# PR-5 - Calculative foundation

Q.1 Represent each student's subject scores as a vector.

→ Represent each students score as: if student scores → english - 80, Maths - 75, physics - 85 write the vector = $x = (80, 75, 85)$.

Q.2 Compute:

- Norm-1 and Norm-2 of vector
- Dot product and angle between two students score vectors.
- Cross product (for 3D selected subjects).

→ · Norm 1 :- is the sum of the absolute values of the vector's component.

for a vector $x = [x_1, x_2 \ldots n] = \|X\|_1 = \sum_{i=1}^{n} |x_i|$.

$x = [80, 75, 85) = \underline{\underline{240}}$

· Norm 2 :- The standard "straight-line" distance from the origin to the vector's endpoint,

for a vector $x = [x_1, x_2 \ldots, x_n] = \|X\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2}$

$x = \sqrt{80^2 + 75^2 + 85^2}$

$x = \sqrt{6400 + 5625 + 7225}$

$x = \sqrt{19{,}250} \approx 138.74$

→ for two vectors a and b, the dot product (a·b) is the product of their lengths (magnitudes) and the cosine of the angle (θ) between them.

$$a \cdot b = |a|\,|b| \cos(\theta).$$

for $x = (80, 75, 85)$, $y = (95, 90, 85)$:

$x \cdot y = 80 \cdot 75 + 75 \cdot 90 + 85 \cdot 85$

$\cos\theta = \dfrac{x \cdot y}{\|x\|_2 \|y\|_2}$, so $\theta =$

$80 \cdot 75 = 6000$
$75 \cdot 90 = 6750$
$85 \cdot 85 = 7225$

Add Them

$x \cdot y = 6000 + 6750 + 7225 = 19975$

$\|x\|_2$ and $\|y\|_2$

$\|x\|_2 = \sqrt{80^2 + 75^2 + 85^2}$
$= \sqrt{6400 + 5625 + 7225} = \sqrt{19250}$
$= \sqrt{19250} \approx 138.74.$

$\|y\|_2 = \sqrt{75^2 + 90^2 + 85^2} = \sqrt{16450}$
$= \sqrt{5625 + 7225 + 3600}$

$\approx 128.280.$

Angle Between Two Vectors
The smallest angle formed when the tails of the two vectors are placed at the same point.

$\cos\theta = \dfrac{x \cdot y}{\|x\|_2 \|y\|_2}$

$\|x_2\| \|y_2\| \approx 139.285 \times 128.280 \approx 17875.5$

$\cos\theta \approx \dfrac{17350}{17875.5} \approx 0.971$

This value near 1 means the vectors point in a very similar direction.

→ The cross product of two 3D vectors $\vec{a}$ and $\vec{b}$ results in a new vector $\vec{c}$ that is perpendicular (orthogonal) to both orginal vector. The direction of the resulting vector is determined by the right-hand rule, and its magnitude is equal to the area of the parallelogram spanned by the original two vectors.

For vector $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ in 3D:

$$x \times y = \begin{vmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} = (x_2 y_3 - x_3 y_2,\ x_3 y_1 - x_1 y_3,\ x_1 y_2 - x_2 y_1).$$

student example:

$x = (80, 75, 85)$ and $y = (75, 90, 85)$:

$i$ comp : $75 \cdot 85 - 85 \cdot 90 = 6375 - 7650 = -1275$

$j$ : $85 \cdot 75 - 80 \cdot 85 = 6375 - 6800 = -425$

$k$ : $80 \cdot 90 - 75 \cdot 75 = 7200 - 5625 = 1575$

$\Rightarrow x \times y = (-1275, -425, 1575)$

Q.3 Find the projection of one vector onto another.

