

**Problem Statement:**

To verify whether the lines passing through the given set of points are parallel or not

**Solution-1** *Using the vector representation*

Given the points,  $A = \begin{pmatrix} 4 \\ 7 \\ 8 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ , and  $C = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$ ,  $D = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$

- Compute the direction vector for the given set of points

$$B - A = \begin{pmatrix} -2 \\ -4 \\ -4 \end{pmatrix} \quad (1)$$

$$D - C = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} \quad (2)$$

- Check whether one of the direction vector is the scalar multiple of the other direction vector

Here, from (1) and (2),  $B - A = k(D - C)$ . In this example,  $k = -1$ .

**Hence, the lines are parallel.**

**Solution-2** *Using the matrix representation and rank of a matrix*

Represent the direction vectors in the matrix form and perform row reduction:

$$i.e., M = (B - A \quad D - C)^T$$

$$M = \begin{pmatrix} -2 & -4 & -4 \\ 2 & 4 & 4 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} -2 & -4 & -4 \\ 0 & 0 & 0 \end{pmatrix}$$

Here, the rank of the matrix is 1. This implies that the lines are parallel.

**Solution-3** *Using the cross product of the vectors*

- Compute the cross product of the direction vectors

The cross product of the direction vectors given in (1) and (2) is:

$$\begin{pmatrix} -2 \\ -4 \\ -4 \end{pmatrix} \times \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -16 + 16 \\ -8 + 8 \\ -8 + 8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The zero vector infers that the lines are parallel.