# Assignment-5

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***Abstract*—In this work, we estimate the area of triangle with vector representation.**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_5

## 1 Problem Statement

Prove that the triangles on the same base (or equal bases) and between the same parallels are equal in area.

## 2 Solution

Let ABC and ABD are the given triangles with the same base AB and between the same parallel lines AB and CD.

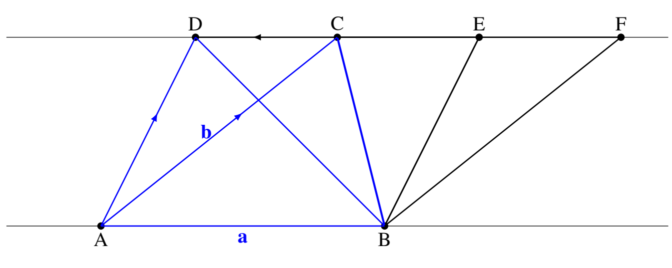


Fig. 1: Triangles on same base

Form a parallelogram, say ABFC, by extending the triangle ABC and ABED by extending the triangle ABD. The parallelograms lie on the same base AB and between the same parallel lines AB and CD.
Let AB = $\mathbf{a}$ and AC = $\mathbf{b}$.
Hence, the area of ▱ABFC is given by

$$(AB \times AC) = \mathbf{a} \times \mathbf{b} \tag{2.0.1}$$

Similarly, the area of ▱ABED is given by

$$\begin{aligned}(AB \times AD) &= (\mathbf{a} \times (AC + CD)) && (2.0.2)\\ &= (\mathbf{a} \times (\mathbf{b} + k\mathbf{a})) && (2.0.3)\\ &= (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times k\mathbf{a}) && (2.0.4)\\ &= (\mathbf{a} \times \mathbf{b}) + k(\mathbf{a} \times \mathbf{a}) && (2.0.5)\\ &= (\mathbf{a} \times \mathbf{b}) \quad [\because \mathbf{a} \times \mathbf{a} = 0] && (2.0.6)\end{aligned}$$

From (2.0.1) and (2.0.6), we can infer that the area of ▱ABFC = area of ▱ABED. We know that the area of triangle is half the area of the parallelogram. Therefore,
Area of △ ABC = $\frac{1}{2}$ (Area of ▱ABFC)

$$= \frac{1}{2}(\mathbf{a} \times \mathbf{b}) \tag{2.0.7}$$

Area of △ ABD = $\frac{1}{2}$ (Area of ▱ABED)

$$= \frac{1}{2}(\mathbf{a} \times \mathbf{b}). \tag{2.0.8}$$

From (2.0.7) and (2.0.8), it is proved that the triangles on the same base and between the same parallels are equal in area.