

ITM(SLS) BARODA UNIVERSITY, VADODARA

**School of Computer Science Engineering
& Technology**

(B.Tech. SEM-VII)

Probability and Statistical Modelling for Computer Science

ASSIGNMENT-3

Define following:

1. Marginal, Conditional probability.
2. Characteristics of the following distribution: Binomial, Poisson, Normal
3. Random variables with its type.

Solve following:

1. From a computer tally based on employer records, the personnel manager of a large manufacturing firm finds that 15 percent of the firm's employees are supervisors and 25 percent of the firm's employees are college graduates. He also discovers that 5 percent are both supervisors and college graduates. Suppose an employee is selected at random from the firm's personnel records, what is the probability of:
(a) selecting a person who is both a college graduate and a supervisor?
(b) selecting a person who is neither a supervisor nor a college graduate?
Ans: 0.05, 0.65
2. An MBA applies for a job in two firms X and Y. The probability of his being selected in firm X is 0.7 and being rejected at Y is 0.5. The probability of at least one of his applications being rejected is 0.6. What is the probability that he will be selected by one of the firms?
Ans: 0.8
3. Suppose your chance of being offered a certain job is 0.45, your probability of getting another job is 0.55, and your probability of being offered both jobs is 0.30. What is the probability that you will be offered at least one of the two jobs?
Ans: 0.70
4. Find the probability of the event of getting a total of less than 12 in the experiment of throwing a die twice.
5. An Economist believes that during periods of high economic growth, the Indian Rupee appreciates with probability 0.70; in periods of moderate economic growth, it appreciates with probability 0.40; and during periods of low economic growth, the Rupee appreciates with probability 0.20. During any period of time the probability of high economic growth is 0.30; the probability of moderate economic growth is 0.50 and the probability of low economic growth is 0.20. Suppose the Rupee value has been appreciating during the present

period. What is the probability that we are experiencing the period of (a) high, (b) moderate, and (c) low, economic growth?

6. Among the sales staff engaged by a company, 60% are males. In terms of their professional qualifications, 70% of males and 50% of females have a degree in marketing. Find the probability that a sales person selected at random will be
(a) a female with degree in marketing
(b) a male without degree in marketing
7. A factory has three units A, B, and C. Unit A produces 50% of its products, and units B and C each produces 25% of the products. The percentage of defective items produced by A, B, and C units are 3%, 2% and 1%, respectively. If an item is selected at random from the total production of the factory is found defective, what is the probability that it is produced by:
(a) Unit A (b) Unit B (c) Unit C

PROBABILITY DISTRIBUTIONS

SECTION-BINOMIAL DISTRIBUTION

Q1. 12% of the tablets produced by a tablet machine are defective. What is the probability that out of a random sample of 20 tablets produced by machine, 5 are defective?

Ans: 0.0567

Q2. The probability that Australia wins a cricket test match against India is given to be $\frac{1}{3}$. If India & Australia play three test matches, what is the probability that
(a) Australia will loose all the three matches (b) Australia will win atleast one match.

Ans: (a) 0.2963 (b) 0.7037

Q3. The probability that an infection is cured by a particular antibiotic drug within five days is 0.75. Suppose four patients are treated by this antibiotic drug, what is the probability that:

(a) No patient is cured (b) Exactly two patients are cured (c) At least two patients are cured

Ans: (a) 0.0039 (b) 0.2109 (c) 0.9492

Q4. Fit a binomial distribution to the following data and compare with actual frequencies.

x	0	1	2	3	4	5
f	2	14	20	34	22	8

Q5. The incidence of an occupational disease in an industry is such that the workers have 20% chance of suffering from it. What is the probability that out of six workers, four or more will come in contact of the disease?

Ans: 0.0170

Q6. A person applies for a personal loan of Rs. 1,50,000 from a nationalized bank to repair his house. The loan offer informed him that over the years, bank has received

about 2920 loan applications per year and that the probability of approval was 0.85.

(a) The person wants to know the average and standard deviation of the number of loans approved per year.

(b) Suppose bank actually received 2654 loan applications per year with an approval probability of 0.82. What are the mean and standard deviation now?

Ans: (a) $\mu = \text{Mean} = 2482$, $\sigma = \text{Standard Deviation} = 19.295$

(b) $\mu = \text{Mean} = 2176.28$, $\sigma = \text{Standard Deviation} = 19.792$

Q7. Out of 320 families with five children each, what percentage would be expected to have (a) Two boys & three girls (b) At least one boy. Assume equal probability for boys & girls.

