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# Optimizing the Operation of a Toll Plaza System Using Simulation: A Methodology

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Many urban areas and states are facing an increase in population density and the industrial base, creating traffic congestion and jams at toll plazas on major highways during rush hours. This article provides a detailed study and analysis of the performance evaluation of a toll plaza using discrete event simulation modeling. Under a given time-dependent traffic flow and a set of specified model parameters, the required optimal number of tollbooths and the resulting mean delay can be estimated with the discrete event simulation model. The results show that a toll plaza that has a shorter delay under light load conditions is more sensitive to variations in service type and the number of toll plazas. The results show that the performance of toll plazas improves as the number of tollbooths increases because less time is spent in queue.

**Keywords:** Modeling and simulation, optimization, toll plaza, transportation engineering and planning

## 1. Introduction

Highways and freeways in urban areas are experiencing severe traffic congestion and jams due to the increasing population and migration of people to the cities and urban areas. This issue is of great concern to city and urban planners and engineers, as well as to transportation engineers and planners. It is important to build high-quality freeways that can be maintained on a regular basis using good methods. It is also important to provide enough funds to do so, and using the toll concept is one way to have enough funds to maintain such highways. Good designs of these toll plaza systems can have a significant impact on the effective use of the infrastructure and can contribute to increasing the standards and quality of living of the residents in urban areas [1-10]. In this study, four types of toll collection methods are considered: E-Z (electronic) pass, toll only, manual or tolls, and full (manual) service. In the electronic (E-Z) pass payment case, the vehicles are provided with a transponder unit by which valid payment can be checked or deducted automatically from the credit card account or checking account of the vehicle owner. This method is also called the dynamic method. Special lanes with transceivers communicating with the transponders are used for these vehicles; other vehicles that use the nondynamic payment method are directed to manual lanes that include toll only, manual or toll, and full service [1-10].

This article provides a detailed simulation study of the performance evaluation of the toll plaza system. The performance metrics considered are delays, number of tollbooths, and type of service. A comparative performance analysis of different types of tollbooths will provide an insight into the advantages and disadvantages of each with respect to performance.

Several published articles cover the topic of simulation of the toll plaza [1-7]. A review of literature on related studies is given next.

In 1954, Edie [5] conducted a study in which he considered traffic delays at tollbooths of homogeneous booths and vehicles. When we have different types of traffic, the problem becomes more challenging as the analysis becomes much more difficult. Among the possible techniques that can be used to analyze the performance of all transportation systems, including tollbooth plazas of highways, are operation research techniques [8]. van Dijk et al. [1] presented different queuing models for the simulation of the toll plaza. Among the variety of commonly used queuing configurations under different scenarios are the ones illustrated in Figure 1.

One option is to separate the lanes and to offer only one type of payment system at every tollbooth (Fig. 1). The disadvantage of a separate-lane system is that certain tollbooths might be underused while others would be overloaded. This can be inefficient in operational usage. Analytical queuing systems can predict the behavior of separate-lane queuing systems well.

Another option would be to offer all payment systems at all tollbooths and have the vehicles queue in one line (see

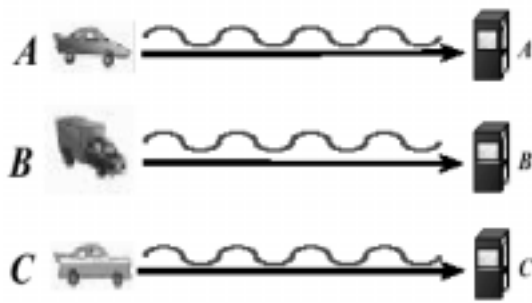


Figure 1. Queuing system 1: Separate line [1]

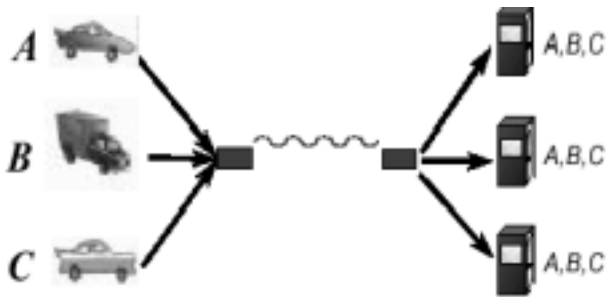


Figure 2. Queuing system 2: One line [1]

Fig. 2). Each tollbooth would accept cash, credit cards, and electronic payment, or it would be a homogeneous-type toll plaza. The advantage in this case is that it provides more operational flexibility while also increasing the efficiency (utilization) of the tollbooths. The disadvantage, however, is that the wide range of service times introduces variability. As variability in service times is one of the main causes for queues to arise, this disadvantage could override the gain in efficiency.

