Selection of UE-based Virtual Small Cell Base Stations using Affinity Propagation Clustering

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Abstract—5G, will require a number of Key Technological Components to meet its very ambitious goals, including Heterogeneous Networks and Small Cells. The Dense Deployment of Small Cell Base Stations (SBSs) will play a major role as they can be installed in a more targeted manner to relieve traffic in hot spot areas, increase coverage, and spectral efficiency. However, the current deployment of the SBS is static (examples include Femto, Pico, and Nano cells), normally deployed on demand around hotspots. In the scenario of unpredictable crowd movement, these static deployment of small cell can be inefficient with high CAPEX and OPEX.

This paper focuses on the scenario where UE-based Virtual Small Cell Base Stations (UE-VBSs) can be dynamically selected among UEs to deal effectively with the non-stationary, non-uniform distribution of mobile traffic with respect to time and space domain. To implement this concept, we propose an efficient clustering technique (Affinity Propagation Clustering) to select the best set of UE-VBSs to supplement a loaded BS. Simulative evaluation highlights salient features of the technique, as well as its limitations with regard to scalability. Further, we discuss the modification of the algorithm according to our objective scenario by which we are focusing on reducing the message passing procedure to and from only a number of eligible UE-VBSs using the power received by a UE from the eligible UE-VBSs as a parameter. The algorithm is implemented and evaluated in MATLAB and, also validated using NS3 simulator.

Index Terms—5G; Ultra-Dense Network; UE-based Small Cell; clustering.

I. INTRODUCTION

5G, the next generation of cellular network, has set a very ambitious set of goals including higher system spectral efficiency, lower energy consumption, better coverage, very high bit rates, much lower delays, and a much higher number of connected devices. To meet these key technical features of 5G [1], the dense deployment of SBSs [2] is expected to play a major role, as part of HetNets and Small Cells [6]. A SBS can serve as a Base Station (BS) to the **User Equipments (UEs)** in a small neighborhood in its proximity. According to DellOro group [3], Small Cells are the primary driver of growth in the overall 5G RAN market. Normally, the deployment of the SBS is static (examples include Femto, Pico, and Nano

cells). Small Cells reduce the physical separation between the UEs and the BS, which leads to: i) higher data rates and throughput; ii) increased spectral efficiency and higher capacity; iii) reduced energy consumption (both from the UEs and the BSs by allowing UEs to communicate at a shorter range with low signaling overhead); iv) keep interference at an adequate level; and v) enable the minimization of power emitted by both the UEs and the BSs [4].

The manual plug-and-play utility of some types of Small Cells such as Femto cell and Pico cell networks [5], can further boost the on-demand capacity. According to the latest literature [6] and [7], Small Cells are bringing huge amount of capacity and data rates in areas where it is needed the most. However, a major technical concern for 5G networks is currently not efficiently addressed in current Small Cells literature due to the restraint abilities of their static deployment, making them inflexible to cope with the changing (short term) dynamics of mobile traffic. The mobile traffic is not always uniformly and symmetrically distributed in both spatial and time domains due to the dynamic activity and mobility of the human factor. It shifts with the mobile user population activities, such as attending a music concert, or a football match, or with the masses location preferences that change through time.

To cope with the mobile traffic dynamics in Ultra-Dense 5G cellular network [8], a UE-based virtual Small Cell formation as a solution is proposed in [9], that provides for a real time and dynamic massive deployment of virtual Small Cells in an effortless and flexible manner. In UE-based virtual Small Cell, a group of UEs acts as UE-VBSs with BS functionalities to facilitate the networking requirements of other UEs at any time and any location. Hence, this solution removed the restrains of existing Small Cells beyond their static positioning. Note that a work expressing a similar idea is presented in [10]. This work presents the analytical modeling in a UE-based virtual Small Cell formation, with the main focus being on finding optimum Small Cell density for minimizing the communication links to the micro cells and increasing capacity.

