CAPE LAB ASSIGNMENT 5

GROUP - E

19CH10047 Sanskar Singh
19CH10030 Pooja Sharma
19CH10071 Shikha Kumari
19CH10049 Shalini Bansal
19CH10016 Dipjoy Basak

(All Group members contributed equally)

PART 1: Stiff Ode

PROBLEM STATEMENT :

1. Stiff ODE

Solve the following system of Ordinary Differential Equations (van der Pol ODE) using both ode45 and ode15s.

$$dy_1/dt = y_2 \qquad y_1(0) = 2$$
$$dy_2/dt = 1000(1 - y_1^2)y_2 - y_1 \qquad y_2(0) = 0$$
$$tspan = [0, 3000]$$

(a) Comment on the efficiency of `ode45` and `ode15s` in solving this system.

(b) Plot y_1 and y_2 as a function of time.

MATLAB CODE

```
function dydt = stiffode(t,y)
%solving using ode45 and ode15s
%system of odes
dydt = [y(2); (1000*y(2).*(1-y(1).^2))-y(1)];




%initial values
y0 = [2;0];
tSpan = [0 3000];
%%% solving using ode45
[tSol, ySol] = ode45(@(t,y) stiffode(t,y), tSpan, y0);
%%%Plotting y1 vs time
plot(tSol, ySol(:,1), '--r');
xlabel('time');
ylabel('y1');
%%%Plotting y2 vs time
plot(tSol, ySol(:,2), '--b');
xlabel('time');
ylabel('y2');

%%% solving using ode15s
[tSol, ySol] = ode15s(@(t,y) stiffode(t,y), tSpan, y0);
```

Efficiency :

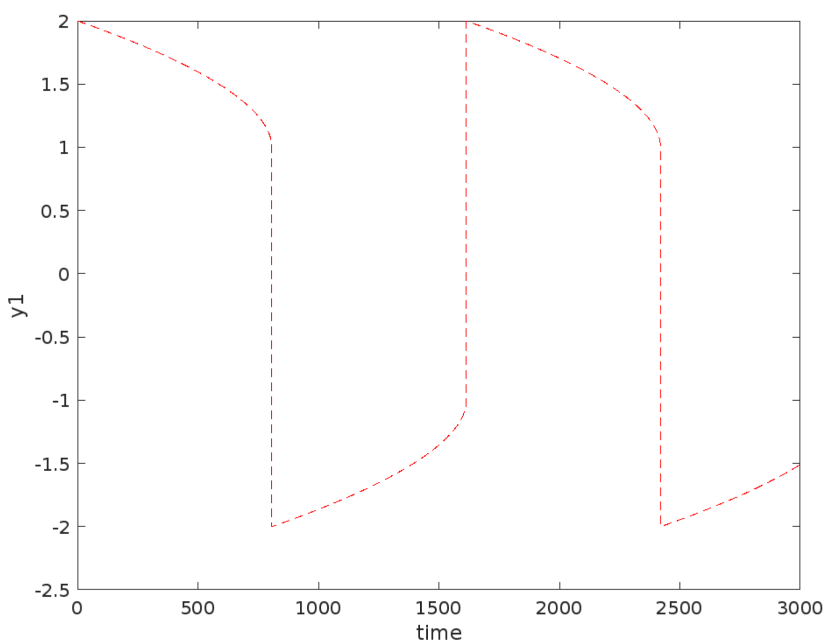
When we are solving the given equations using the ode45 in MATLAB it takes very large number of steps (6735953) to solve the eqns, this is because it is taking very small steps in the order of 10^{-4} at every stage because ode45 is an explicit method and it becomes unstable if we take larger steps

While when we are solving it using ode15s it takes 592 steps, as it is an implicit method there is no restriction on the step sizes, initially, we take small steps but when the one value stabilizes we can take large steps.

