Assignment-16

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***Abstract*—In this document, we demonstrate the usage of rank nullity theorem.**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_16

1 Problem Statement

Let $\mathbf{A}$ be a 4×4 matrix. Suppose that the null space $N(\mathbf{A})$ of $\mathbf{A}$ is

$$\left\{(x, y, z, w) \in \mathbf{R}^4 : x + y + z = 0, x + y + w = 0\right\} \tag{1.0.1}$$

Then which one of the following is correct

1) dim(column space($\mathbf{A}$)) = 1
2) dim(column space($\mathbf{A}$)) = 2
3) rank($\mathbf{A}$) = 1
4) $\mathbf{S} = \{(1, 1, 1, 0), (1, 1, 0, 1)\}$ is a basis of $N(\mathbf{A})$

2 Solution

From the rank nullity theorem we know that,

$$dim(\mathbf{R}) = rank(\mathbf{A}) + dim(N(\mathbf{A})) \tag{2.0.1}$$

The nullspace is given by

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.2}$$

The nullspace is defined as the solution space of two equations $x + y + z = 0$ and $x + y + w = 0$ in 4 variables x, y, z and w. Thus there are 2 free variables and hence we have,

$$dim(N(\mathbf{A})) = 2 \tag{2.0.3}$$

Using (2.0.1), we get

$$rank(\mathbf{A}) = 2 \quad [\because dim(\mathbf{R}) = 4] \tag{2.0.4}$$

Hence we get,

$$dim(C(\mathbf{A})) = 2 \quad [\because dim(C(\mathbf{A})) = rank(\mathbf{A})] \tag{2.0.5}$$