In fact, by combining both insights, a third option might even be more attractive by using specialized lanes to keep the variability per lane to a minimum and by also allowing "overflow" when one or more ticket booths are temporarily underused (see Fig. 3). Among all three options, the more efficient option 2 (one-line system) has an overall better performance [1].

Chao [4] demonstrated several design issues in the toll plazas of highway systems. The author analyzed several design issues to search for the optimal layout when the number of each type of toll collection booth is fixed. In the simulation result, Chao explained that the average delay (i.e., mean waiting time in the toll plaza for all travelers) is not affected by the relevant positions (i.e., the layout of the different toll collection booths); the difference lies in the variance of waiting times. The reasoning behind that fact is the following. If the different methods of toll-collecting booths are relatively spread out, the different classes of

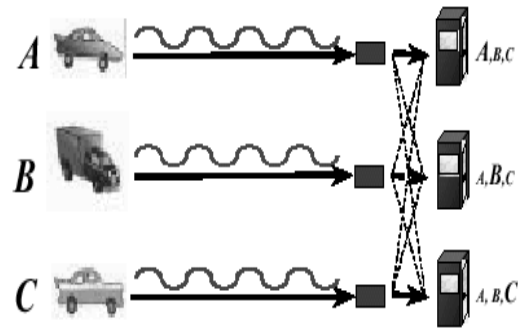


Figure 3. Queuing system 3: Specification and overflow [1]

vehicles have to watch very carefully to figure out which lanes to join. It happens very regularly that some vehicles have to move to the left and some to the right. As a result, different classes of vehicles have to slow down and give way to some other vehicles, yielding longer delays. On the other hand, such confusion would also create relatively less traffic-intense territories, which would allow a portion of the "opportunistic" vehicles to get to their desired booths faster. Furthermore, the disorder created by the vehicles gives opportunities to aggressive drivers, who can find ways to go through the toll plaza in a shorter amount of time, which might be unfair to the average traveler. A better design gives rise to smaller variance, and a poor design gives rise to larger variance [4].

Matstoms [2] presented an animation scheme by which it is possible to analyze the result of a simulation in detail. Vehicles were shown as small moving dots arriving at the toll station, waiting in queue, getting service, and leaving. Furthermore, with animation, it is possible to dynamically open and close lanes, change the type (manual/automatic) of lane, and modify the set of accepted vehicle types. This animation is a useful tool for verifying the simulation model. Figure 4 is an example of such an animation.

In this article, we present a simulation study to optimize the operation of a toll plaza on a highway. The goal is to investigate the relation between the average delay versus the number of tollbooths in the toll plaza for different types of service, the delay versus the arrival rates of cars, the mean number of busy tollbooths versus the arrival rate of cars for all types of service, and the delay versus peak traffic period. Such results will allow us to make decisions on the number and type of tollbooths, thereby optimizing the overall system operation.

## 2. Simulation Model

To analyze a tollbooth plaza system, we can use (a) analytic queuing modeling, (b) real experimentation, and (c) simulation modeling. The real experimentation approach