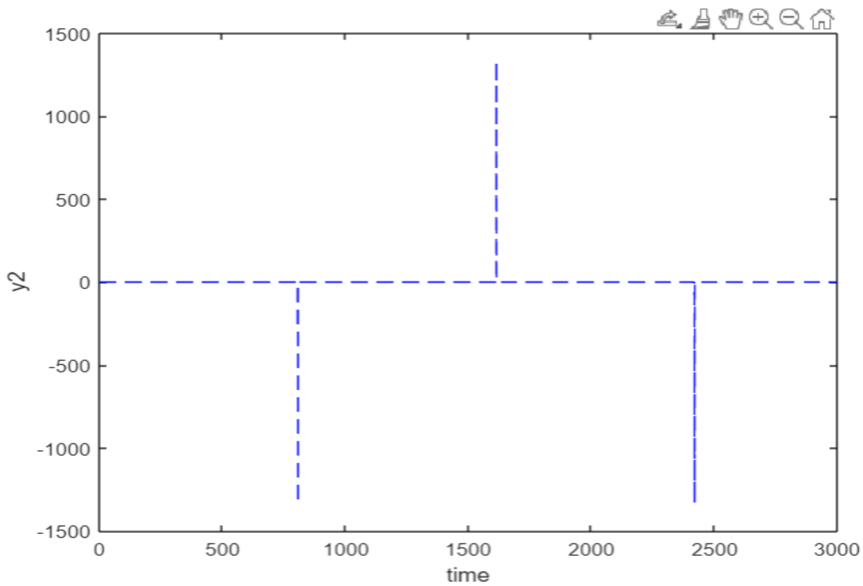
So, we can say ode15s is more efficient in solving stiff odes.

PLOTS:

Plotting y1 vs time



Plotting y2 vs time



PART 2: Modelling and Simulation of Binary Distillation Column

PROBLEM STATEMENT:

2. Modelling and Simulation of a Binary Distillation Column

Consider the binary distillation column as shown in the figure. Liquid hold-up at each tray is constant and assume constant molar overflow in the column. The following data are given.

Nomenclature: Consider Condenser as Tray-1, Reboiler as Tray-10

Number of trays = 10, Relative volatility (α) is constant

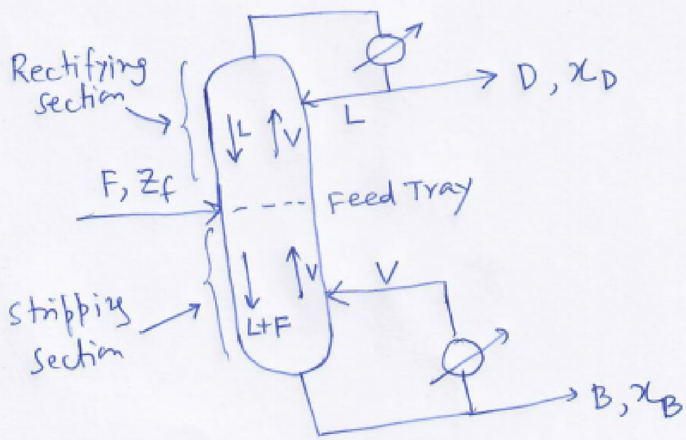
$F = 50$ mol/s, $D = 25$ mol/s, $z_f = 0.5$, $V = 60$ mol/s

Hold-up: For tray, $M = 400$ mol, Condenser, $M_C = 4000$ mol, Reboiler, $M_R = 4000$ mol

Initial condition: Compositions in all trays, $x_f = 0.5$

(a) Solve the dynamic equations to find steady state compositions when feed enters at Tray-4, Tray-5, and Tray-6. Assume $\alpha = 2.5$

(b) Consider feed enters at Tray-5. Find the steady state compositions at each tray for $\alpha =$ 1.5, 3, and 4. Explain the effect of magnitude of relative volatility on separation.



$$y_i = \frac{\alpha x_i}{1+(\alpha-1)x_i}$$

EQUATIONS:

Assumptions:

1. Liquid hold-up at each tray is constant
2. constant molar overflow in the column
3. Relative volatility (α) is constant
4. Number of trays = 10, Tray 1 - condenser
   Tray 10 - Reboiler