Ans: (a) $5/16 = 31.25\%$ (b) $31/32 = 97\%$

Q8. A fair coin is tossed six times. Find the probability of getting

(a) Exactly two heads (b) At least four heads (c) No heads (d) At least one head
Ans: (a) $15/64$ (b) $11/32$ (c) $1/64$ (d) $63/64$

SECTION-POISSON DISTRIBUTION

Q9. In a glass manufacturing process, it is known that on the average 1 in every 1000 of the items produced has one or more bubbles. What is the probability that a random sample of 5000 will yield less than two times possessing bubbles?

Ans: 0.0404

Q10. The frequency of accidents per shift in a factory is as shown in the table:

Accidents	0	1	2	3	4
Frequency	190	82	22	5	1

Construct a Poisson distribution table by calculating Expected Frequencies & comparing them with actual frequencies.

Hint: Find the Mean from the above data & and equate to mean of Poisson distribution $\text{Mean} = \lambda$ for applying formula of Poisson distribution for $x=0$ to 4

Q11. For a Poisson distribution, if $P(X=1)$, $P(X=2)$ find $P(X \geq 1)$.

Ans: 0.8647

Q12. A manufacturer who produces medicine bottles, find that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contain (a) No defectives (b) At least two defectives

Ans: (a) 0.6065 & 61 boxes (b) 0.0903 & 10 boxes

Q13. Find the probability that at most 5 defective bolts will be found in a box of 200 bolts if it is known that 2% of such bolts are expected to be defective.

Ans: 0.7844

Q14. An insurance company has discovered that only 0.1% of the population is involved in a certain type of accident every year. If its 1000 policy holders are selected at random from the population, what is probability that not more than 5 of its clients are involved in such accident next year?

Ans: 0.9994

Q15. The number of telephone calls arriving on an internal switch board of an office is 90 per hour. Find the probability that the number of accidents are (a) At least one (b) At most one

Ans: (a) 0.8347 (b) 0.4628

Q16. A new automated production process has had an average of 1.5 breakdowns per day. Because of the cost associated with a breakdown, management is concerned about the possibility of having three or more breakdowns during a day. Assume that the breakdowns occur randomly, that the probability of a breakdown is the same for any two time intervals of equal length and that breakdowns in one period are independent of breakdowns in other periods. What is the probability of having three or more breakdowns during a day?

Ans: 0.1912

SECTION-NORMAL DISTRIBUTION

Q17. A customer accounts of certain department store have an average balance of Rs. 120 & a standard deviation of Rs. 40. Assuming that the account balances are normally distributed, what percentage of account

(a) is over Rs. 150? (b) is between Rs. 100 & Rs. 150? (c) is between Rs. 60 & Rs. 90? Hint: Multiply probability answer by 100 to get percentage

Ans: (a) 22.66% (b) 16.49% (c) 15.98%

Q18. If $\mu=50$ & $\sigma=10$ find

(a) $P(50 \leq X \leq 80)$ (b) $P(60 \leq X \leq 70)$ (c) $P(30 \leq X \leq 40)$ (d) $P(40 \leq X \leq 60)$ Ans:

(a) 0.4987 (b) 0.1359 (b) 0.1359 (d) 0.6826

Q19. An aptitude test for selecting officers in a bank was conducted on 1000 candidates. The average score is 42 and the standard deviation of the scores is 24. Assuming normal distribution for the scores, find

(a) The number of candidates whose scores exceeds 58

(b) The number of candidates whose scores lie between 30 and 66.

Ans: (a) 0.2524 & 252 candidates (b) 0.5328 & 533 candidates

Q20. Time taken by a crew, of a company, to construct a small bridge is a normal variate with mean 400 labour hours and standard deviation of 100 labour hours. What is the probability that the bridge gets constructed between 350 to 450 labour hours?

Ans: 0.3830

Q21. 1000 light bulbs with a mean life of 120 days are installed in a new factory and their length of life is normally distributed with standard deviation of 20 days. How many bulbs will expire in less than 90 days?

Ans: 67 bulbs

Q22. If a sample of 120 tablets in a factory, the mean and standard deviation of active ingredient content were 11.35 and 3.03 respectively, find the percentage of tablets having active ingredient content between 9 and 17 in the whole sample, assuming that active ingredient content is normally distributed.

Ans: 0.7509 & 90 tablets, 75%

Q24. In a normal distribution, 31% of the items are under 45 and 8% of the items are over 64. Find the mean and standard deviation.

Ans: Mean=49.95 & Standard deviation=10

Q26. Convert the following binomial distribution problems to normal distribution problems and solve:

a. $P(x \leq 16 | n = 30 \text{ and } p = .70)$

b. $P(10 < x \leq 20) | n = 25 \text{ and } p = .50)$

c. $P(x = 22 | n = 40 \text{ and } p = .60)$

d. $P(x > 14 | n = 16 \text{ and } p = .45)$