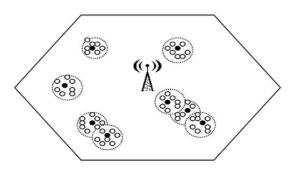


Fig. 1. Network consisting of one BS where active UE-VBSs are the Black Nodes and UEs are the White Nodes

To illustrate our ideas, a simple scenario of a network consisting of one BS and eight (8) active UE-VBSs is depicted in Fig. 1. In this scenario, a BS needs to serve a number of UEs. However the BS does not have the capacity to meet their requirements in terms of number of connections, power and/or bandwidth. To minimize the heavy load traffic problem, it is proposed that the BS it can select a subset of the UEs served by the BS (predefined as capable and willing to offer their services) so as to serve a small neighborhood in their proximity by becoming UE-VBSs. These UE-VBSs are served by the BS and they in turn serve a number of UEs in their proximity.

Another noteworthy example of proposing a similar approach (i.e., using UEs with e-NB functionality), albeit limited to direct Device to Device (D2D) communication within a cell is provided in the patent [12]. More specifically, in [12] a UE is enabled to operate or function temporarily as a personal Femto cell BS. The result creates a Personal Cell (PC) that enables UEs within its nearby physical neighborhood to communicate directly with each other without the need to establish a communication path through an evolved base station.

In the scenario illustrated in Fig. 1, it is assumed that a number of UEs have the required hardware and software specification to become the eligible UE-VBSs. There is Service Level Agreement (SLA) between the service providers and those eligible UE-VBSs, that once activated, the latter will provide services as BS. In return, the active UE-VBSs will get some incentive for their voluntary service. The price of service provided by the active UE-VBSs can be discussed in future. The selection of UE-VBSs for activation among the eligible UE-VBSs is based on the distribution of UEs and, the UEs will associate with the active UE-VBS based on the maximum received power.

Thus, the main objective of this paper is the selection of active UE-VBSs among all the eligible UE-VBSs, and the dynamic network formation using virtual Small Cells. For the dynamic formation of virtual Small Cells (clusters) and the activation of UE-VBSs (cluster centers), affinity propagation clustering [13] technique is adopted. The parameter and the

message passing procedure used by the original affinity propagation clustering is modified according to the above given scenario in Fig. 1. The modified algorithm is implemented and the results are validated with respect to system performance using network simulation.

This paper is organized as follows: section II summarizes the explanation of original affinity propagation clustering. Section III focuses on the discussion of the adoption and usage of affinity propagation clustering in our objective scenario followed by a simulative evaluation and discussion in section IV. Finally, some concluding remarks are presented in section V.

II. AFFINITY PROPAGATION CLUSTERING

The process of clustering involves organizing objects into groups whose members are similar in one way. There are popular clustering methods such as K-Means [14], K-Medoids [15] and K-Center [16] which depend on initial choice of randomly chosen centers. In contrast, affinity propagation clustering technique considers all data points as probable exemplars or candidate cluster centers and gradually identifies clusters [13]. Real valued messages are recursively transmitted until a good set of exemplars with the corresponding clusters emerge. During the process, at any time, the magnitude of each message reflects the current affinity that one data point has for choosing another data point as its cluster center, so this method is called affinity propagation. It can avoid many of the poor solutions caused due to unlucky initializations.

Also, the number of clusters (K) is required in K-Means or K-Medoids or K-Center clustering techniques initially as input. However, in our objective scenario, it is difficult to predict the number of clusters beforehand due to dynamic mobile traffic. Thus, it also leaves no place for random selection of initial cluster centers. Therefore, methods like K-means, K-Medoids, K-center cannot be used in our proposed scenario. Furthermore, the resultant cluster centers of K-Means may not be from the data points. Again, in our scenario, the cluster centers need to be among the data points as we have to select the active UE-VBSs among the eligible UE-VBSs. The affinity propagation clustering technique eradicates these problems as the cluster centers are from the data points and it does not require "K" as an input or any random initialization. Thus, we choose affinity propagation clustering as it does not have any of the above constraints unsuitable for our proposed scenario.