$F = 50$ mol/s

$D = 25$ mol/s

$V = 60$ mol/s

$z_f = 0.5$

Holdups

$M = 400$ mol

$M_C = 4000$ mol

$M_R = 4000$ mol

$\alpha = 2.5$

Initial condition $(t=0)$ $x_f = 0.5$

for ith tray:

$$\frac{d}{dt}(M x_i) = V_{i+1} y_{i+1} + L_{i-1} x_{i-1} - V_i y_i - L_i x_i$$

Condenser:

$$\frac{d}{dt}(M_c x_D) = V y_2 - D x_D - L x_D$$

Rectifying section:

$$\frac{d}{dt}(M_i x_i) = (V_{i+1} y_{i+1} + L_{i-1} x_{i-1}) - (V_i y_i + L_i x_i)$$

Feed Tray:

$$\frac{d}{dt}(M_f z_f) = (F z_f + L_{i-1} x_{i-1} + V_{i+1} y_{i+1}) - (V_i y_i + (L+F)_i x_i)$$

Stipping section

$$\frac{d}{dt}(M_i x_i) = ((L+F)_{i-1} x_{i-1}) + V_{i+1} y_{i+1}) - (V_i y_i + (L+F)_i x_i)$$

Reboiler:

$$\frac{d}{dt}(M_R x_R) = x_a (L+F) - y_r V - x_r (F-D)$$

Code MATLAB:

```
function fval = binarydis(X)
V = 60;
F = 50;
D = 25;
zf= 0.5;
M = 400;
Mr = 4000;
Mc = 4000;
alpha = 2.5;% Can be changed accordingly(for b part)
L = V-D;
B = F-D;
feedTray = 4; % Can be changed accordingly
Y = alpha*(X)./(1+(alpha-1)*X);
%condenser eqn
fval(1) = (V*Y(2)-D*X(1)-L*X(1))/Mc;
%Rectifying Section
for i = 2:(feedTray-1)
   fval(i) = (V*Y(i+1)+L*X(i-1) - V*Y(i)-L*X(i))/M;
end
%feed tray
fval(feedTray) = (F*zf + L*X(feedTray-1) +
V*Y(feedTray+1)-V*Y(feedTray)-(L+F)*X(feedTray))/M;
%Stripping Section
for i = (feedTray+1):9
```

```
   fval(i) = ((L+F)*X(i-1)+V*Y(i+1)-V*Y(i)-(L+F)*X(i))/M;
end
%reboiler
fval(10) = ((L+F)*X(9) - V*Y(10) - B*X(10))/Mr;
end


X0 =[0.5;0.5;0.5;0.5;0.5;0.5;0.5;0.5;0.5;0.5];
Xsol = fsolve(@(X) binarydis(X), X0);
```

For Feed Tray = 4

```
Xsol =

    0.8931
    0.7697
    0.6474
    0.5452
    0.5168
    0.4682
    0.3937
    0.2966
    0.1940
    0.1069
```

For Feed Tray 5 :

```
Xsol =

    0.9057
    0.7935
    0.6778
    0.5764
    0.4992
    0.4458
    0.3674
    0.2707
    0.1735
    0.0943
```

For Feed Tray 6:

```
Xsol =

    0.9055
    0.7931
    0.6773
    0.5758
    0.4987
    0.4461
    0.3679
    0.2711
    0.1738
    0.0945
```

2b)

For alpha = 1.5

```
Xsol =

    0.7030
    0.6121
    0.5532
    0.5164
    0.4939
    0.4752
    0.4485
    0.4112
    0.3611
    0.2970
```

For alpha = 3

```
Xsol =

    0.9487
    0.8604
    0.7442
    0.6184
    0.5081
    0.4357
    0.3295
    0.2111
    0.1136
    0.0513
```

For alpha = 4

```
Xsol =

    0.9835
    0.9372
    0.8460
    0.7002
    0.5295
    0.4198
    0.2629
    0.1259
    0.0492
    0.0165
```

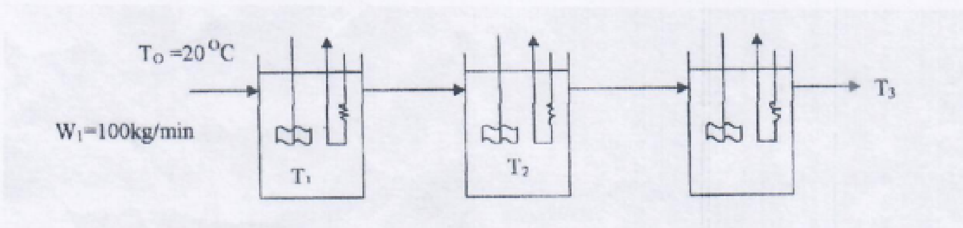
The larger the value of α, the greater the degree of separability, i.e. the easier the separation.