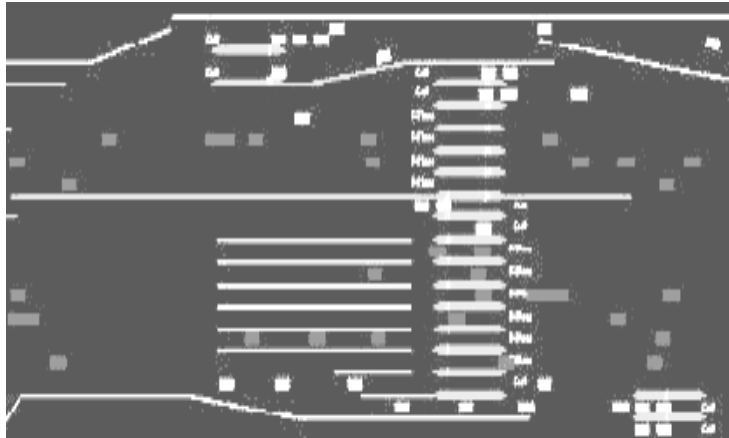


Figure 4. Example animation of a toll plaza simulation (www.kdlassociates.com)

is considered expensive and disruptive to the system, although it is the most accurate technique. The analytic modeling approach is considered difficult due to the nature and complexity of the toll plaza system. Therefore, we adopted simulation modeling to study this system because simulation modeling is the most flexible technique for the performance prediction and analysis of such complex systems. Moreover, simulation allows us to (a) conduct any simulation experiment without fear of failure, using any arbitrary input distribution; (b) conduct a proper sensitivity analysis for the critical input parameters on the performance metrics; and (c) conduct an experiment without incurring costs since computational power these days is abundant.

The following assumptions have been made in our detailed study:

1. The arrival rate at the tollbooth is Poisson distributed; therefore, interarrival processes follow an exponential distribution.
2. Homogeneous traffic and homogeneous tollbooths are assumed.
3. It is assumed that all tollbooths are always active.
4. Average service time for different service types is as assumed to have the following values:  
E-Z pass = 3.8 seconds,  
Token only = 7.5 seconds,  
Token and manual = 10 seconds, and  
Manual only = 20 seconds.  
Such values are reasonable and have been considered in previous work (e.g., Pursula [3]).
5. We assumed that, conceptually, the traffic forms one queue (see Fig. 2), and cars can move from one lane to another if there is more than one lane. In our simulation study, we assumed that the time for jockeying is negligible.

An example of a toll plaza is shown in Figure 5, while Figure 6 shows the flowchart of the simulation program that we used in our study.

The current practice in most tollbooth systems, such as the Garden State Parkway or the New Jersey Turnpike, is to have four types of toll collection schemes, which are as follows: the E-Z (electronic) pass, token only, exact change or token, and full service (cash with receipt). The main advantage of such a configuration is that every vehicle has access and can adjust to each type of tollbooth service. It has been argued that each traveler in such a scheme has to find the particular lane that suits him or her. This may result in slow traffic and some delays, especially during rush hours. The issues that have to be addressed in the design of a toll plaza system include the following: (a) what is the relationship between traffic delay and the number of different types of tollbooths and layouts? (b) How many booths of each type should be used? (c) What kind of layout should the booths have? (d) What other traffic control schemes can be used to ease traffic congestion and jams?

In this study, we will attempt to answer some of these research questions. The flowchart shown in Figure 6 summarizes the main tasks of our simulator. At the beginning, we need to invoke the initialization routine, in which the simulation clock, the system state, statistical counters, and the event list are all initialized. Then, control is passed to the main program to invoke the timing routine to find out the most imminent event. In this case, in the arrival event routine, we have to generate the arrival of vehicles. We assume that the interarrival times follow the exponential distribution. This means that

$$F(X) = \text{CDF} = \text{Cumulative Distribution Function} \\ = 1 - e^{-\lambda X}.$$

This means that we can use  $U = 1 - e^{-\lambda X}$ ; therefore, the exponential random variate  $x$  can be written as



Figure 5. An example tollbooth (www.kdlassociates.com)

$$X = -1/\lambda \ln(1 - U).$$

To reduce the complexity, we can write the last expression as

$$X = -1/\lambda \ln(U).$$

This is correct since  $(1 - U)$  and  $U$  are both random variates from 0 to 1, and using  $U$  will provide less computational complexity. Therefore, we used this last expression to generate the exponential random variate of the interarrival times of the vehicles arriving at the toll plaza booths.