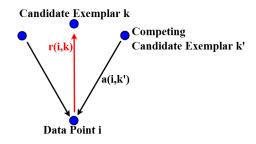


Fig. 2. Sending Responsibility

In the original affinity propagation clustering, for N data points, firstly, the similarity matrix S (i.e., N * N) is computed

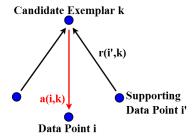


Fig. 3. Sending Availability

which represents the similarity between the data points. For that, the negative euclidean distance is considered between the data points. The diagonal elements of the similarity matrix S is the median among all the distances. This is considered to achieve the optimal number of clusters. The affinity propagation clustering works with three matrices:

- Similarity matrix: s(i, j) represents the negative euclidean distance between i^{th} and j^{th} data point.
- Responsibility matrix: In Fig. 2, r(i, k) reflects how well suited k^{th} data point is to serve as a cluster center or exemplar for i^{th} data point as compared to all other data points k', $k' \neq k$.
- Availability matrix: In Fig. 3, a(i,k) reflects to what degree the k^{th} data point is available as exemplar for the i^{th} data point compared to the other data points (i'), $i' \neq i$.

Initially, the responsibility matrix (R) and availability matrix (A) are considered as zero. In each iteration, the matrix R and A will be updated using Eqs. 1 and 2, respectively. The self-availability a(k,k) is updated according to the Eq. 3.

In the first iteration, the value of A is zero. When some data points are effectively assigned to other exemplars in the subsequent iterations, their availabilities will become negative. These candidate centers will be removed from competition, as these negative availabilities will decrease the effective values of some of the input similarities s(i,k') in the above rule. The resultant matrix E is the addition of the responsibility matrix and the availability matrix, i.e., E = R + A. The exemplars or cluster centers are extracted from this matrix whose diagonal values are positive.

$$r(i,k) \longleftarrow s(i,k) - \max_{(k^{'},k^{'} \neq k)} (a(i,k^{'}) + s(i,k^{'})) \quad (1)$$

$$a(i,k) \longleftarrow \min(0, r(k,k) + \sum_{i', i' \notin (i,k)} \max(0, r(i',k)) \quad (2)$$

$$a(k,k) \longleftarrow sum_{i^{'}s.t.i^{'} \neq (k)} max(0,r(i^{'},k)) \tag{3}$$

The clustering procedure will be terminated after a fixed number of iterations or after the local decisions stay constant for a pre-decided number of iterations. The affinity propagation clustering is used to select the optimal number of UE-VBSs for activation among the eligible and available UE-VBSs. The association of UEs with the UE-VBS is based on the Received Signal Strength from eligible UE-VBSs.

III. SELECTION OF ACTIVE UE-VBSs USING AFFINITY PROPAGATION CLUSTERING

The dynamic network formation of virtual Small Cells is initiated by the BS, dependent on its loading/congestion status. If a BS is congested then it will broadcast a message. The available eligible UE-VBSs in the radio range of BS, will receive the message and will reply back to the BS. For simplicity, it is assumed that the available UE-VBSs are static for a time duration. After acknowledgement to the BS, the available eligible UE-VBSs will start broadcasting a CPICH and their node ID in their coverage area.

There are two possible approaches by which the UEs will provide their context to BS:

- Provide it directly to the BS, with which the UEs already have a connection. However, in order to avoid excessive signaling overhead (since we are considering about ultradense networks), an efficient context requesting approach should be developed.
- Provide it to the available UE-VBSs with the strongest signal strength and then the available UE-VBSs will aggregate the context received from the UEs and send it to the BS. This can avoid excessive signaling overhead and congestion and also reduce interference since the UEs are close to the available UE-VBSs. However, new connections have to be established (either using WiFi, Bluetooth) with each UE and available UE-VBS. This is something that we will study in future work.

