

Assignment-1

Pooja H
AI20MTECH14003

Abstract—This assignment finds whether the lines passing through the given points are parallel or not.

Download all python codes from

svn co https://github.com/poojah15/EE5609_AI20MTECH14003

1 PROBLEM STATEMENT

To show that the line passing through the points $\begin{pmatrix} 4 \\ 7 \\ 8 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ is parallel to the line through the points $\begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$

2 THEORY

Let the lines be parallel and the first two points pass through $\mathbf{n}^T \mathbf{x} = c_1$
i.e.

$$\mathbf{n}^T \mathbf{x}_1 = c_1 \Rightarrow \mathbf{x}_1^T \mathbf{n} = c_1 \quad (2.0.1)$$

$$\mathbf{n}^T \mathbf{x}_2 = c_2 \Rightarrow \mathbf{x}_2^T \mathbf{n} = c_2 \quad (2.0.2)$$

and the second two points pass through $\mathbf{n}^T \mathbf{x} = c_2$
Then

$$\mathbf{n}^T \mathbf{x}_3 = c_3 \Rightarrow \mathbf{x}_3^T \mathbf{n} = c_3 \quad (2.0.3)$$

$$\mathbf{n}^T \mathbf{x}_4 = c_4 \Rightarrow \mathbf{x}_4^T \mathbf{n} = c_4 \quad (2.0.4)$$

Putting all the equations together, we obtain

$$\begin{pmatrix} \mathbf{x}_1^T \\ \mathbf{x}_2^T \\ \mathbf{x}_3^T \\ \mathbf{x}_4^T \end{pmatrix} \mathbf{n} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} \quad (2.0.5)$$

Now if this equation has a solution, then \mathbf{n} exists and the lines will be parallel.

3 EXAMPLE

Given the points, $\mathbf{A} = \begin{pmatrix} 4 \\ 7 \\ 8 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix},$ and $\mathbf{C} =$

$$\begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$$

Applying the row reduction procedure on the coefficient matrix:

$$\begin{aligned} & \begin{pmatrix} 4 & 7 & 8 \\ 2 & 3 & 4 \\ -1 & -2 & 1 \\ 1 & 2 & 5 \end{pmatrix} \xrightarrow[r_3+r_4]{r_1-2r_2} \begin{pmatrix} 4 & 7 & 8 \\ 0 & 1 & 0 \\ -1 & -2 & 1 \\ 0 & 0 & 6 \end{pmatrix} \\ & \xrightarrow[r_3-6r_4]{r_1-7r_2} \begin{pmatrix} 4 & 0 & 8 \\ 0 & 1 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & 6 \end{pmatrix} \xrightarrow[r_1-8r_4]{r_4/6} \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -2 & 1 \\ 0 & 0 & 1 \end{pmatrix} \\ & \xrightarrow[r_3+r_4]{-r_3-2r_2} \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{r_1-4r_3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

Here, the number of non-zero rows are three and hence the rank of the matrix is 3 which implies that the solution exists. Therefore the lines passing through \mathbf{A}, \mathbf{B} and \mathbf{C}, \mathbf{D} are parallel.