Then the program will check if there is any queued vehicle. If there is any, then we check to see if the tollbooth is full. If it is not full, then we generate the departure event and invoke the corresponding departure event routine and increment the number of used tollbooths. If the new vehicle finds a queued vehicle in front of it, then we update the queue length and time accordingly. This process is repeated until the simulation time is finished.

We used in our simulation analysis the next-event advance mechanism to advance the simulation clock, as it is more accurate than the fixed-increment time advance mechanism. The latter scheme has the following disadvantages: (a) errors are introduced by processing the event at the end of the interarrival time in which it occurs, and (b) it is necessary to decide which event to process first when events that are not simultaneous in reality are treated as such by the fixed-increment time advance scheme. Various system configurations and simulation experiments were considered in our study.

### 3. Simulation Results and Discussion

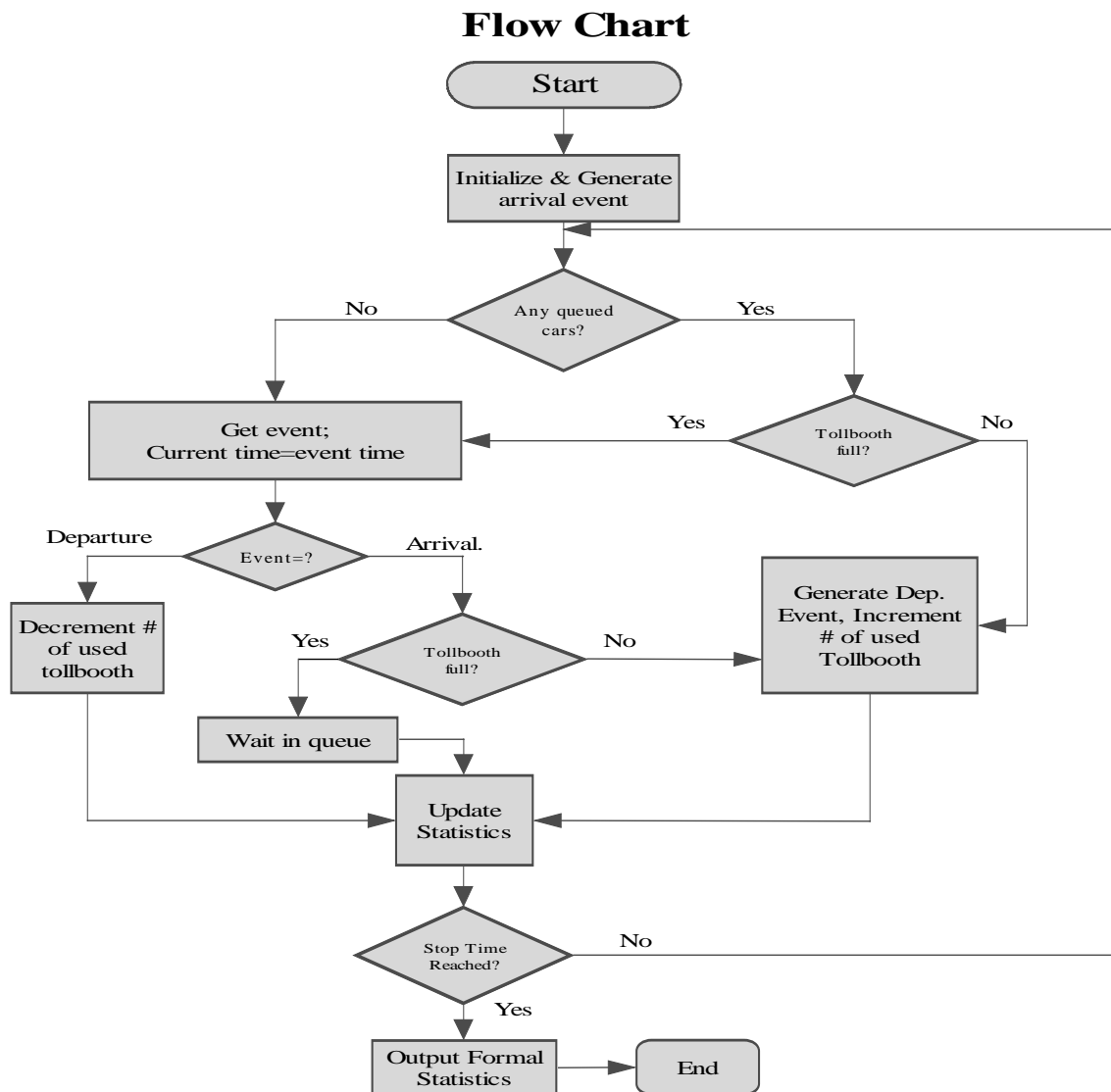
We conducted several simulation experiments to analyze the system, optimize its design, and predict the values of the performance metrics. Figure 7 shows the average delay as a function of the number of tollbooths for different service types. From the figure, we observe that the delay increases as the number of tollbooths decreases, and the delay decreases as the number of tollbooths increases. In the case of manual service, the delay is higher than in any other service type. Delay is lowest for the E-Z pass (electronic or automatic) service. Performance of manual plus token service is better than manual service, but performance of token service is better than manual plus token service.

