Assignment-2

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***Abstract*—In this work, we compute the modulus (norm) of the complex numbers.**

Download all python codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_2

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_2

1 Problem Statement

If $\mathbf{z}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathbf{z}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, find $\left\| \frac{\mathbf{z}_1+\mathbf{z}_1+1}{\mathbf{z}_1-\mathbf{z}_2+1} \right\|$

2 Solution

Let us consider $\frac{\mathbf{z}_1+\mathbf{z}_1+1}{\mathbf{z}_1-\mathbf{z}_2+1}$, then

$$\mathbf{z}_1 + \mathbf{z}_1 + 1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.1}$$

$$= \begin{pmatrix} 5 \\ -2 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{z}_1 - \mathbf{z}_2 + 1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.3}$$

$$= \begin{pmatrix} 2 \\ -2 \end{pmatrix} \tag{2.0.4}$$

$$\frac{\mathbf{z}_1 + \mathbf{z}_1 + 1}{\mathbf{z}_1 - \mathbf{z}_2 + 1} = \frac{\begin{pmatrix} 5 \\ -2 \end{pmatrix}}{\begin{pmatrix} 2 \\ -2 \end{pmatrix}} \tag{2.0.5}$$

In general, the complex number $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ can be represented in the form of matrix as:

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} a_1 & -a_2 \\ a_2 & a_1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.6}$$

Therefore using (2.0.6), (2.0.5) can be represented as:

$$\frac{\mathbf{z}_1 + \mathbf{z}_1 + 1}{\mathbf{z}_1 - \mathbf{z}_2 + 1} = \begin{pmatrix} 5 & 2 \\ -2 & 5 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ -2 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.7}$$

$$= \begin{pmatrix} 5 & 2 \\ -2 & 5 \end{pmatrix} \begin{pmatrix} 1/4 & -1/4 \\ 1/4 & 1/4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.8}$$

$$= \begin{pmatrix} 7/4 & -3/4 \\ 3/4 & 7/4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.9}$$

Using (2.0.6) we get,

$$\frac{\mathbf{z}_1 + \mathbf{z}_1 + 1}{\mathbf{z}_1 - \mathbf{z}_2 + 1} = \begin{pmatrix} 7/4 \\ 3/4 \end{pmatrix} \tag{2.0.10}$$

The modulus of a complex number $\begin{pmatrix} a \\ b \end{pmatrix}$ is defined as $\sqrt{a^2 + b^2}$. Therefore,

$$\left\| \frac{\mathbf{z}_1 + \mathbf{z}_1 + 1}{\mathbf{z}_1 - \mathbf{z}_2 + 1} \right\| = \sqrt{(7/4)^2 + (3/4)^2} \tag{2.0.11}$$

$$= \frac{\sqrt{58}}{4} \tag{2.0.12}$$