→ shadow of one 3D vector on the direction of the other.

$$\text{Proj}_y(x) = \frac{x \cdot y}{y^2}\, y$$

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$$x \cdot y = 80 \cdot 75 + 75 \cdot 90 + 85 \cdot 85$$
$$6000 + 6750 + 7225$$
$$= 19{,}975$$

$$y^2 = 75^2 + 90^2 + 85^2 = 5625 + 8100 + 7225$$
$$= 20{,}950$$

Scalar coefficient is
$$= \frac{x \cdot y}{y^2} = \frac{19975}{20{,}950} \approx 0.9536$$

$$\text{Proj}_y(x) = 0.9536 \cdot (75, 90, 85)$$

component-wise:

- first : $0.9536 \cdot 75 \approx 71.52$
- second : $0.9536 \cdot 90 \approx 85.82$
- Third : $0.9536 \cdot 85 \approx 81.06$

$$y(x) \approx (71.52, 85.82, 81.06)$$

PART-B Matrix operations

Q.4 Form a matrix of students x subject. Perform
- Matrix addition and multiplication
- Transpose and Inverse (if possible).
- Determinant.

→ Matrix add.

$$A = \begin{bmatrix} 80 & 75 & 85 \\ 75 & 90 & 85 \\ 70 & 60 & 80 \end{bmatrix}, B = \begin{bmatrix} 5 & 0 & 5 \\ 0 & 5 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

Matrix Addition

$$A + B \quad \begin{bmatrix} 85 & 75 & 90 \\ 75 & 95 & 85 \\ 72 & 63 & 81 \end{bmatrix}$$

Matrix Multiplication

$$AA^T = \begin{bmatrix} 19250 & 1995 & 16900 \\ 19975 & 20950 & 17450 \\ 16900 & 17450 & 14900 \end{bmatrix}$$

b) Transpose of A is

$$A^T = \begin{bmatrix} 80 & 75 & 70 \\ 75 & 90 & 60 \\ 80 & 85 & 80 \end{bmatrix}$$

- Rows become columns and columns become row.

c) Determinant

$$\begin{bmatrix} 80, & 75, & 85 \\ 75, & 90, & 85 \\ 70, & 60, & 80 \end{bmatrix}$$

$$\det(A) = 80\begin{bmatrix} 90 & 85 \\ 60 & 80 \end{bmatrix} - 75\begin{bmatrix} 75 & 85 \\ 70 & 80 \end{bmatrix} + 85\begin{bmatrix} 75 & 90 \\ 70 & 60 \end{bmatrix}$$

$(90 \cdot 80 - 85 \cdot 60) = 7200 - 5100 = 2100$

$(75 \cdot 80 - 70 \cdot 85) = 6000 - 5950 = 50$

$(75 \cdot 60 - 90 \cdot 70) = 4500 - 6300 = 1800$

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$\det(A) = 80(2100) - 75(50) + 85(-1800)$
$= 168000 - 3750 - 153000 = 112500$

since,
$\det(A) \neq 0$, A is invertible.

Part C Linear Transformation & Geometry

Q.5. Explain line, plane and hyperplane with respect to your dataset dimensions.

→ line : in 2D (two subject)
- consider maths score $x$ and physical score $y$.
- A line in this 2D space has equation $ax + by = c$.

Ex - "All students whose Math + Physics = 160" is

$$x + y = 160$$

The point (80,80) lies on this line because $80 + 80 = 160$; the point (95,60) does not as $95 + 60 = 155$.

→ • Plane in 3D (three subject) : Math $x$, Physics $y$, English $z$.

- A Plane in 3D has equation $ax + by + cz = d$

Ex :- "All students whose total in three subjects is 240" is

$$x + y + z = 240$$

(80, 75, 85) lies on this plane because $80 + 75 + 85 = 240$. Another student (75, 90, 85) lies on a parallel plan $x + y + z = 250$ since $75 + 90 + 85 = 250$.

→ Hyper plan in higher dimensions (many sub)

- with $n$ subjects, a student's scores from a vector $x = (x_1, \ldots, x_n)$.

- A hyperplane is the higher-dimensional ~~generational~~ generalization of a plane, with equation

$$w_1 x_1 + w_2 x_2 + \ldots + w_n x_n = b$$

- Example: with 5 subjects and weights $w =$ $(0.3, 0.2, 0.2, 0.15, 0.15)$, the hyperplane

$$0.3x_1 + 0.2x_2 + 0.2x_3 + 0.15x_4 + 0.15x_5 = 70$$

represents all students whose weighted average score is exactly 70; point on one side have weigted score $> 70$ (eg., Above Average) and on the other side $< 70$ ("Below Average").

