Assignment-14

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***Abstract*—In this document, we find the matrix of T in the ordered basis B**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_14

1 Problem Statement

Let T be the linear operator on $\mathbf{R}^2$ defined by

$$T\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}-x_2\\x_1\end{pmatrix} \tag{1.0.1}$$

What is the matrix of T in the ordered basis $\mathbf{B}=\{\alpha_1,\alpha_2\}$, where $\alpha_1=\begin{pmatrix}1\\2\end{pmatrix}$ and $\alpha_2=\begin{pmatrix}1\\-1\end{pmatrix}$?

2 Solution

Let $\mathbf{B'}$ be the standard ordered basis for $\mathbf{R}^2$. Then,

$$T(\alpha'_1)=T\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}=0\alpha_1+1\alpha_2 \tag{2.0.1}$$

$$T(\alpha'_2)=T\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}-1\\0\end{pmatrix}=-1\alpha_1+0\alpha_2 \tag{2.0.2}$$

Hence, the matrix of T in the standard ordered basis $\mathbf{B'}$ is

$$T_{\mathbf{B'}}=\begin{pmatrix}0&1\\-1&0\end{pmatrix} \tag{2.0.3}$$

Given,

$$\alpha_1=\begin{pmatrix}1\\2\end{pmatrix} \tag{2.0.4}$$

$$\alpha_2=\begin{pmatrix}1\\-1\end{pmatrix} \tag{2.0.5}$$

Then,

$$\alpha_1=1\alpha_1+2\alpha_2 \tag{2.0.6}$$

$$\alpha_2=1\alpha_1-1\alpha_2 \tag{2.0.7}$$

So the $\mathbf{P}$ matrix is

$$\mathbf{P}=\begin{pmatrix}1&1\\2&-1\end{pmatrix} \tag{2.0.8}$$

And

$$\mathbf{P}^{-1}=\begin{pmatrix}\frac{1}{3}&\frac{1}{3}\\\frac{2}{3}&-\frac{1}{3}\end{pmatrix} \tag{2.0.9}$$

Hence

$$[T]_{\mathbf{B}}=\mathbf{P}^{-1}[T]_{\mathbf{B'}}\mathbf{P} \tag{2.0.10}$$

$$=\begin{pmatrix}\frac{1}{3}&\frac{1}{3}\\\frac{2}{3}&-\frac{1}{3}\end{pmatrix}\begin{pmatrix}0&1\\-1&0\end{pmatrix}\begin{pmatrix}1&1\\2&-1\end{pmatrix} \tag{2.0.11}$$

$$=\begin{pmatrix}-\frac{1}{3}&\frac{1}{3}\\\frac{1}{3}&\frac{2}{3}\end{pmatrix}\begin{pmatrix}1&1\\2&-1\end{pmatrix} \tag{2.0.12}$$

$$[T]_{\mathbf{B}}=\begin{pmatrix}\frac{1}{3}&-\frac{2}{3}\\\frac{5}{3}&-\frac{1}{3}\end{pmatrix} \tag{2.0.13}$$

Hence, $[T]_{\mathbf{B}}$ is the required matrix for the given ordered basis $\mathbf{B}$.