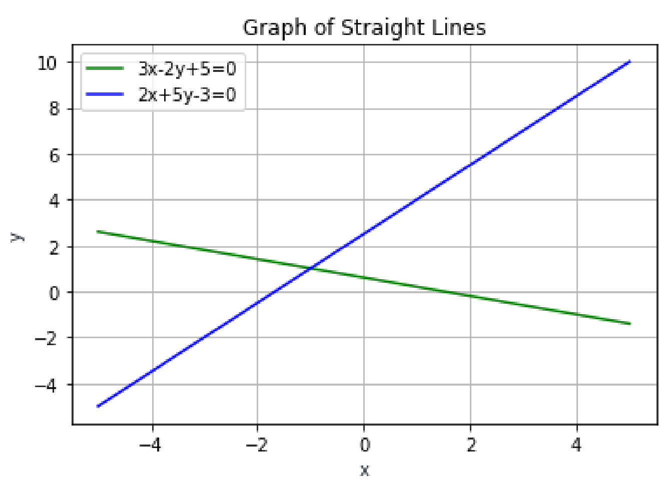

Fig. 1: Plot of two straight lines.