The objective of this paper is to select a set of UEs, among the eligible to act as UE-VBS. The aim is to select the optimal number of UE-VBSs for activation considering their spatial distribution which follows the PCP model. Unlike [10], the work in [11] assumed a Poisson Cluster Process (PCP) [111] for the UEs distribution in a micro cell, as UEs can be densely deployed in certain positions, e.g., a football crowd. The PCP model is practically more suitable because it represents the data points that are denser at certain positions just like our objective scenario (e.g for some occasions, such as a going for lunch, the crowd will get together at certain locations). On the other hand, Poisson Point Process (PPP) model [17] represents scattered data points which is not suitable for our objective scenario.

In our objective scenario, the UEs are clustered: each cluster represents a Small Cell and one cluster center is chosen to serve as the UE-VBS and cater to all the UEs in that cluster. The affinity propagation clustering is used to select the optimal number of active UE-VBSs. In this paper, the "Received Signal Strength (RSS)" in Eq. 4 is taken as the similarity parameter between the eligible UE-VBSs and UEs which represent the similarity matrix (S). The UE will associate with the UE-VBS from which it receives the maximum power. In Eq. 4, P_{ij}^r is the received power (dBm) by the i^{th} UE from the j^{th} eligible UE-VBS and, P_j^t is the transmit power of the j^{th} eligible UE-VBS.

$$P_{ij}^r = P_j^t - PL(d_{ij}) \tag{4}$$

$$PL(d_{ij}) = \alpha + \beta 10log_{10}(d_{ij}) + \sigma$$

Where d_{ij} is the distance from the i^{th} UE to the j^{th} UE-VBS and σ is the noise. The values of parameters α and β are channel dependent.

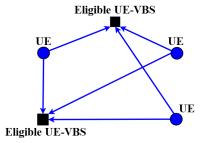


Fig. 4. Sending Responsibility.

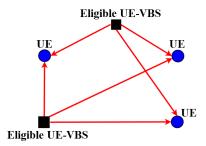


Fig. 5. Sending Availability.

Section II states that affinity propagation clustering gives equal priority to all the data points to be the cluster center or exemplar. In this scenario, all UEs can not be the active UEVBS, only some UEs are considered as eligible UE VBSs. Therefore, the responsibility values, i.e., how well suited the k^{th} UE-VBS is to serve as a cluster center or exemplar for i^{th} UE as compared to all other UE-VBSs (k'), are passed from each UE (non-eligible UE-VBS) to each eligible UE-VBS which is represented in Fig. 4. Similarly, the availability values, i.e., to what degree point the k^{th} UE-VBS is available as exemplar for the i^{th} UE compared to the other UEs (i'), are passed from every eligible UE-VBS to every UE (non-eligible UE-VBS) in Fig. 5.

The UEs will measure the RSS value received from each eligible UE-VBS in their proximity which represents the responsibility matrix (R) of affinity propagation clustering. Each time the matrix R will be updated based on the Eq. 1. For simplicity, it is assumed that the UEs are also in fixed positions. The UE, which is beyond the reach of any cluster heads will directly associate with the BS. We can select the minimum of the distances between the resultant cluster centers that is called "Min". Based on this "Min" value the two different clusters emerge. If the distance between an active UE-VBS and the associated UE is more than the "Min" value then the UE (considered as an outlier) will directly associate with the BS.

TABLE I Values at diagonals of eligible UE-VBSs in E matrix

Iteration	V4	V7	V11	V15	V19
1	-19.0848	-30.0392	-11.1577	-32.6982	-14.4291
2	-24.8935	-28.2268	-7.7063	-34.2332	-14.2790
3	-25.2562	-16.0677	1.8722	-25.0298	-12.3157
4	-25.9278	-7.7418	10.7124	-16.8921	-10.7813
5	-28.4569	-2.0547	15.4047	-11.1993	-9.6086
6	-30.6466	2.5439	15.8634	-8.0535	-9.9782
7	-31.6360	6.1571	14.2830	-7.4657	-11.6367
8	-31.1786	8.6213	12.9646	-8.7255	-14.5683
9	-29.7785	10.6703	13.8895	-10.4981	-17.4560
10	-28.5141	12.5390	16.0572	-12.4296	-20.2011
15	-27.3244	21.8213	23.6779	-21.8324	-31.7589
20	-27.3952	28.8717	24.4507	-30.5640	-33.6138
25	-27.4021	30.6844	24.4905	-35.2674	-33.6729
30	-27.4023	30.8107	24.4923	-36.2697	-33.6747
35	-27.4023	30.8169	24.4923	-36.3771	-33.6747
40	-27.4023	30.8171	24.4923	-36.3852	-33.6747
45	-27.4023	30.8171	24.4923	-36.3857	-33.6747