Figure 8 shows the relation between the average delay and the arrival rate for different service types. From the figure, we observe that the delay increases as the arrival rate increases. In the case of manual service, the delay is higher than any other service type. Delay is lowest for the E-Z pass service. Performance of manual plus token service is better than only manual service, but performance of token service is better than manual plus token service.

Figure 9 shows the relation between the mean number of busy tollbooth stations and the arrival rate. It is seen from the figure that the mean number of busy tollbooths increases as the arrival rate increases. From Figures 8 and 9, we observe that the mean number of busy tollbooths and delays increases as the arrival rate of vehicles increases.

Table 1 shows the results obtained from our simulation analysis that summarizes the average delay versus peak traffic time from 6:30 to 10:30 a.m.

The time frame for observing delay characteristics is 6:30 to 10:30 a.m. From the results, it is observed that as



**Figure 6.** A general flowchart of the simulation program for the simulation of a toll plaza

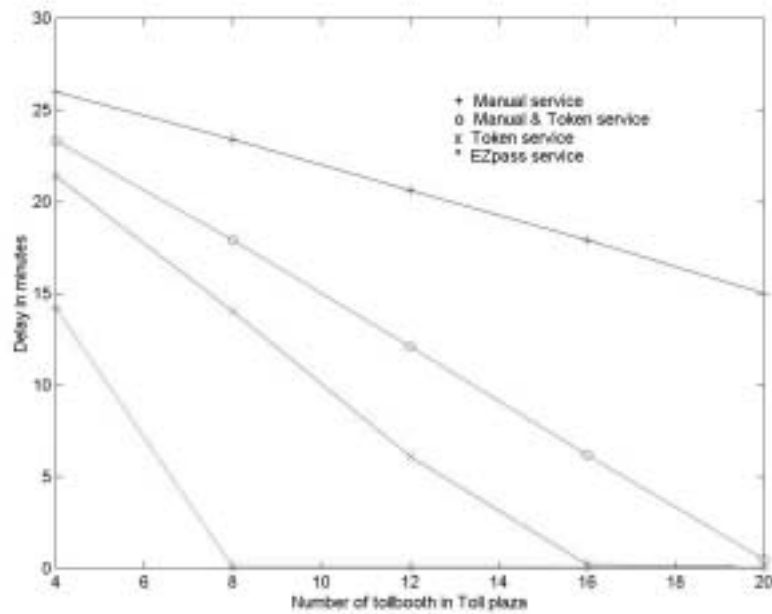
time passes during the peak traffic period, the arrival rate increases, and so delays increase. After a certain period, the arrival rate decreases, but there is some existing traffic in the queue and, therefore, average delays continue to increase. As the traffic starts decreasing and the queued vehicles are processed, the average delay decreases significantly. This result is also shown in Figure 10.

