Assignment-14

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***Abstract*—In this document, we find the matrix of T in the ordered basis B**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_AI20MTECH14003/tree/master/Assignment_14

1 Problem Statement

Let T be the linear operator on $\mathbf{R}^2$ defined by

$$T(x_1, x_2) = (-x_2, x_1) \tag{1.0.1}$$

What is the matrix of T in the ordered basis $\mathbf{B} = \{\alpha_1, \alpha_2\}$, where $\alpha_1 = (1, 2)$ and $\alpha_2 = (1, -1)$?

2 Solution

Applying the transformations on α_1 and α_2 we get,

$$T(\alpha_1) = (-2, 1) \tag{2.0.1}$$

$$T(\alpha_2) = (1, 1) \tag{2.0.2}$$

In order to write T(α_1) and T(α_2) in terms of α_1 and α_2, we row reduce the augmented matrix

$$\begin{pmatrix} 1 & 1 & -2 & 1 \\ 2 & -1 & 1 & 1 \end{pmatrix} \xleftrightarrow[R_2 \leftarrow \frac{-R_2}{3}]{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & 1 & -2 & 1 \\ 0 & 1 & -\frac{5}{3} & \frac{1}{3} \end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{5}{3} & \frac{1}{3} \end{pmatrix} \tag{2.0.4}$$

Hence, the matrix T in ordered basis $\mathbf{B}$ is

$$[T]_{\mathbf{B}} = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} \\ -\frac{5}{3} & \frac{1}{3} \end{pmatrix} \tag{2.0.5}$$