PART 3: Dynamic Simulation of Stirred Tank Heater

PROBLEM STATEMENT:

3) Dynamic Simulation of stirred tank heater

There are three stirred tanks with internal heating arrangement connected in series. Each tank is initially filled with 100kg of oil at 50°C. Saturated steam at 250°C condenses inside heating coil immersed in each tank. The oil fed into first tank at the rate of 1OOkg/min and overflows into 2nd and 3rd tank at same flow rate. The temperature of oil fed to first tank is 20 °c. The CP of the oil is 2KJ/kg^{0}c. For each tank, rate of heat transfer may be assumed as UA= 10 K.J/min°c.



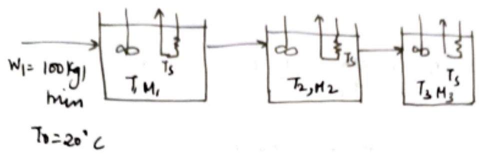
a. Develop the dynamic model equations.

b. Determine the steady state temperature in all 3 tanks.

c. What time interval will be required for T3 to reach 99% of steady state value during start up?

d. Show the "step response" of all the temperature till the new steady state, if the T_0 is suddenly increased by 50% from its original value.

3a,b)



Initial value of T_1, T_2, T_3 in tank

$T_1 = 50°C$, $T_2 = 50°C$ $T_3 = 50°C$

given, $UA = 10$ kJ/min°C

$C_p = 2$ kJ/kg°C

$W = 100$ kg/min

$T_s = 250°C$

Mass present inside the tank is constant

$M_1 = M_2 = M_3 = M$

from Energy Balance,

$$\frac{d}{dt}(MC_pT_1) = WC_p(T_0 - T_1) + UA(T_s - T_1)$$

$$\frac{d}{dt}(MC_pT_2) = WC_p(T_1 - T_2) + UA(T_s - T_2)$$

$$\frac{d}{dt}(MC_pT_3) = WC_p(T_2 - T_3) + UA(T_s - T_3)$$

(a) The dynamic Model Equations are :-

$$\frac{dT_1}{dt} = \frac{W}{M}(T_0 - T_1) + \frac{UA}{MC_p}(T_s - T_1) \quad — (1)$$

$$\frac{dT_2}{dt} = \frac{W}{M}(T_1 - T_2) + \frac{UA}{MC_p}(T_s - T_2) \quad — (2)$$

$$\frac{dT_3}{dt} = \frac{W}{M}(T_2 - T_3) + \frac{UA}{MC_p}(T_s - T_3) \quad — (3)$$

(b) At steady state $\frac{d}{dt} \to 0$

$$\frac{dT_1}{dt} = \frac{dT_2}{dt} = \frac{dT_3}{dt} = 0$$

f_1: $WC_p(T_0 - T_1) + UA(T_s - T_1) = 0$

f_2: $WC_p(T_1 - T_2) + UA(T_s - T_2) = 0$

f_3: $WC_p(T_2 - T_3) + UA(T_s - T_3) = 0$

solving the above eq^ns using fsolve in MATLAB.

Code in MATLAB:

```
function fval = stirrtank(T)
% Given values
T0 = 20;
UA = 10;
Cp = 2;
W = 100;
Ts = 250;
% Equations
fval(1,1) = W*Cp*(T0 -T(1)) + UA*(Ts-T(1));
fval(2,1) = W*Cp*(T(1)-T(2)) + UA*(Ts-T(2));
fval(3,1) = W*Cp*(T(2)-T(3)) + UA*(Ts-T(3));
```

```
Command Window :
to = [50;50;50];
tsol = fsolve(@(T) stirrtank(T), to)
```

Ans:

