Mackey-Glass Time Series Prediction using LMS algorithm

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***Abstract*—In this report, we investigate the predictability of the Mackey Glass Time Series using Least Mean Square Algorithm. After inspection of Mackey Glass equations, prediction using Least Mean Square (LMS) Algorithm is explored.The Mackey-Glass Time Series generates 5000 data points which is split in 80:20 train to test ratio.The training set helps to learn the weights which is used later on the testing set to predict the series.[Mur12]**

***Index Terms*—Mackey-Glass Time Series,Least Mean Square.**

I. Introduction

Data is a continually expanding progression of numerical information. Every information can be represented as a signal which in turn can be numerically represented. The prediction of future evolution of these numbers is one of the main topics of discussion in the scientific community.Signals in nature are often analog and continuous. However such signals can be elaborated and represented as a period arrangement of numerical data through the sampling technique, which transform analog flows in digital ones.[Mur12]

Prediction consists in taking advantage of the time-series history to forecast the future values in the series. Since nature is non-linear by definition, there have been efforts and attempts to create efficient algorithms able to manage and predict non-linear data mappings. Several algorithms have been developed always taking into account a fair trade-off between performance and cost.The prediction in the field of stock market, weather forecast is highly demanded in the market now.[Mur12]

In the following discussion of Least Mean Square algorithm on Mackey-Glass time series' prediction is described. This time-series, which well represents a chaotic and periodic dynamics, was initially used to model physiological control systems such as electrolytes, oxygen, and glucose in the blood. Mackey Glass time-series used in the experiments is generated using a non-linear time-delay differential equation, discretized for time step equal to 1. Prediction is based on past inputs, but since time is constantly moving forward, it's clear that past inputs are infinite. However the predictor input space must be finite, hence the input's history should be truncated to a certain finite number of previous samples. The choice of the time-window size is a critical step in the performance of the prediction, and determines its memory on the past events.[Mur12]

Least Mean Square is a well-known and robust algorithm based on the minimization of the mean square error. It is a stochastic gradient descent method; in fact it can be derived by using the instantaneous gradient of the cost function. Applying the method of steepest descent we can formulate the update step of the LMS algorithm by the weight formula.

II. Model Parameters

Mackey-Glass Time series is generated using Eq(1) where b, a, τ and n are real numbers ,and x_i represents the value of x at (t-τ). Depending on values of the parameters, the equation results in a range of periodic and chaotic dynamics.[MM09]

In this paper, the respective parameters are 0.1,0.2,20 and 10 respectively with an initial condition of 1.2.It is then numerically solved using Runge-Kutta method for 4^{th} order using Eq(2) .

$$dx/dt = ax_\tau/(1 + x_\tau^n) - bx \tag{1}$$

$$x(t + \Delta t) = x(t) + k1/6 + k2/3 + k3/6 + k4/6 \tag{2}$$

III. Methods

The Least Mean Squares Algorithm used for prediction uses 5 taps with recurrent ARMA modelling with forced desired input after defined time steps.The weights are initialized with random values,these are updated later during the training phase using the formula in Eq(3). The updated weights are then utilized for the testing stage.

$$W(i + 1) = W(i) + \eta x(i)e(i) \tag{3}$$

where ,

$$e(i) = x(i) - Y(i) \tag{4}$$

where ,

x(i) = ith value in the training set ,
Y(i) = Predicted Output,
η = learning rate ,
W = weights vector

The Minimum Square Error is calculated using Eq(5) for each data point during training and test stages.

$$MSE = e(i)^2 \quad (5)$$

where, n = number of samples

IV. Results

In this section,time series prediction is inspected by varying η and number of taps in the LMS prediction model.