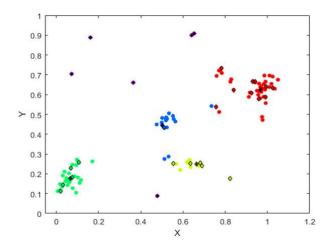


Fig. 6. Selection of active UE-VBSs using affinity propagation clustering.

IV. EVALUATION AND DISCUSSION

Table I shows the convergence behavior of the algorithm where 5 UEs (V4, V7, V11, V15 and V19) have been considered as eligible UE-VBSs out of 20 UEs which is presented in Fig. 6. The termination point in this scenario is the 45th iteration, as the responsibility matrix (R) and availability matrix (A) are identical to their corresponding matrices in the previous iteration. The summation of responsibility and availability matrix is stored in the E-matrix. The eligible UE-VBSs who hold positive values at the diagonals of E-matrix are considered as active UE-VBSs after the final iteration. Indicatively for the given scenario using a laptop the convergence time was 0.040393 seconds. As shown in Table I, the eligible UE-VBS V11 gets a positive value in the 3rd iteration at 0.013691 seconds and eligible UE-VBS V7 gets a positive value in the 6th iteration at 0.015461 seconds. A difference of 0.00177 seconds can be observed between the convergence times of V11 and V7 to get a positive value.

TABLE II CONVERGENCE TIME BASED ON THE ELIGIBLE UE-VBSS.

No. of eligible UE-VBSs	Time taken to converge	Resultant no. of active UE-VBSs)
	(seconds)	active OE-VB3s)
10	0.3139	3
20	0.7810	5
25	1.07210	4
30	2.17446	8
40	1.83497	5
50	6.00591	11
60	3.11173	10
70	5.65878	15

For a preliminary evaluation, the affinity propagation clustering with our required modifications was executed on 100 UEs, using a different number of randomly chosen eligible UE-VBSs from the full set of UEs together with their corresponding generated transmission powers, also chosen randomly. The results are presented in Table II, and we notice no consistent increase in the time taken to converge and the resultant number of active UE-VBSs with respect to the increase in the number of eligible UE-VBSs. Thus, it can be concluded that the number of active UE-VBSs depend on the spatial distribution of the UEs and the eligible UE-VBSs together with their corresponding transmission powers. To make the system model more practical, we have specifically focused on the result when there are 25 eligible UEs to act as UE-VBSs among the 100 UEs served by the BS and presented the scenario in Fig. 6.

Its convergence time and resultant number of active UE-VBSs are highlighted in bold in Table II. The 100 number of UEs are distributed based on the PCP [11] in a grid of 1100 meters. The eligible UE-VBSs are selected randomly, represented with the diamond symbol. After execution of affinity propagation clustering in MATLAB with our required modifications by considering the received signal as input, 4 cluster centers (active UE-VBSs) are generated (marked as "+") and 6 UEs are considered as outliers (marked as "*") as depicted in Fig. 6.

The result obtained from this scenario, i.e., 4 selected active UE-VBSs among the 25 eligible UE-VBSs, which is presented in Fig. 6 is validated using the network simulator NS-3.27 [18]. The simulation is executed and the average values are plotted. Each simulation is executed for 100 seconds. The simulation area is considered as $1,100\times1,100~m^2$. The transmission power of BS is 50 dBm with 20 MHz bandwidth and 0.01 ms delay. For simplicity, it is assumed that the bandwidth of Uplink and Downlink are equal. Every UE uses a the packet size of 1,500 bytes and, the transmission power for each UE is randomly distributed between the range of 20 dBm to 25 dBm.

