Assignment-6

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Abstract—In this document, we find the value of k such that the equation represents a pair of straight lines.

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1 PROBLEM STATEMENT

Find the value of k such that $6x^2 + 11xy - 10y^2 + x + 31y + k = 0$ represent pairs of straight lines.

2 SOLUTION

Given,

$$6x^2 + 11xy - 10y^2 + x + 31y + k = 0 \quad (2.0.1)$$

From (2.0.1) we get,

$$V = \begin{pmatrix} 6 & \frac{11}{2} \\ \frac{11}{2} & -10 \end{pmatrix} \quad (2.0.2)$$

$$u = \begin{pmatrix} \frac{1}{2} \\ \frac{31}{2} \end{pmatrix} \quad (2.0.3)$$

$$f = k \quad (2.0.4)$$

Compute the slopes of lines given by the roots of the polynomial $-10m^2 + 11m + 6$

$$i.e., m_i = \frac{-b \pm \sqrt{-|V|}}{c} \quad (2.0.5)$$

$$\implies m = \frac{\frac{-11}{2} \pm \frac{19}{2}}{-10} \quad (2.0.6)$$

$$\implies m_1 = \frac{-2}{5}, m_2 = \frac{3}{2} \quad (2.0.7)$$

Let the pair of straight lines be given by

$$n_1^T x = c_1 \quad (2.0.8)$$

$$n_2^T x = c_2 \quad (2.0.9)$$

Here,

$$n_1 = k_1 \begin{pmatrix} -m_1 \\ 1 \end{pmatrix} = k_1 \begin{pmatrix} \frac{2}{5} \\ 1 \end{pmatrix} \quad (2.0.10)$$

$$n_2 = k_2 \begin{pmatrix} -m_2 \\ 1 \end{pmatrix} = k_2 \begin{pmatrix} \frac{-3}{2} \\ 1 \end{pmatrix} \quad (2.0.11)$$

We know that,

$$n_1 * n_2 = \begin{pmatrix} a \\ 2b \\ c \end{pmatrix} \quad (2.0.12)$$

Substituting (2.0.10) and (2.0.11) in the above equation, we get

$$k_1 \begin{pmatrix} \frac{2}{5} \\ 1 \end{pmatrix} * k_2 \begin{pmatrix} \frac{-3}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \\ -10 \end{pmatrix} \quad (2.0.13)$$

$$\implies k_1 k_2 = -10 \quad (2.0.14)$$

By inspection, we get the values, $k_1 = 5, k_2 = -2$. Substituting the values of k_1 and k_2 in (2.0.10) and (2.0.11) respectively, we get

$$n_1 = \begin{pmatrix} 2 \\ 5 \end{pmatrix} \quad (2.0.15)$$

$$n_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (2.0.16)$$

Using Teoplitz matrix representation, the convolution of n_1 with n_2, is as follows:

$$\begin{pmatrix} 2 & 0 & 5 \\ 5 & 2 & 0 \\ 0 & 5 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \\ -10 \end{pmatrix} = \begin{pmatrix} a \\ 2b \\ c \end{pmatrix} \quad (2.0.17)$$

Hence, n_1 and n_2 satisfies (2.0.12). We have,

$$c_2 n_1 + c_1 n_2 = -2u \quad (2.0.18)$$

Substituting (2.0.15), (2.0.16) in (2.0.18), we get

$$\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix} \begin{pmatrix} c_2 \\ c_1 \end{pmatrix} = -2 \begin{pmatrix} \frac{1}{2} \\ \frac{31}{2} \end{pmatrix} \quad (2.0.19)$$

Solving for c_1 and c_2, the augmented matrix is,

$$\begin{pmatrix} 2 & 3 & -1 \\ 5 & -2 & -31 \end{pmatrix} \xleftarrow[R_2 \leftarrow R_2 - 5R_1]{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & \frac{3}{2} & \frac{-1}{2} \\ 0 & \frac{-19}{2} & \frac{-57}{2} \end{pmatrix} \quad (2.0.20)$$

$$\xleftarrow[R_1 \leftarrow R_1 - \frac{3}{2}R_2]{R_2 \leftarrow \frac{R_2}{-19/2}} \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & 3 \end{pmatrix} \quad (2.0.21)$$

Hence we obtain,

$$c_1 = 3, c_2 = -5 \quad (2.0.22)$$

We know that,

$$f = k = c_1 c_2 \quad (2.0.23)$$
$$\implies \boxed{k = -15} \quad (2.0.24)$$

Hence the solution. Using (2.0.8) and (2.0.9), the equation of pair of straight lines is given by,

$$\begin{pmatrix} 2 & 5 \end{pmatrix} \mathbf{x} = 3 \quad (2.0.25)$$
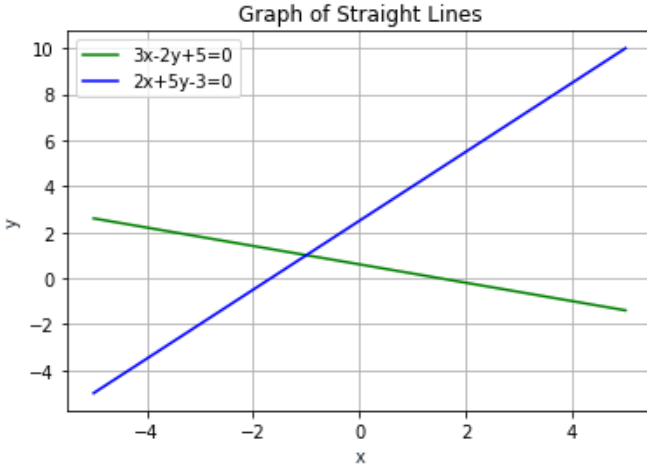$$\begin{pmatrix} 3 & -2 \end{pmatrix} \mathbf{x} = -5 \quad (2.0.26)$$

Graphically,

Fig. 1: Plot of two straight lines.