Table 2 summarizes the result obtained for the time period from 4:00 to 8:00 p.m. At the beginning (i.e., at 4:00 p.m.), the arrival rate is 10 cars/minute. Then the arrival rate increases, and so does the average delay. The arrival rate reaches a peak from 5:00 to 5:30 p.m., but delay is not highest at that period. Delay is highest in the 6:00 to 6:30 p.m. period because that is when many cars have

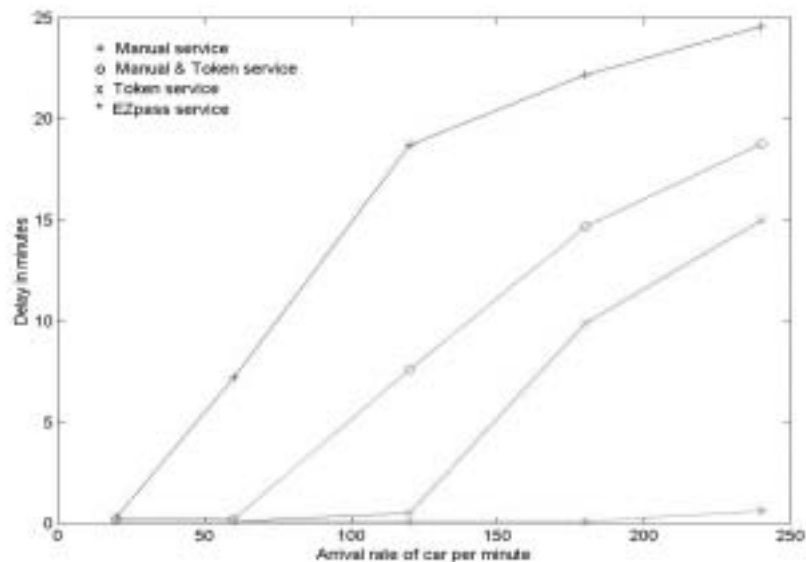
already been waiting in the queue. These results are also shown in a bar plot in Figure 11.

#### 4. Conclusion

To conclude, we have presented a discrete event simulation study of the operation of a toll plaza system on a highway. Our simulation results have shown that the toll plaza has a shorter delay under light load conditions and is more sensitive to variations in service type and the number of toll plaza systems under heavy load (traffic) conditions. The results show that the performance of the toll plaza improves as the number of tollbooths increases because less time is spent

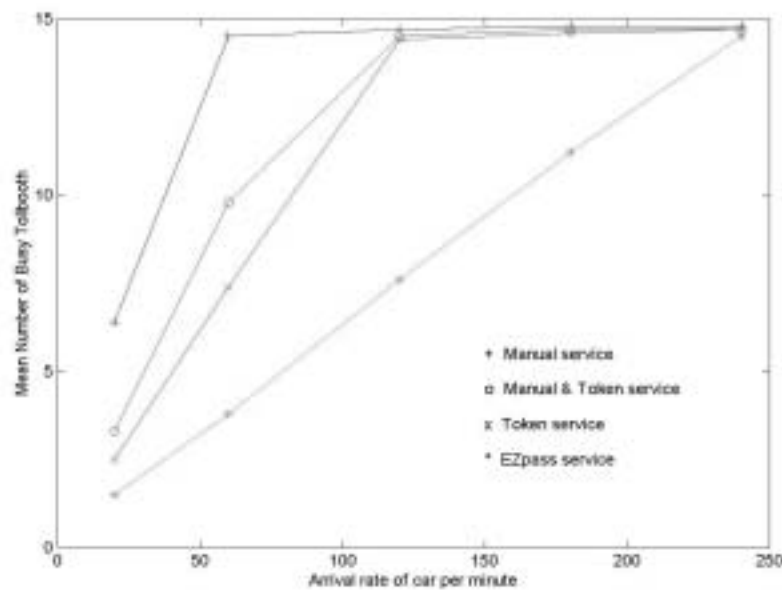


**Figure 7.** Delay (in minutes) versus number of tollbooth stations. Here, the arrival rate is 120 car/minutes. Service time for manual service = 20 seconds, token service = 7.5 seconds, manual plus token service = 10 seconds, and E-Z pass = 3.8 seconds.