Figs. 7, 8, 9 present the system performance such as network throughput, average delay and jitter with respect to the number of active UE-VBSs. The popular K-Medoids clustering algorithm is used for choosing the UE-VBSs for

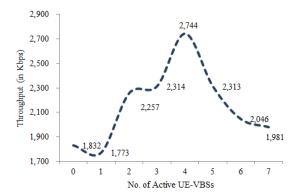


Fig. 7. The network throughput (Kbps) with respect to number of active UE-VBSs. The y-axis indicates the network throughput average delay, while the x-axis indicates the number of active UE-VBSs.

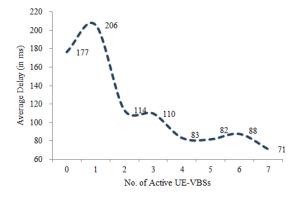


Fig. 8. The average delay (ms) with respect to number of active UE-VBSs. The y-axis indicates the average delay, while the x-axis indicates the number of active UE-VBSs.

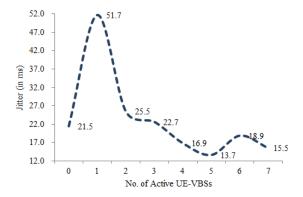


Fig. 9. The jitter (ms) with respect to number of active UE-VBSs. The y-axis indicates the jitter (in ms), while the x-axis indicates the number of active UE-VBSs.

different number of active UE-VBSs (K), i.e., K=0 to 7. When K=0, it represents that there is no UE-VBS and all the UEs are connected to BS.

From Fig. 7, it is observed that the throughput does not increase monotonically. The maximum throughput is achieved when there are 4 UE-VBSs activated and slowly decreases

as the number of active UE-VBSs increases. It justifies the result which is obtained by implementing affinity propagation clustering (with our required modifications) that 4 number of UE-VBSs is the optimal solution. Fig. 7 also rightly points out that when there is only one UE-VBS, the throughput is the least because it has lesser transmission power than the BS and it serves every UE alone. But, as illustrated in Fig. 7, apart from the scenario where there is only one active UE-VBS, the throughput is always more when the number of active UE-VBSs are greater than 1 compared to the scenario when there is no UE-VBS. Thus, we can also conclude that when multiple UE-VBSs serve the UEs, it boosts up the network performance as compared to the scenario where only the BS works.

Figs. 8 and 9 show the average delay and jitter which are obtained with different number of active UE-VBSs. It is observed from Fig. 8 that the delay decreases with the increment in the number of active UE-VBSs and, almost saturates when it reaches beyond 4 active UE-VBSs. Similarly, Fig. 9 presents the jitter with a different number of active UE-VBSs. The jitter decreases with the increment in the number of active UE-VBSs and is towards the least when the number of active UE-VBSs is 4. When the number of the active UE-VBSs is 1, the delay and jitter are at their peak because of the high congestion in the network due to only one UE-VBS whose transmission power is less than a BS.

Based on the results presented in Fig. 7, Fig. 8, and Fig. 9, it can be concluded that the proposed approach with respect to the system performance selects the optimal number of active UE-VBSs which is four (4) for the specific scenario presented in Fig. 6.

V. CONCLUSION

We have used the affinity propagation clustering to dynamically select UE-VBSs from some eligible UE-VBSs to act like BSs so that they can serve the requirements of UEs in that Small Cell (cluster). Affinity propagation clustering generally uses the similarity measure as a parameter but according to our objective scenario, we have adopted the power received by a UE from an eligible UE-VBS as a parameter for the clustering purpose. Also, instead of the general process of passing messages among all the nodes, we have restricted the message passing procedure between the UEs and the eligible UE-VBSs. A major drawback of affinity propagation clustering is that it will not perform well in large-scale data

set. Thus, we aim to divide the data set into multiple groups on whom affinity propagation clustering will work simultaneously which would give rise to better performance. Furthermore, we can formulate the system using the concept of SLA and network pricing for the encouragement of consumers to serve as voluntary UE-VBSs.

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