Problem Statement:
To verify whether the lines passing through the given set of points are parallel or not

Solution-1 *Using the vector representation*
Let $A(a_1, a_2, a_3) = $ A(4, 7, 8) and $B(b_1, b_2, b_3) = $ B(2, 3, 4)
Let $C(c_1, c_2, c_3) = $ C(-1, -2, 1) and $D(d_1, d_2, d_3) = $ D(1, 2, 5)
●Compute the direction vector for a given set of points

$$v_1 = (b_1 - a_1, b_2 - a_2, b_3 - a_3) = (-2, -4, -4) \quad ...(1)$$
$$v_2 = (d_1 - c_1, d_2 - c_2, d_3 - c_3) = (2, 4, 4) \quad ...(2)$$

●Check whether v_1 is scalar multiple of the vector v_2
Here, from (1) and (2), $v_2 = -1.v_1$
Hence, the lines are parallel

Solution-2 *Using the matrix representation and rank basis* Represent the direction vectors in the matrix form:
i.e., $M = [v1v2]^T$

$$M = \begin{bmatrix} -2 & -4 & -4 \\ 2 & 4 & 4 \end{bmatrix}$$

Perform: R2 = R1 + R2 = **0**
Hence, Row reduced echelon matrix is:

$$M = \begin{bmatrix} -2 & -4 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

Here, the rank of the matrix is 1. This implies that the lines are parallel.

Solution-3 *Using the cross product of vectors*
●Compute the cross product of the directional vectors

$$det(M) = \begin{vmatrix} i & j & k \\ -2 & -4 & -4 \\ 2 & 4 & 4 \end{vmatrix}$$

= i(-16 + 16) - j(-8+8) + k(-8+8)
= (0, 0, 0) The zero vector infers that the lines are parallel.