**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:**

**Given:** *μ* = 45, *σ* = 8

P = probability that service manager can meet his commitment.

q = probability that service manager cannot meet his commitment

Car is dropped for 1 hr and work starts after 10 min. Thus time for successful delivery of commitment is 60 -10 = 50min. Delivery after this time means they cannot meet the commitment

So, q = P(x>=50)

=P((x - *μ)/σ >= (50* - 45)/8)

=P( Z *>= 0.625 )*

*=1* - P( Z *<= 0.625 ) {as Z table and python gives less than type probability (<=) I.e left*

*Hand side probability. So we convert >= into <= }*

***{python code : import scipy.stats as st***

***st.norm.cdf(0.625) =***  0.7340144709512995 ***}***

*=1* - 0.7340144709512995

=0.26598552904870054

**So option B is correct answer.**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:**

*μ* = 38 *σ* =6.

Let x be the age of Employee.

P(X=x) = probability of age

**A:** More employees at the processing center are older than 44 than between 38 and 44.

Probability of employee that is older than 44 = P(x >= 44)

=P((x - *μ)/σ >= (44*- 38)/6)

=P( Z *>= 1 )*

*=1* - P( Z *<= 1 )*

*=0.15865525393145707*

**So , P(x >= 44) = *0.15865525393145707***

Probability of employee that is older than 44 is *0.15865525393145707 or* ***15.87%***

Probability of employee is between 38 and 44. =P( 38 <= x <= 44)

=P( 38 <= x <= 44)

= P( x *<=* 44 *)* **-** P( x *<=* 38  *)*

*=P(z <= (44*- 38)/6 *)* **-** *P(z <= (38*- 38)/6 *)*

=P( Z *<= 1 )* - P( Z *<= 0)*

*=0.8413447460685429***-** 0.5

*{python code*

*st.norm.cdf(1)-st.norm.cdf(0) = 0.3413447460685429}*

**=***0.3413447460685429*

**So, P( 38 <= x <= 44) =*0.3413447460685429***

Probability of employee is between 38 and 44 is *0.3413447460685429* or **34.13%.**

**As P(x >= 44) < P( 38 <= x <= 44)**

**Which suggests that, the statement “**More employees at the processing center are older than 44 than between 38 and 44.**” Is false.**

B : A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Probability that A training program for employees under the age of 30 is P(x<=30).

P(x<=30)=P( Z <= (30**-** 38)/6 )

=P( Z <= (30**-** 38)/6 )

=P( Z <= **-**1.33 )

=st.norm.cdf(-1.33)

=0.09175913565028077

So, P(x<=30) =0.09175913565028077 I.e. 9.17 %

To find total no of employee under 30 ,

We know n = 400

So, total no of employee under 30

=0.09175913565028077 \*400

=36.70365426011231

So employees under 30 are approximatly 36 .

**The statement “**A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.” **is True.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:**

**Given:**

If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) and both are independently and identically distributed.

Here parameters=> mean =2μ , variance =2^2\*σ2 = 4σ2

**then 2 *X*1 ~ *N*(2μ, 4σ2) ,**

and

For *X*1 + *X*2 ,  parameters=> mean =μ + μ =2μ, variance =2^2\*σ2 = σ2 +σ2 =2σ2

**Then *X*1 + *X*2 ~ *N*(2μ , 2σ2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:**

**Given:**

X ~ N(100, 202)

μ= 100

σ2= 20

P= P(a<= x <=b)= 0.99

Q = 1 **-** P =0.01 =>this will be the complimentary of our desired area(area at the 2 tails)

Q/2 =0.005 => 1 tail

Lets find Z score for 0.005 probability from python table.

Z score = -2.575

To find a, b from both side, we have formula z = (x **-** μ)/σ ,here x = a,b

(As we know, a positive Z-score indicates that the value is above the mean(right hand side of mean ), while a negative Z-score indicates that the value is below the mean.(left hand side of mean ))

Thus,

a = μ + (Z \* σ) = 100 + (-2.575 \* 20) = 48.5

b = μ - (Z \* σ) = 100 - (-2.575 \* 20) = 151.5

**a, b = (48.5 , 151.5) option D.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:**

**Given:**

Profit1 ~ N(5, 32) Profit2 ~ N(7, 42)

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

To find the combine distribution

MEAN = Mean1+ Mean2 = 5+7 =12\*45(in rupees) = **540**

STD DEV = sqrt(VARIANCE)= sqrt(32 + 42) =5\* 45 = **225**

95% probability which is just 95% confidence interval and we know the critical values for this

**{Python program =**

st.norm.interval(0.95 ,540, 225) = (99.00810347848784, 980.9918965215122)}

Therefore,

**Range is Rs (99.00810347848784, 980.9918965215122) in Millions**

1. Specify the 5th percentile of profit (in Rupees) for the company

The 5th percentile corresponds to 1.65 standard deviations below the mean

So Z = 1.645

X =MEAN - (Z\*STD DEV)

=**540** - 1.645 \***225**)

**X = 170.0 Millions**

C .Which of the two divisions has a larger probability of making a loss in a given year?

Profit1 ~ N(5, 32) Profit2 ~ N(7, 42)

For Profit1:

Probability of making a loss for Profit1 = P(Profit1 < 0) = CDF(0, 5, 3) [=cdf(0, mean , sd)]

{stats.norm.cdf(0,5,3)}

=**0.040059156863817086**

**=4%**

For Profit2:

Probability of making a loss for Profit2 = P(Profit2 < 0) = CDF(0, 7, 4)

{stats.norm.cdf(0,7,4)}

=**0.0477903522728147**

**=4.7%**

**Thus , second division has larger probability of loss which is 4.7%**