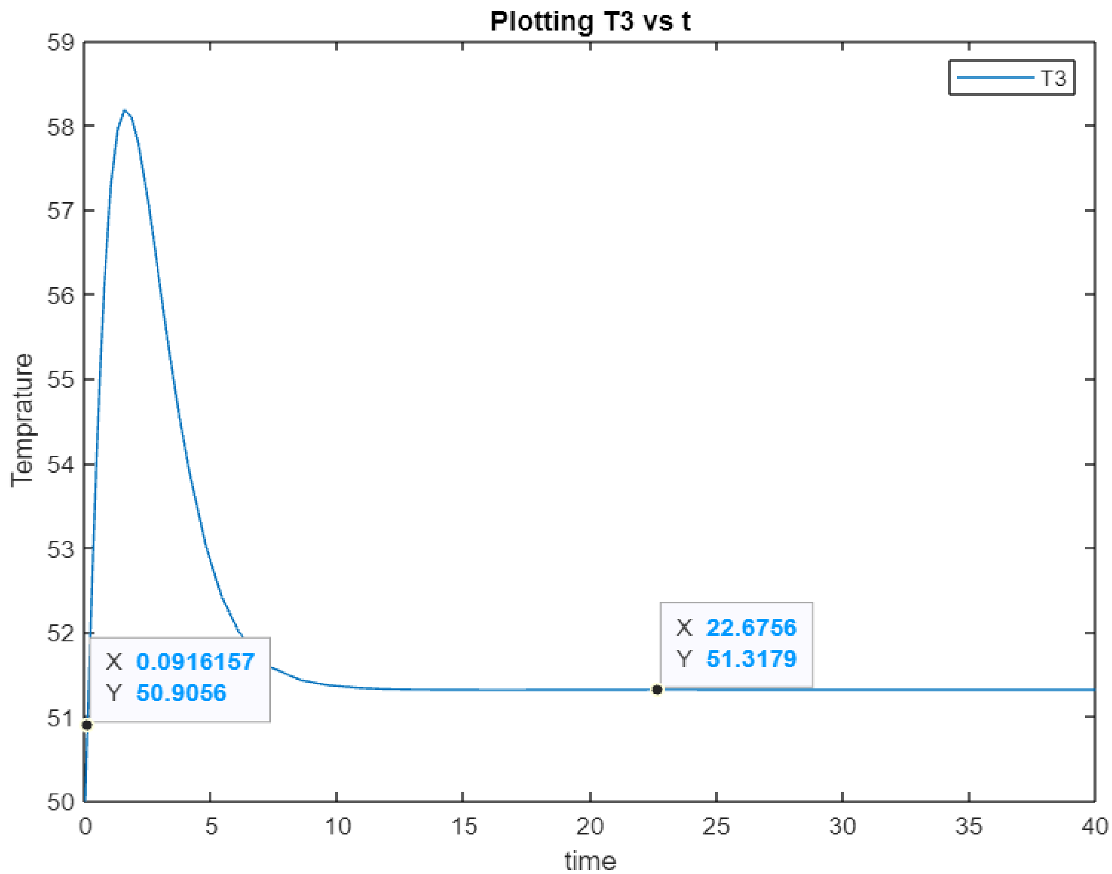
```
  T1 =30.9524
  T2 =41.3832
  T3 =51.3174
```

3c)

MATLAB Code:

```
function dT_dt = stirrtank2(t, T)
% Given values
T0 = 20;
UA = 10;
Cp = 2;
W = 100;
Ts = 250;
M = 100;
dT_dt = [W*(T0 -T(1))/M + UA*(Ts-T(1))/(M*Cp);
        W*(T(1)-T(2))/M + UA*(Ts-T(2))/(M*Cp);
        W*(T(2)-T(3))/M + UA*(Ts-T(3))/(M*Cp);];
end


tSpan = [0 40];
To = [50;50;50];
[t, T] = ode15s(@stirrtank2, tSpan, To);
plot(t,T(:,3));
xlabel('time');
ylabel('Temprature');
legend('T1');
title('Plotting T3 vs t');
```