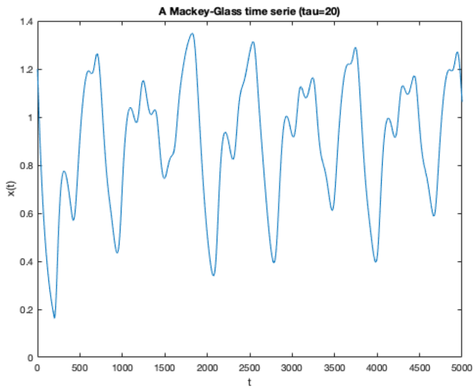
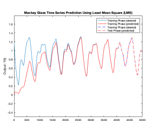
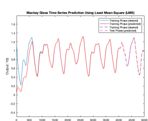
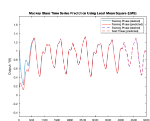


Fig. 1: Mackey-Glass Time Series with b=0.1,a=0.2,τ=20

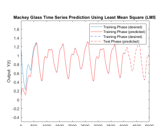



(a) Prediction using M=1 (b) Prediction using M=5

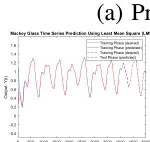
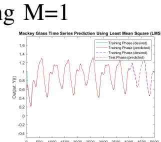


(c) Prediction using M=10

Fig. 2: η=1e^{-3}



(a) Prediction using M=1

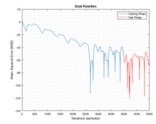
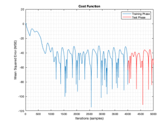



(b) Prediction using M=5 (c) Prediction using M=10

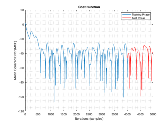
Fig. 3: η=5e^{-3}

The code to obtain these graphs is published online[Kha18; Coc09].

From the graphs and metrics obtained, it's quite obvious

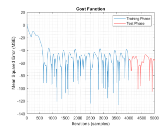



(a) MSE using M=1 (b) MSE using M=5

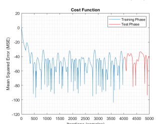
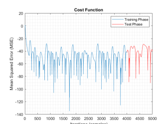


(c) MSE using M=10

Fig. 4: η=1e^{-3}



(a) MSE using M=1

(b) MSE using M=5 (c) MSE using M=10

Fig. 5: η=5e^{-3}

M	1	5	10
MSE Training(in dB)	-12.029	-16.648	-18.874
MSE Testing(in dB)	-54.836	-41.619	-35.149
Training Time(in secs)	0.01618	0.02536	0.01659
Testing Time (in secs)	0.00492	0.00886	0.00855

(a) η=1e^{-3}

M	1	5	10
MSE Training(in dB)	-18.986	-23.075	-24.269
MSE Testing(in dB)	-52.904	-39.305	-35.281
Training Time(in secs)	0.01418	0.01123	0.01211
Testing Time (in secs)	0.00359	0.00362	0.0430

(b) η=5e^{-3}

TABLE I: Metrics for learning rates of 1e^{-3} and 5e^{-3}

that η=1e^{-3} is not recommended as initial values are either higher or lower than the desired value. Thus, affecting the MSE during training and testing.

On the other hand, with a learning rate 5e^{-3}, the predicted and desired time series are overlapping each other. Also, the MSE of training and testing are close except when M = 1. When M=1, there are only two weights, which is susceptible to error.

V. Conclusions

In this paper, Least Mean Square algorithm with η=5e^{-3} and 10 taps is used. The above model results in an MSE of -21.082dB and -27.435dB during training and testing respectively. The primary objective of this work is restricted to predict value of the Mackey Glass time series with the least error. In future, parameters such as b and a can be determined from the expected values such that comparison between the actual and predicted parameters can be performed. According to [Mur12] variants of LMS such as KLMS and NLMS-FL can be implemented for this work to find the shortcomings of LMS while exploring KLMS and NLMS-FL. KLMS which stands for Kernel Least Mean Square was introduced in order to deal with the high non-linearity present in natural phenomena on mapping between input and output. The main idea is to map input data with high dimensional feature space, using a transformation function. This method is the simplest among others. The other method is NLMS-FL which stands for Normalized Least Mean Square Functional Links. It is characterized by a normalization factor which scales the overall error signal. This is the most innovative method used for prediction. KLMS is still a good solution for nonlinear time series at least better than LMS. NLMS-FL is said to be the best performer as there is a great difference in performance at low noise. Thus, resulting in higher speed of adaptation and rapid convergence of the learning curve.

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