

Problem Statement:

To verify whether the lines passing through the given set of points are parallel or not

Solution

Let the lines be parallel and the first two points pass through $n^T x = c_1$ i.e.

$$n^T x_1 = c_1 \Rightarrow x_1^T n = c_1, \quad n^T x_2 = c_2 \Rightarrow x_2^T n = c_2 \quad (1)$$

and the second two points pass through $n^T x = c_2$ Then

$$n^T x_3 = c_3 \Rightarrow x_3^T n = c_3, \quad n^T x_4 = c_4 \Rightarrow x_4^T n = c_4 \quad (2)$$

Putting equations (1) and (2) together, we obtain

$$\begin{pmatrix} x_1^T \\ x_2^T \\ x_3^T \\ x_4^T \end{pmatrix} \vec{n} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} \quad (3)$$

Now if this equation has a solution, then \vec{n} exists and the lines will be parallel.