Assignment-2

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Abstract—In this work, we compute the modulus (norm) of the complex numbers.

The modulus of a complex number $\begin{pmatrix} a \\ b \end{pmatrix}$ is defined as $\sqrt{a^2 + b^2}$. Therefore,

$$\|\mathbf{z}_1 + \mathbf{z}_1 + 1\| = \sqrt{5^2 + (-2)^2} \qquad (2.0.6)$$

$$= \sqrt{29} \qquad (2.0.7)$$

$$\|\mathbf{z}_1 - \mathbf{z}_2 + 1\| = \sqrt{2^2 + (-2)^2} \qquad (2.0.8)$$

$$= \sqrt{8} \qquad (2.0.9)$$

Putting together (2.0.7) and (2.0.9), we have

$$\left\| \frac{\mathbf{z}_1 + \mathbf{z}_1 + 1}{\mathbf{z}_1 - \mathbf{z}_2 + 1} \right\| = \frac{\sqrt{29}}{\sqrt{8}} \qquad (2.0.10)$$

1 Problem Statement

If $\mathbf{z}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, $\mathbf{z}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, find $\left\| \frac{\mathbf{z}_1 + \mathbf{z}_1 + 1}{\mathbf{z}_1 - \mathbf{z}_2 + 1} \right\|$

2 Solution

Let us consider $\frac{\mathbf{z}_1 + \mathbf{z}_1 + 1}{\mathbf{z}_1 - \mathbf{z}_2 + 1}$, then

$$\mathbf{z}_1 + \mathbf{z}_1 + 1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (2.0.1)$$

$$= \begin{pmatrix} 5 \\ -2 \end{pmatrix} \qquad (2.0.2)$$

$$\mathbf{z}_1 - \mathbf{z}_2 + 1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (2.0.3)$$

$$= \begin{pmatrix} 2 \\ -2 \end{pmatrix} \qquad (2.0.4)$$

$$\frac{\mathbf{z}_1 + \mathbf{z}_1 + 1}{\mathbf{z}_1 - \mathbf{z}_2 + 1} = \frac{\begin{pmatrix} 5 \\ -2 \end{pmatrix}}{\begin{pmatrix} 2 \\ -2 \end{pmatrix}} \qquad (2.0.5)$$