Q.6 Show how dimensionality increases from 2D → 3D → higher dimensions with hyperplanes.

→ Line: (2D ~~(~~space)

→ In 2D score space (two subjects: Math $= x$, Physics $= y$), a line is a 1D object defined by $ax + by = c$

$>$ ~~two~~ ~~sub~~.

Plane (3D space)

In 3D score space (three subject: Math $= x$, Physics $= y$, ~~Physics~~ english $= z$), a plane is 2D object defined by $ax + by + cz = d$.

Hyperplane (nD space): (many)
In a n-dimensional score space (n subject), a hyperplane is an (n-1)D object defined by $w_1 x_1 + w_2 x_2 + \dots w_n x_n = b$.

PART D Eigenvalues & Decomposition

Q.7 compute the eigenvalues and eigenvector of the covariance matrix.

→ Eigenvalues shows that how much variance exists in each direction.

To find eigenvalue : $\det(C - \lambda I) = 0$.

our matrix is $A = \begin{bmatrix} 80 & 75 & 85 \\ 75 & 90 & 85 \\ 70 & 60 & 80 \end{bmatrix}$

· compute covariance matrix
Means : Math = 75, Physics = 75, English = 83.33

The Eigenvalue :

$$|C - \lambda I| = 0$$

$$\begin{bmatrix} 80 & 75 & 85 \\ 75 & 90 & 85 \\ 70 & 60 & 80 \end{bmatrix} - \lambda \begin{bmatrix} 01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{matrix} + & - & + \end{matrix}$$

$$\begin{bmatrix} 80-\lambda & 75 & 85 \\ 75 & 90-\lambda & 85 \\ 70 & 60 & 80-\lambda \end{bmatrix} = 0$$

Let's find determinate first :

$$(80-\lambda)\begin{vmatrix} 90-\lambda & +85 \\ 60 & +80-\lambda \end{vmatrix}$$

$$\therefore (80-\lambda)\left[(90-\lambda)(80-\lambda) - 5100\right]$$
$$-75 \quad \left[75(80-\lambda) - 5950\right]$$
$$85 \quad \left[70(90-\lambda) - 4500\right] = 0$$

$$\therefore (80-\lambda)\left[7200 - 90\lambda - 80\lambda + \lambda^2 - 5100\right]$$
$$-75\left[6000 - 75\lambda - 5950\right]$$
$$85\left[6300 - 70\lambda - 4500\right]$$

Q.8. Perform LU, Decomposition of the dataset matrix.

→ Matrix $= A = \begin{bmatrix} 80 & 75 & 85 \\ 75 & 90 & 85 \\ 70 & 60 & 80 \end{bmatrix}$

here :

$L$ = lower triangle

$U$ = upper triangle

$L = \begin{bmatrix} 80 & 75 & 85 \\ 75 & 90 & 85 \\ 70 & 60 & 80 \end{bmatrix}$ $U = \begin{bmatrix} 80 & 75 & 85 \\ 75 & 90 & 85 \\ 70 & 60 & 80 \end{bmatrix}$

Q.9 Perform Singular Value Decomposition (SVD) and explain its role in dimensionality reduction.

→ Any matrix $A = U \Sigma V^T$

where, $U, V$ are orthogonal, and $\Sigma_i$ is diagonal with singular value $\sigma_i = \sqrt{\lambda_i}$

Here, $U$ → left singular vector

$\Sigma$ → Singular value

$V^T$ → Right singular vectors

Part E Dimensionality Reduction

Q.10. Apply (PCA) to reduce the dataset from multiple subject to 2 dimensions.

→ What it does : Take 3 subject → Reduces to 1-2 "summary scores" that capture most variance.

Steps :

1. Standardize - scores A → The dataset
2. Covariance - Compute the Matrix
3. Eigenvectors - Select top 2
4. Project dataset [keep top 2]

Q. 11 Apply LDA to classify students into "Above Avg" and "Below Avg" category.

→ finds hyperplane that best separates "Above Avg" vs "Below Avg" students.

> LDA → finds Max difference / separation between Above / Below Avg students
> used for classification.

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