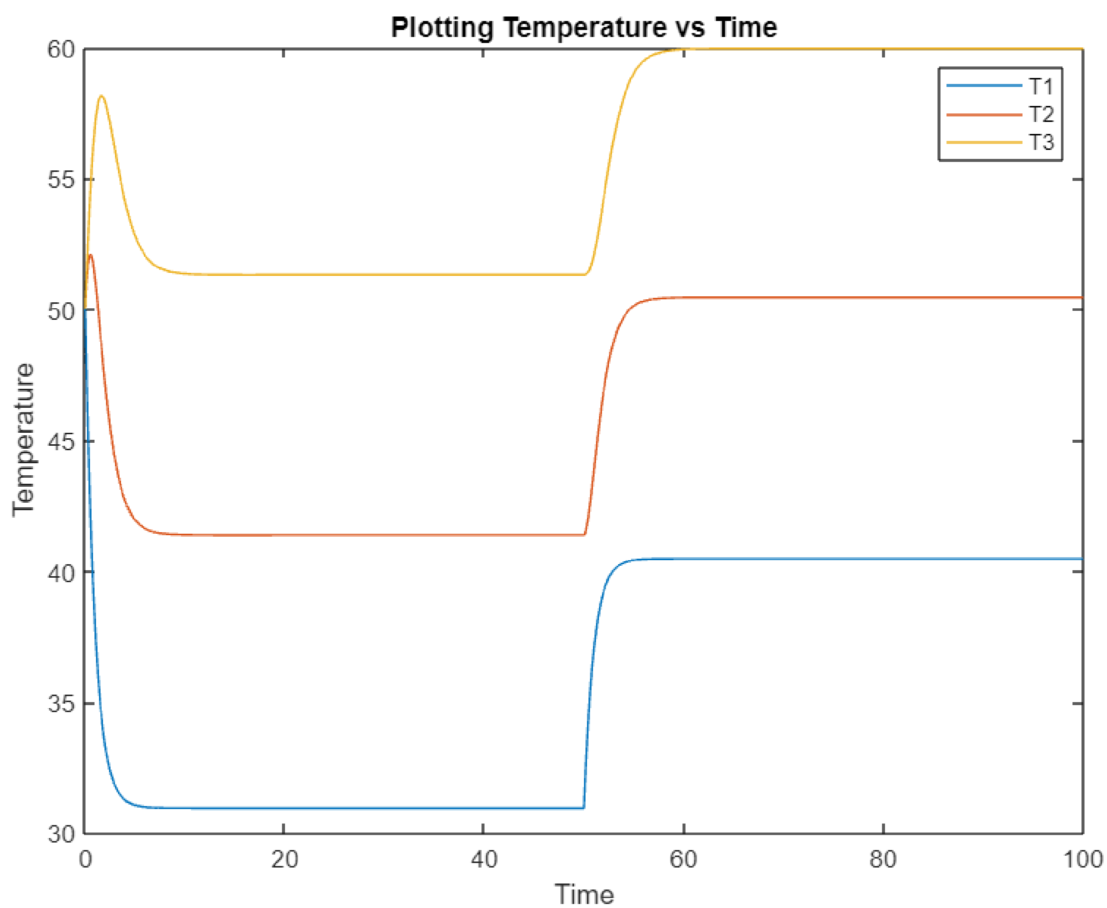
Steady State value of T3 = 51.3179 ℃
99 % of Steady State Value = 50.8 ℃
Time Interval of approx [0, 0.08] is required to reach 99% of steady state value as we can see from the above plotted graph.

3d)
MATLAB Code:

```
function dT_dt = stepresponse(t, T)
% Given values
T0 = 20;
UA = 10;
Cp = 2;
W = 100;
Ts = 250;
M = 100;
```

```
if (t>50)
   T0 = T0 +0.5*T0;
end
dT_dt = [W*(T0 -T(1))/M + UA*(Ts-T(1))/(M*Cp);
         W*(T(1)-T(2))/M + UA*(Ts-T(2))/(M*Cp);
         W*(T(2)-T(3))/M + UA*(Ts-T(3))/(M*Cp);];
end
```

```
tSpan = [0 100];
To = [50;50;50];
[t, T] = ode15s(@stepresponse, tSpan, To);
plot(t,T);
xlabel('Time');
ylabel('Temperature')
title('Plotting Temperature vs Time')
legend('T1', 'T2', 'T3')
```



New Steady State values we can see from the graph and from the workspace data:
T1 = 40.4762℃
T2 = 50.4535℃
T3 = 59.9557℃