Q 5 (a)

$$l(\theta) = \log \prod_{i=1}^{m} P(x^{(i)} \mid y^{(i)}; \theta)\, P(y^{(i)})$$

$$= \log \prod_{i=1}^{m} \left[\prod_{j=1}^{n} P(x_j^{(i)} \mid y^{(i)}; \theta \right] P(y^{(i)})$$

$$= \left[\sum_{i=1}^{m} \sum_{j=1}^{m} \log P(x_j^{(i)} \mid y^{(i)}; \theta) \right] + \sum_{i=1}^{m} \log P(y^{(i)})$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{m} \log \left(\binom{P(i)}{x_j(i)} \alpha_{j|y=y(i)}^{x_j^{(i)}} (1-\alpha_{j|y=y(i)})^{P(i)-x_j^{(i)}} \right)$$

$$+ \sum_{i=1}^{m} \log (P_y(i))$$

↳ Prior probability

$$L = \sum_{i=1}^{m} \sum_{j=1}^{m} \log \binom{P(i)}{x_{j(i)}} + \log \alpha_{j|y=y(i)}^{x_j(i)} + \log (1-\alpha_{j|y})^{P(i)-x_j(i)}$$

To take the derivative of the log likelihood we can ignor $\sum_{j=1}^{m}$ since we are taking derivative with respect to $\alpha_{i|y=y(j)}$

$$\frac{\partial L}{\partial \alpha_{i|y=j}} = \sum_{i=1}^{m} 0 + \frac{x_j(i)}{\alpha_{j|y=y(i)}} + \frac{P(i)-x_j(i)}{1-\alpha_{j|y=y(i)}}$$

Equating this derivative to 0 we get

$$\sum_{i=1}^{m} x_j(i)(1-\alpha_{j|y=y(i)}) + P(i)-x_j(i)\,\alpha_{j|y=y(i)} = 0.$$

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$$\alpha = \frac{\sum_{i=1}^{m} x_j(i)}{\sum_{i=1}^{m} x_j\cancel{(i)} + P(i) - x_j\cancel{(i)}}$$

} considering $a = x_j(i)$

$b = P(i) - x_j(i)$

$$= \frac{\sum_{i=1}^{m} x_j(i)}{\sum_{i=1}^{m} P(i)}$$

Hence $\sum a x(1-x) + b x$

$x = \frac{\sum a}{\sum a+b}$.

Hence $\alpha = \frac{\sum_{i=1}^{m} x_j(i)}{\sum_{i=1}^{m} P(i)}$