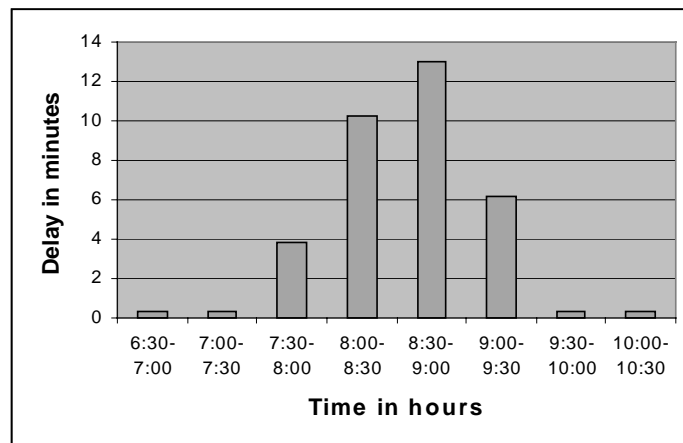


**Figure 8.** Delay (in minutes) versus arrival rate (car per minute). Here, the number of tollbooth stations is 15. Service time for manual service = 20 seconds, token service = 7.5 seconds, manual plus token service = 10 seconds, and E-Z pass = 3.8 seconds.





**Figure 9.** Mean number of busy tollbooths versus arrival rate (car per minute). Here, the number of tollbooths is 15. Service time for manual service = 20 seconds, token service = 7.5 seconds, manual plus token service = 10 seconds, and E-Z pass = 3.8 seconds.



**Figure 10.** Average delay versus peak traffic period from 6:30 to 10:30 a.m. Here, the total number of tollbooth stations is 15.

in queue. This simulation program can be potentially used in the design and analysis of actual toll plaza systems. Our method is flexible, detailed, and easy to use. It can be modified easily to study any toll plaza for a highway. Finally, it is essential to use simulation before starting an infrastructure project such as tollbooth plaza on a highway or an intersection of a road in a city or urban area. This allows us to have a cost-effective design and enables us to predict the behavior and performance of such complex systems before investing the time, efforts, and money in their construction.

Also, the team that is supposed to design such projects should include—in addition to city and urban planners, transportation engineers/planners, and architects—experts in modeling and simulation. Such experts could be city planners or transportation engineers/planners who know the operation of these systems as well as how to model and simulate them. With early use of simulation at the design stage of infrastructure projects, pitfalls in the design could be avoided, and capacity planning, tuning, and determination could be done properly.



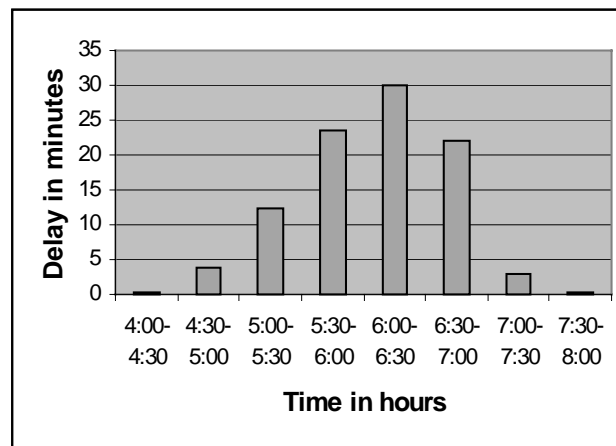


Figure 11. Delay versus peak traffic period from 4:00 to 8:00 p.m. The total number of tollbooth stations is 15.

Table 1. Delay versus peak traffic time from 6:30 to 10:30 a.m.

Time	Arrival Rate (car/min)	Average Delay Minutes Per 30 min. Period
6:30-7:00	2	0.305710
7:00-7:30	20	0.341434
7:30-8:00	60	3.828410
8:00-8:30	50	10.248000
8:30-9:00	40	12.964400
9:00-9:30	30	6.193480
9:30-10:00	10	0.360386
10:00-10:30	5	0.346879

Table 2. Delay versus peak traffic time from 4:00 to 8:00 p.m.

Time	Arrival Rate (car/min)	Average Delay Minutes Per 30 min. Period
4:00-4:30	10	0.338124
4:30-5:00	60	3.959830
5:00-5:30	70	12.378000
5:30-6:00	50	23.500800
6:00-6:30	30	30.018500
6:30-7:00	20	22.084400
7:00-7:30	10	2.968420
7:30-8:00	5	0.370038

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