Assignment-18

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Abstract—**In this document, we explore the properties of determinant and eigenvalues of similar matrices.**

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1 PROBLEM STATEMENT

Let **A** and **B** be $n \times n$ matrices over **C**. Then,

1) **AB** and **BA** always have the same set of eigenvalues.
2) If **AB** and **BA** have the same set of eigenvalues then **AB** = **BA**
3) If $\mathbf{A}^{-1}$ exists, then **AB** and **BA** are similar
4) The rank of **AB** is always the same as the rank of **BA**.

2 ANSWERS FOR DIFFERENT CASES

AB and **BA** always have the same set of eigenvalues.	**True**. Let λ be an eigenvalue of **AB**, and **x** be a corresponding eigenvector. Then $$\mathbf{ABx} = \lambda \mathbf{x}$$ Left-multiplying by **B**: $$\mathbf{B(AB)x} = \mathbf{B}(\lambda\mathbf{x})$$ $$\mathbf{(BA)(B)x} = \lambda(\mathbf{Bx}) \text{ (by associativity of multiplication)}$$ $\implies \lambda$ is an eigenvalue of **BA** with **Bx** as the corresponding eigenvector, assuming **Bx** is not a null vector. If **Bx** is null, then **B** is singular, so that both **AB** and **BA** are singular, and $\lambda = 0$. Since both the products are singular, 0 is an eigenvalue of both.
If **AB** and **BA** have the same set of eigenvalues then **AB** = **BA**	**False**. Let $$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$ then $$\mathbf{AB} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{BA} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$ $\implies$ Similar eigenvalues, but $\mathbf{AB} \neq \mathbf{BA}$
If $\mathbf{A}^{-1}$ exists, then **AB** and **BA** are similar	**True**. Given that $\mathbf{A}^{-1}$ exists and hence, $$\mathbf{AB} = \mathbf{A}^{-1}(\mathbf{AB})\mathbf{A} = (\mathbf{A}^{-1}\mathbf{A})\mathbf{BA} = \mathbf{BA}.$$ Hence, $\mathbf{AB} \simeq \mathbf{BA}$
The rank of **AB** is always the same as the rank of **BA**.	**False**. From the given above **AB** and **BA**, it is noted that the rank(**AB**) = 0 and rank(**BA**)=1. Hence the rank of **AB** need not always be same as rank of **BA**.